Wonderous Wavelengths: Applications of Modelling Periodic Functions to Analyze Phenomena in Light

by Dalton Cooper, 2022 CTI Fellow
East Mecklenburg High School

This curriculum unit is recommended for NC Math 3

Keywords: NC Math 3, Trigonometry, Trigonometric Functions, Sine Function, Cosine Function, Light, The Doppler Effect, Reflection, Refraction

Teaching Standards: See Appendix 1 for teaching standards addressed in this unit.

Synopsis: Following Thomas Young's double-slit experiment in 1802, physicists began to favor the wave model of light, using the properties of a transverse wave to model phenomena observed in nature, such as refraction, reflection, and the Doppler Effect. Mathematically, trigonometric functions can be used to model periodic phenomena, such as light. My curriculum unit uses phenomena that change the amplitude and wavelength of light in order to discuss transformations of the sine and cosine functions, as well as using transformations to the sine and cosine functions to model phenomena in the natural world.

Introduction

School Demographics

East Mecklenburg High School is located on Monroe Road and draws its student body from southeast Charlotte; for this reason, it is a socioeconomically diverse school as the area consists of working-class and low-income neighborhoods, immigrant communities, as well as upper-middle class neighborhoods. The geography reflects itself in the student body. Thirty-six percent of students qualify for free or reduced lunch, including McKinney-Vento students; this amounts to 38.3% of students being designated as *economically disadvantaged*, which effectively falls in-line with Charlotte-Mecklenburg Schools (38.5%) and the state of North Carolina (38.9%). The student body provides a reflection of the city of Charlotte's demographics, with 38.1% of the student body is black (compared to 35.5% in the city), 32.8% Hispanic (compared to 14.6% in the city), and 19.8% white (compared to 40.7% in the city), and 7.1% Asian (compared to 6.6% in the city); like the city of Charlotte, East Mecklenburg High School has become increasingly diverse in recent years, seeing an influx of immigrants from Central and South America, as well as Southeast and East Asia.

East Mecklenburg High School has a reputation as being strong academically, supported by key metrics for college and career readiness. Of East Mecklenburg students, 71.3% are designated "college ready" by their ACT score (relative to 61.0% in CMS and 55.2% in the state of North Carolina). Additionally, East Mecklenburg boasts the largest and oldest International Baccalaureate (IB) programs in Charlotte-Mecklenburg Schools, with 10.2% of students enrolled in IB and an additional 16% of the student body seeking advanced coursework in Advanced Placement (AP) classes. This can be partially attributed to an experienced teaching staff, with 93.9% of teachers having 3 or more years of experience (relative to 84.2% in CMS), as well as 22 National Board Certified Teachers.

Despite East Mecklenburg's successes and strengths, educators have little opportunity to rest on their laurels as numerous challenges present themselves, specifically in the math content area where I will be writing my curriculum unit. In NC Math 1, only 14.2% of East Meck students were grade level proficient (GLP) (relative to 19.4% GLP in CMS and 25.1% in the state). While Math 1 EOC metrics lag the district in state, this is not to say that math instruction isn't a strength at East Mecklenburg High School. 56.6% of students earned GLP scores in NC Math 3 there (relative to 51.3% in CMS and 44.6% in the state), indicating that strength of math instruction is making a positive impact when viewed from the perspective of long-term trends. With all of this in mind, one of the primary goals of my curriculum unit is to provide all students at East Mecklenburg High School access to the opportunities it boasts in advanced coursework; the challenges to this include tailoring instruction to scaffold and support students coming in below grade level and meet the needs of the school's growing English language learner (EL) population while simultaneously creating opportunities for rigorous instruction, application, and assessment for students labeled as "college and career ready."

Rationale

Mathematical instruction ideally prepares students equally for conceptual, procedural, and applicational rigor. Conceptual rigor encapsulates student knowledge and understanding of mathematical relationships, principles, and conventions, including their ability to justify approaches and view areas of mathematics as a coherent body as opposed to a series of unrelated problems. Procedural rigor describes a student's ability to execute operations fluently and accurately. Applicational rigor encapsulates students' ability to apply conceptual and procedural knowledge and skills to leverage mathematics in real-world context. A good mathematics curriculum will balance the three aspects of mathematics instruction through varying instructional approaches and activities.

As such, my curriculum unit's goals are threefold. First, my unit will look to build a coherent student understanding of trigonometric functions; within the context of North Carolina Math 3's relatively short unit, this will mean being able to describe the connection between right triangles, the unit circle, and the sine and cosine functions. To meet my first goal, my unit will include necessary instructional scaffolds to quickly fill in procedural knowledge related to right triangle trigonometry from NC Math 2. This will support my unit's second goal of building student confidence by providing opportunities to build procedural fluency so that classes may see better engagement with higher-rigor conceptual and applicational rigor. Finally, my unit's third goal is to make explicit connections between periodic functions (including their transformations) and their applications in analysis of light wavelength in astronomy, meteorology, and military and civilian uses.

Background Research

North Carolina Math 3 will be students' first experience with periodic functions in seeing the sine and cosine function (see Appendix 1 for teaching standards). Within the scope and sequence of North Carolina math standards, to this point students have utilized the sine, cosine, and tangent functions for right triangle trigonometry the year prior in NC Math 2, earlier in the year in NC Math 3, students will also encounter radians measures for the first time when looking at sector area and arc length, and will also have generalized rules of transformations of functions (vertical/horizontal shift, vertical/horizontal stretch/compression, and reflections across the *x*/*y*-axis) from earlier in the year in NC Math 3 as well. The short trigonometry unit in NC Math 3 builds these three prerequisites into a seven-day unit that consists of the unit circle and parameters of sine and cosine functions (period, vertical shift, amplitude, and frequency; phase shift and the tangent and reciprocal trig functions are not covered until NC Pre-Calculus). Periodic functions are useful for modelling aspects of waves, such as light, and additionally the transformations made to periodic functions closely ties to observable phenomena in light.

Light is a form of energy caused by a disturbance in electromagnetic fields (and, as such, is as electromagnetic waves or electromagnetic radiation); just as a rock dropped into a still pond would cause ripples to emanate, certain sources of energy can create ripples in electromagnetic fields[1]. When describing and understanding light, three central characteristics can be visualized using the ripple/wave analogy. Wavelength describes the length of a single cycle of a light wave, as measured from peak to peak or trough to trough (note that in the field of mathematics, the word *period* is used interchangeably with wavelength to describe the same characteristic of periodic functions); wavelength determines the type of light that is emitted from a source. While most people typically think of light that falls within the visible spectrum (7×10^{-5} to 4×10^{-5} cm)[2], short x-rays that one would have taken at a doctor's office that can penetrate flesh and bone (10×10^{-6} to 10×10^{-9} cm) as well as radio waves that can be longer than the earth itself also are forms of electromagnetic radiation, or light. The difference between these is the wavelength of the light (see Figure 1). The flipside of the wavelength coin is the frequency of light; simply put, frequency is how many oscillations a wave (such as light) will make within a given time (measured in Hertz, Hz, or oscillations per second). Frequency has an inverse relationship with wavelength, as shorter waves will yield faster oscillations; light's frequency is $f = \frac{c}{\lambda}$, where *c* is the speed of light in a vacuum and λ is the wavelength of the wave. The third and final characteristic of light is its amplitude, or the distance from the midline of the wave to a peak or trough (it can be also helpful to think of amplitude as half the distance from the peak to the trough).

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[1] Arcand and Watzke, *Light: the Visible Spectrum and Beyond,* 12
[2] Ibid, 86

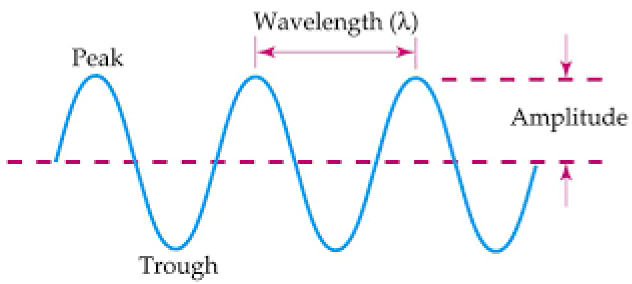

Figure 1: A simple diagram of a light wave, and its features[3]

The characteristics of light waves can be modeled using a trigonometric function, such as a sinusoidal wave. In NC Math 3, students are expected to describe the characteristics of a sine or cosine function in $f(x) = a \cdot \sin sin\ (bx - c)\ + d$ form, including the period (i.e. wavelength), frequency, and amplitude. As such, mathematical transformations to a sin function, such as a horizontal compression, can be utilized to represent natural occurrences that would affect the wavelength, frequency, or amplitude of light. As such, my scientific research primarily revolves around phenomena that would alter the wavelength and amplitude of light so that students can utilize mathematical modelling to describe changes in electromagnetic waves.

With the benefit of modern scientific knowledge, even most high school students understand the speed of light to be constant, *c*, typically in the ballpark 3.0×10^8 meters per second, depending on precision and units. For centuries, Western mathematical and scientific thought was dominated by the ancient Greeks, whose views on astronomy and science derived from what could be observed and justified using formal logic. Aristotle, for instance, said that "…light is due to the presence of something, but it is not a movement," for if something is not moving then it cannot possibly have speed[4]. Through the thirteenth century, understanding light as omnipresent and without speed, as postulated by Aristotle was the dominant consensus; while a handful of scientists and mathematicians such as Ibn Al-Haytham and Roger Bacon argued otherwise, they did not have the ability to empirically measure light and as such, all discussion around the speed of light was an exercise of the imagination[5]. With the advent of the Scientific Revolution, however, came a paradigm shift where the speed of light existed but was understood to be infinite; this approach to light was developed by Johannes Kepler, Francis Bacon, and Rene Descartes, the latter of which described light's speed as being a stiff stick being pushed. While the person at the other end of the stick would feel the push regardless of how long the stick is, there is no observable movement at any point along the stick[6]. During this time, there were notable dissenters who proposed a finite speed of light such as Galileo Galilei and Isaac Beeckman, the latter of whom attempted to measure light's speed with his flash-and-mirror

[3] "Light Waves." *Khadley*
[4] MacKay and Olford, "Scientific Method, Statistical Method, and the Speed of Light," p.255
[5] Ibid. 255
[6] Ibid, 255-256

experiment; however, experiments such as Beeckman's were hamstrung by the accuracy of scientific tools of the time. Descartes gave a critique of Beeckman's experiment reasoning that if light travelled at Beeckman's calculated speed, then a noticeable delay in visible light would occur following the end of a lunar eclipse[7]; while the benefit of hindsight allows us to explain that light's speed and the moon's proximity makes this delay imperceptible, theories of a finite speed would remain on the fringe until the seventeenth century.

The speed of light as we understand it today started with the Danish astronomer Ole Romer. While visiting an observatory near Copenhagen in 1676, Romer spent significant time observing Jupiter and its moons. After 8 months of data, Romer began to notice discrepancies with the eclipses of Jupiter and its moons based on the orbits of the Earth, the moons, and the planets; for instance, when the Earth approached Jupiter, the delay observed in an ellipse was noticeably shorter. Romer deduced that light travelling at a finite speed could explain this phenomenon[8]. While Romer significantly advanced scientific understanding of the speed of light, it was not until fifty-three years later in 1729, that English astronomer James Bradley would offer the conclusive proof of light's speed as a constant that would become scientific consensus. At the time, Bradley was measuring the distance to nearby stars using a technique called a parallax. In a parallax, the angle between the earth, the sun, and a nearby star is measured twice a year; using the two measurements, it is then possible to apply the Law of Sines to calculate the distance to the star. While performing parallaxes, Bradley noticed semiannual variation in the position of stars based on the earth's movement, or stellar aberration; Bradley arrived at the same conclusion as Romer that the earth's movement and the finite speed of light could explain these seasonal variations[9]. From his observations, Bradley estimated that light takes eight minutes and twelve seconds to travel from the sun to the earth, or 300,267,640 meters per second. As scientific equipment improved, so did the ability to measure the speed of light, or the constant of c, that we take for granted today.

I had previously borrowed Arcand and Watzke's analogy that light as a wave is a ripple in electromagnetic fields, like how a rock can create ripples in a still pond. For a wave to travel, it requires a surface, or medium, in which to create a disturbance. Some waves can't exist outside of certain media (e.g., dispersion waves, like those found at the ocean, need a body of water or fluid), while others can travel through a variety of substances. When James Bradley first observed stellar aberration in 1729, he determined that the earth's own movement as well as the finite speed of light was the cause of apparent seasonal variation in observed stars' position. In visualizing light travelling at a finite speed, Bradley described light as particles travelling at velocity[10]. While Bradley visualized light as raindrops travelling between stars and the earth, the

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[7] Ibid, 256
[8] Ibid, 257
[9] Ibid, 257
[10] Bradley, "THE REV. JAMES BRADLEY ON THE MOTION OF THE FIXED STARS (Reprinted from the Philosophical Transactions of 1727.)" p.175

19th century saw the rise of the wave theory of light, which raised new questions about a medium through which light would travel. Wave theory of light dates to the seventeenth century from Christiaan Huygens and even passed through Swiss mathematician Leonhard Euler, who in 1749, conducted experiments with light and optical lenses. In his research, Euler dispersed white light into a spectrum of colors using prisms and made mathematical arguments relating the color of light to its wavelength, making the analogy to sound waves and harmonics[11]. As a mathematician, not a scientist, there were notable holes in his theory. First, Euler could not accurately calculate the wavelength of light meaning that his mathematical proofs were effectively an exercise in imagination. Second, Euler's comparison of light waves to sound waves assumed that it would need a medium to travel through; Euler assumed an elastic ether[12]. While Euler made a numerous errors in that would prevent wave theory from gaining traction, early in the 19th century many of his misconceptions would be soon corrected.

While light was theorized to behave as a wave for two hundred years prior, wave theory backed by scientific rigor is commonly attributed to English scientist Thomas Young. A child prodigy, Young shared Euler's interest in light and sound and similarly theory and by the age of thirty began to experiment in earnest. In his double-slit experiment, Young observed that light shined through two parallel apertures would produce a different pattern than light shined through a single aperture; the double-slit pattern, Young observed, coincided with interference between light waves as ripples in a pond would interfere with each other[13]. While Young had proven that light behaves as a wave, scientists still needed an explanation for a medium through which it travelled. Unlike sound, light could travel through a vacuum, leading a luciferous ether to emerge as the explanation for a medium. Luciferous ether remained the scientific consensus until the Mickelson-Morley Experiment of 1887, which instead of gaining insight into the ether by measuring interference patterns and changes to the speed of light by splitting and recombining beams, saw no changes to either outside of the acceptable error[14]. Following the Mickelson-Morley experiment, Swiss physicist Albert Einstein's 1905 Theory of Relativity would become the foundation of the understanding for light as a disturbance in electromagnetic fields[15].

Understanding the scientific basis of light as a wave is central to examining phenomena that can affect its wavelength. While the speed of light, c, is typically held constant, it can be affected by the medium that it travels through. In a vacuum, light travels at 299,792,456 meters per second; however, in transparent materials such as air or glass it is understood to be slower. This is a result of a property called refraction, in which light changes its speed as it changes between media; the ability of a medium to alter the speed of light is called its *refractive index*, which can be found

[11] Moller, "Leonhard Euler's Wave Theory of Light."
[12] Ibid
[13] Gittinger, "Thomas Young: The Foundations of Light, Color, and Optics"
[14] Roy, Amit. "The Experiment of Michelson and Morley: Experiment That Ruled Out Ether."
[15] Mermin, N. David. *It's About Time: Understanding Einstein's Relativity*, P. 185-186

with the relationship $n = \frac{c}{v}$, where n is the refractive index, c is the speed of light in a vacuum, and v is the speed of light in a medium[16]. For example, the refractive index of water is typically assumed to be 1.33[17]; as such, light moving from a vacuum into water would slow down to approximately ¾ of its original speed (see Appendix 5: Refraction and Wavelength for the mathematics behind this estimate). The relationship between the wavelength of light and its frequency is $f = \frac{c}{\lambda}$, or $\lambda = c \cdot f$. By decreasing or increasing the speed of light by changing media while holding constant frequency, the wavelength of light can be increased or decreased (see Figure 2). This phenomenon can be modelled mathematically with the *b* parameter in a trig function of the format $f(x) = a \cdot \sin sin\,(bx - c) + d$.

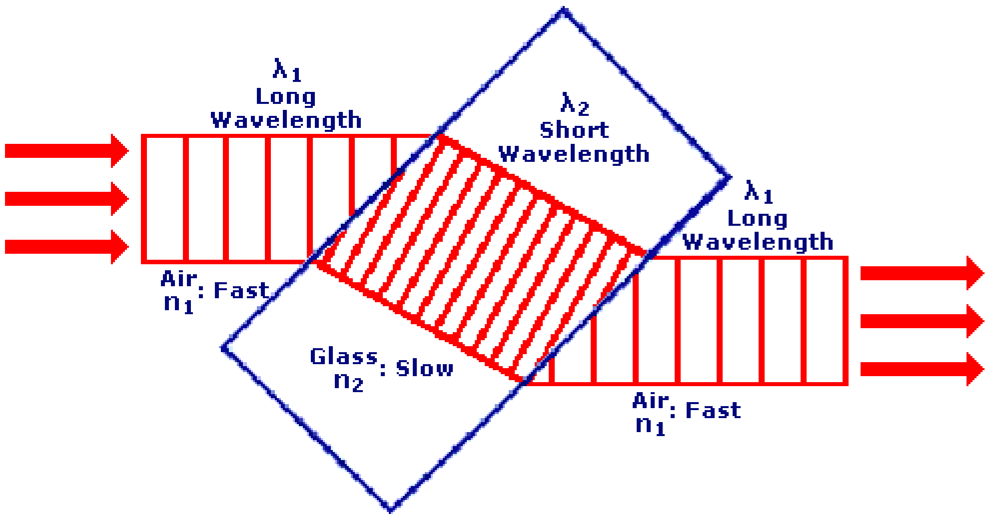

Figure 2: Refraction and Wavelength[18]

Observing astronomical objects in motion contributed to early understandings of light, and is where another phenomenon that can affect the wavelength of light can be seen. The Doppler Effect is the compression of stretching of waves by an object in motion. The siren of an ambulance is commonly used as a place where one can observe the Doppler effect; while the siren emits sound at a constant pitch, the ambulance's siren will be heard at a higher pitch as it approaches and a lower pitch after it passes. This occurs because the sound waves from the front of the ambulance's siren are being shortened by the forward motion of the vehicle, causing an apparent increase in frequency, while the waves emitted at the back of the siren are being stretched by the vehicle's motion away from the wave, causing an apparent decrease in frequency (see Figure 3). With measurement of light becoming significantly more precise, the Doppler Effect can be observed and applied when analyzing electromagnetic waves for a number of purposes.

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[16] Born, Max, and Emil Wolf. *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, 13
[17] Ibid, 14
[18] Montgomery, Ted. "Is the color of light determined by its wavelength or frequency?" *Ted Montgomery*

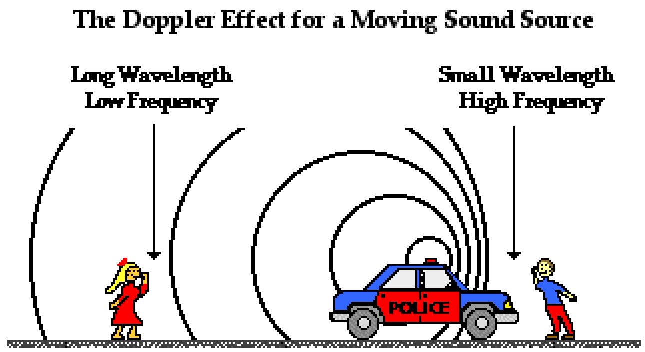

Figure 3: An illustration of the doppler effect from a moving source[19]

When observing bodies in the solar system, astronomers like Romer and Bradley could visibly see the direction of motion of the planets and moons they were observing; however, beyond a certain distance any changes in position are imperceptible to even the best telescopes. In being able to precisely measure the wavelength of emitted light, such as through spectroscopy, slight shifts towards the blue (shorter) or red (longer) end of the visible light spectrum can be measured (although these changes are imperceptible to the naked eye)[20]. Through this analysis, astronomers can determine if bodies such as stars and nebulae are moving towards or away from the earth. On the surface of the earth, the Doppler effect and light are utilized in meteorology with Doppler radar (see Figure 3). Operating on the principle of the Doppler effect, a piece of radar equipment will emit pulses of microwaves towards weather objects, such as a cloud or storm. Based on the frequency and wavelength of the reflected microwaves, the instrument can determine if the reflected waves have been shortened by an approaching object or lengthened by a departing object[21].

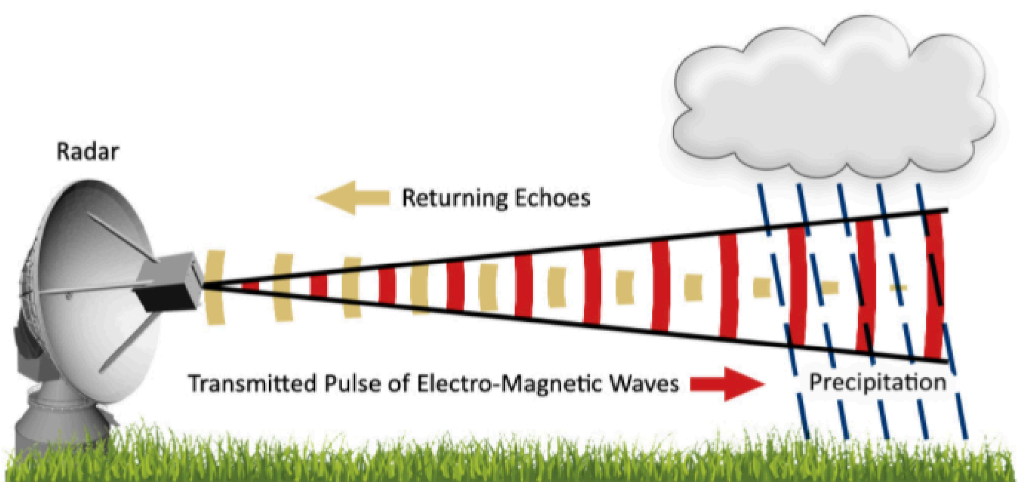

Figure 4: Diagram of Doppler radar tracking a weather event[22]

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[19] "The Doppler Effect," *The Physics Classroom.*
[20] "Red shift and the Doppler effect." *UNESCO Courier*, September 1984, 16. *Gale General*
[21] Arcand and Watzke, 48
[22] "Doppler Radar – UPSC Prelims" *IA4Sure*.

In sports and science, measuring the speed of objects has become easier due to applications of the Doppler effect as well. Historically, an observer would need to record a moving object using a high rate of frame camera with a scale of measurement in the background, or have an object move through electronic gates which would record the time to travel a set distance[23]. Small and fast objects (such as a bullet), or larger objects that could not fit through electronic gates were especially challenging to measure[24]. Radar guns were developed to overcome the challenge of measuring speed and work using the principle of the Doppler effect; radio waves transmitted at a known frequency (and, by extension, wavelength) are emitted and reflected by the gun. The gun then measures the frequency of the reflected waves and calculates the speed of the approaching object[25]. Radar guns have the advantage of being small, inexpensive, portable, and relatively reliable and have found uses in sports (such as for measuring the speed of a pitcher's throw), law enforcement, and science.

With lengthening and shortening wavelengths in the wave model of light come several applications for mathematical modelling with periodic functions, many of which can be done using high school mathematics that will be the basis for activities and practice problems in my curriculum unit.

Instructional Methods

Charlotte-Mecklenburg Schools has moved towards utilizing Open Up curriculum throughout secondary mathematics; as of Fall 2022 every school is using Open Up in Math 1, Math 2 is being piloted at Ardrey Kell High School, Olympic High School, Independence High School, and Harding University High School, and Math 3 is due to be piloted in the 2023 school year. As such, while Math 3 students in the Spring 2023, Fall 2023, and Spring 2024 semesters might not necessarily experience the Open Up curriculum, they will be familiar with the routines and instructional practices from Math 1 and/or Math 2. My unit will look to build consistency with the instructional methods of Math 1 curriculum, which is currently being utilized district wide.

At its core, Open Up is problem-based learning and prioritizes students *doing mathematics*, as opposed to *learning about mathematics*[26]. Simply put, "Students learn mathematics as a result of solving problems. Mathematical ideas are the outcomes of the problem-solving experience rather than the elements that must be taught before problem solving.[27]" This inverts the traditional "I Do-We Do-You Do" model of direct instruction that most teachers and adults experienced as secondary math students, wherein students are led through example problems in a graduated-release model where the teacher chunks instruction and provides immediate feedback

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[23] Angell, Jim. "THE DOPPLER EFFECT."
[24]
[25] ibid
[26] Charlotte-Mecklenburg Schools. "CMS Math 1: What is a Problem-Based Curriculum." 2022
[27] Ibid

to students on their procedural mistakes and conceptual misconceptions. Teachers' role is to facilitate, rather than instruct, except for quickly addressing gaps in prerequisite knowledge hindering student engagement with tasks. Serving as a facilitator for learning, the teacher's responsibilities shift to 1.) ensure students understand the context and what is being asked, 2.) ask questions to advance student thinking, 3.) encourage the exchange of student ideas by encouraging productive dialogue, and 4.) synthesize student learning at the close of activities[28]. A good math teacher can balance procedural, applicational, and conceptual rigor in their instruction; problem-based learning and prioritizing student dialogue allows students to build their conceptual understanding and develop frameworks and strategies for application tasks, while a direct instruction-heavy approach typically sacrifices those for procedural questions. As such, most of my effort in planning resources (see Appendix 2: Pacing Guide and Instructional Activities & Resources) are problem-based tasks which can be completed in 10-20 minutes, target key aspects of what students should be able to know, understand, and do in the standards of NC Math 3 (see Appendix 1: Teaching Standards), and create opportunities for student partner or group work and mathematical discussion.

To balance consistency with the Open Up Math 1 curriculum in use throughout the district, as well as the Math 2 curriculum currently being piloted, my activities are to be as consistent with the design principles of Math 1 as possible. CMS Math 1 prioritizes balancing application, conceptual, and procedural mathematics with several consistent threads in its structure. Prerequisite knowledge is assessed with a pre-test (or Check Your Readiness) prior to each unit; this allows teachers to decide which resources built into each lesson will be necessary to address gaps in language, concepts, and skills throughout the unit[29]. As students become exposed to a number of approaches, they can consolidate conceptual knowledge and build procedural fluency to solve problems quickly and accurately; practice assignments throughout the unit repeatedly expose students to prior learning throughout the course to build or maintain mastery[30]. My content research on the science of light will be integrated throughout the unit in order to give students access to applications of their mathematical knowledge and make connections with their learning in science classes, as well as real-world connections that they observe in their everyday lives, such as checking the weather radar when deciding if they need an umbrella when going to school.

While Open Up curriculum does a number of things very well, there are a number of practices in my own classroom that I will not abandon when implementing this unit, so as to meet the goals of the Math 1 design, while streamlining some of the components that can be time-consuming and can eat into instructional time on a very tight pacing calendar. In my own practice, I have typically avoided giving students a pre-test when entering a unit, as carefully selecting one or

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[28] ibid
[29] Charlotte-Mecklenburg Schools. "CMS Math 1 Design Principles." 2022

[30] ibid

both of my daily warm-up questions almost always gives me enough information to tell how much key prerequisite knowledge and skills I will need to address with my students in a lesson. The CT3 framework of a Preview and Review question has been my preferred format for number of years. In this model, students are given two warm-up questions: the first is a review of a topic from earlier in the school year[31]; I typically use this to address my data needs from previous assessments for questions that have a simple, prevalent misconception that can be easily cleared. The second, preview question is used to assess prerequisite knowledge for the day's topic, sometimes presented in an unfamiliar context (albeit, still at a relatively low rigor)[32]. Having 53 standards to teach in 90 days (some standards taking multiple days, and days in a semester being lost to testing, inclement weather, etc.), collecting good information on student mastery without losing excessive time to cumbersome pre-tests can be beneficial and, as such, my unit will include preview questions for warm-ups so that teachers may assess key prerequisite concepts prior to a lesson and make an adjustment in the moment, in addition to a pre-test. Since teachers' individual data needs may vary for a review question, my unit will only have written preview questions.

Finally, throughout my teaching career I have heavily relied on the Teach Like a Champion strategy of double planning. In double-planning, guided notes, example problems, and some practice is consolidated into student handouts, allowing the teacher to save time (as students don't need to copy down word problems, graphs, and other time-consuming activities without instructional benefit), hold students accountable to specific tasks, and give students a document to refer back to in the future (I like to three-hole punch student handouts so that everything can be consolidated into a binder for the course). This is analogous to the class structure of Charlotte-Mecklenburg Schools' Open Up Curriculum, which utilizes double-planned student workbooks that correspond with a daily slide deck. With this being noted, I have not truly double-planned entire lessons, as the activities (and their slide decks), handouts, and practice assignments have not been consolidated into a single file; this will make it easier for teachers utilizing my unit to pick and choose which activities they would like to facilitate in addition to their own, but will require slightly more work (and significantly more printing) for teachers wanting to utilize my entire curriculum unit with fidelity.

Charlotte-Mecklenburg Schools is a diverse school district, with more than 25,000 students being Multilingual Learners (formerly referred to in CMS as English Learners or English Language Learners), representing over two hundred languages. As such, the district has made an effort to provide teachers with amplifications for teachers in each unit to better meet the needs of students for whom language barriers may affect content mastery. As opposed to altering curriculum or standards, teachers in Charlotte-Mecklenburg Schools have been pushed to *amplify* key concepts, notation, vocabulary, and ideas using graphic organizers, visuals, and representations that

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[31] CT3 Education. "Pedagogy Matters! #1: Do Nows and Exit Tickets – The Essential Bookends to a Lesson"
[32] ibid

highlight critical information. As such, my unit will have a number of vocabulary-focused resources for multilingual learners; these resources also will benefit non-EL students as vocabulary and mathematical notation can be a challenge even for native English speakers to grasp (see Appendix 3: Amplifications for Multilingual Learners).

Instructional Implementation

My curriculum unit is intended to be taught in seven class days, for a 90-minute block schedule, as suggested by the Charlotte-Mecklenburg Schools math curriculum team. These seven days include a unit test and review day in addition to five days of content instruction. The pacing, North Carolina Standards, and unpacked objectives for each day are outlined in Appendix 2: Pacing Guide and Cycle Plan for Instructional Resources. Some teachers favor giving students a diagnostic assessment prior to the implementation of the unit, which is designated on day 00.
In addition to standards and unpacked objectives, the cycle plan in Appendix 2 contains links to instructional activities, including a unit assessment, in Google Drive for teachers to utilize. Teachers may use the unit in its entirety, replace my activities with their own, or only implement one or two activities for a particular aspect of trigonometric functions in their own room.

Throughout the course of the unit, students may begin to develop misconceptions and gaps in their knowledge of trigonometry (in addition to outstanding gaps and misconceptions in prerequisite knowledge). Appendix 4 outlines instructional strategies that teachers may apply to respond to data indicating incomplete learning. Some of the strategies identified (such as using a bridge or preview/review format of the warm-up) are included in the activities in Appendix 2, while others are outlined, but not planned in my curriculum unit, as they respond to very specific issues that different teachers may have (for example, I could not create Chart-the-Error slides for every possible misconception that students may develop in trigonometry).

Finally, Appendix 5 is an extended look at the inverse proportional relationship between frequency and wavelength, and how refraction will change the speed (and by extension, wavelength) of a light wave when changing media. I have provided a specific example using blue light and glass; however, teachers wanting to write scientifically-accurate trig modelling questions involving refraction in context.

Appendix 1: Teaching Standards

My curriculum unit addresses the following standards, as outlined by the NC Department of Public Instruction for NC Math 3:

- NC.M3.F-IF.7: Analyze piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.
- NC.M3.F-TF.1: Understand radian measure of an angle as: 1.) The ratio of the length of an arc on a circle subtended by the angle to its radius, 2.) A dimensionless measure of length defined by the quotient of arc length and radius that is a real number, 3.) The domain for trigonometric functions.
- NC.M3.F-TF.2: Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions.
    - a.) Interpret the sine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its y coordinate. b.)
    - b.) Interpret the cosine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its x coordinate
- NC.M3.F-TF.5: Use technology to investigate the parameters, a, b, and h of a sine function, $f(x) = a \cdot sin(b \cdot x) + h$, to represent periodic phenomena and interpret key features in terms of a context.

Appendix 2: Pacing Guide and Cycle Plan for Instructional Activities & Resources
My curriculum unit follows East Mecklenburg High School's pacing guide for CMS Unit 8 (Trigonometry); in total, there are seven total allotted days (assuming 90-minute blocks), including five instructional days, one review day, and one day for the unit test.

Teachers may make a copy of the unit cycle plan to share with their professional learning community for planning purposes via Google Drive.

Day	Standard(s)	SWBAT Objective(s)	Links to Activities & Resources
00	PK	SWBAT demonstrate mastery on the unit pre-test	-Unit Pre-Test (Note that there is a fair amount of overlap with the warm-up questions)
01	NC.M3.F-TF.1	SWBAT convert between degrees and radians. SWBAT find coterminal and reference angles	-Unit Warm-ups* ● Weekly Warm-up Template; copy Preview questions from unit Warm-up into document and write your own review questions based on student data -Bridge: Circumference of a Circle & Central Angles -Activity #1: What's in a Radian? A Hands-On Activity[33] (10-15') ● Directions ● Slide Deck -Activity #2: Converting Degrees & Radians Guided Notes -Activity #3: Coterminal & Reference Angle Spinners[34] -Extension: Radians and Gradians -Practice: ● Partner Practice: Degrees vs Radians War ● Independent Practice: Converting Degrees & Radians Maze ● Independent Practice: Radians, Coterminal, and Reference Angles Quizizz -Exit Ticket
02	NC.M3.F-TF.2	SWBAT describe the relationship between an angle in standard	-Bridge: Special Right Triangles -Activity #1: Intro to the Unit Circle (Desmos Activity)

[33] Carter, Sarah. "What is a Radian? A Hands-On Activity." *Math Equals Love*.
[34] Carter, Sarah. "Angle Spinner for Sketching Angles in Standard Position." *Math Equals Love*.

		position, (x, y) coordinate, and sine/cosine function SWBAT use the unit circle to find the exact values of trig functions	-Activity #2: Connecting the Unit Circle to Triangle Trig -Activity #3: The Hand Trick • Slide Deck • Blank Unit Circle • Practice Quizizz -Extension: Bug on a Fan -Practice: • Independent Practice: Kuta Worksheet • Independent Practice: Unit Circle Maze -Exit Ticket
03	NC.M3.F-IF.7, NC.M3.F-TF.2, NC.M3.F-TF.5	SWBAT explain the shape of the sine and cosine function SWBAT describe the parameters (period, amplitude, midline, vertical shift, intercepts, and maxima/minima) of a sine or cosine function	-Bridge: Sine, Cosine, and the Unit Circle -Activity #1: Spaghetti Graphs Activity • Graphs Handout • Unit Circle & Questions Handout • Directions -Activity #2: Key Features of Trig Functions Desmos Activity -Activity #3: Interpreting Equations and Graphs Math Lib • Slide Deck • Student Handout -Activity #4 (Extension): Phase Shift Desmos Activity -Practice: • Small Group Practice: Gallery Walk • Independent Practice: Parameters of Trig Functions worksheet -Exit Ticket
04	NC.M3.F-IF.7, NC.M3.F-TF.2, NC.M3.F-TF.5	SWBAT describe the parameters (period, amplitude, midline, vertical shift, intercepts, and maxima/minima) of a sine or cosine function from its graph or equation.	-Station #1: Checkpoint Quiz -Station #2: Trig Function Pictionary • Slide Deck • Student Handout -Station #3: Card Sort • Note that this activity has tangent functions and phase shift; modify, cut or replace it if you have not covered these concepts.

		SWBAT write the equation of a sine or cosine function from its parameters or graph	-Station #4: Quizizz Independent Practice -Station #5: DDI Remediation ● Teachers can address skill/knowledge gaps from Days 1-3 or prior units in small group -Extension #1: Tangent Function -Extension #2: Marbleslide Desmos Activity -Exit Ticket
05	NC.M3.F-IF.7, NC.M3.F-TF.2, NC.M3.F-TF.5	SWBAT use sine and cosine functions in context.	-Bridge: -Activity #1: Modelling Sunspots ● Student Handout ● Sunspot Calculator Spreadsheet ● Completed Spreadsheet (estimate*) ● Slide Deck -Activity #2: Trig & Light ● Slide Deck ● Student Handout -Activity #3: -Practice: ● Partner Work: ● Independent Work: -Exit Ticket
06	PK	SWBAT review for Unit Test	-Unit Test Review Guide -Unit Test Review Quizizz -Trigonometry Jeopardy Slide Deck
07	PK	SWBAT demonstrate mastery on Unit Test	-Checkpoint Quiz -Unit Test -Unit Test (alternate version for retake) -Parallel Problems (for test corrections or pre-work for re-takes)
* My preferred format for warm-ups is the Preview/Review (see Instructional Methods). As such, I have only written preview questions on prerequisite knowledge for students with the unit as each teacher's individual needs to address their data for the review question. To use my warm-ups, teachers should make a copy of the unit warm-ups and write their own review questions, or simply only pull the preview questions and give students a single warm-up question.			

Appendix 3: Amplifications for Multilingual Learners

Charlotte-Mecklenburg Schools is a diverse school district, with more than 25,000 students being identified as multilingual learners representing over two hundred languages. To provide teachers with instructional supports aligned to CMS's goal of amplifications, I have provided several materials in order to help teachers accent key vocabulary and concepts, as opposed to altering the core material or simply providing a translation. The benefits to the amplifications approach are multiple: first, altering the scope or the rigor of the course under the pretense of *meeting students where they are* due to language or content barriers will exacerbate gaps, as opposed to accelerating student academic growth[35]. Second, comes the practicality of accommodating multiple languages in the same classroom; while East Mecklenburg High School and Charlotte-Mecklenburg Schools does see Spanish comprise its largest native language next to English, simply translating content into Spanish is not sufficient when students' native language may include French, Portuguese, Arabic, Vietnamese, or any of the other languages represented. Finally, instructional best practices for Exceptional Children and Multilingual Learners benefits all students, even those without the designation.

As such, the goal of my accommodations is primarily to provide targeted support to the area of vocabulary, including connecting multiple representations that accent key concepts of trigonometry.

Unit Vocabulary	• Contains all vocabulary for the unit, which can be given to students to fill throughout lessons. • Loosely based on the Frayer Model[36]; however, examples and non-examples have been replaced with a sketch to align more consistently with principles of amplification and Universal Design for Learning.
Anchor Charts	• All high schools in Charlotte-Mecklenburg Schools have access to poster printing through the school's media specialist; I have formatted the chart(s) as a .pdf document since that is what media coordinators I've worked with have asked for. I ask for anchor charts to be printed at half-size.

[35] Datnow, Amanda. *NEPC Review: The Opportunity Myth.* 21
[36] WDPI. "Frayer Model" *Wisconsin Department of Public Instruction*

Appendix 4: Resources and Activities for Data-Driven Instruction and Remediation

There are multiple data touchpoints throughout my curriculum unit; teachers may utilize some combination of the pre-test and/or warm-ups to assess student mastery of prerequisite knowledge prior to each lesson. Additionally, each lesson utilizes an exit ticket to assess student mastery for that day, a short checkpoint quiz to help teachers identify gaps in understanding of trig functions prior to the unit test, and the unit test itself. My goal is to provide teachers with guidance on how and when to utilize this data, as well as provide strategies for addressing gaps to avoid being *data-rich, information-poor*.

Resource	**Area of Need**	**Notes**
Bridge	Gaps in prerequisite knowledge identified by the pre-test or daily warm-up.	-Each day of my curriculum unit includes a bridge, which can be used to quickly fill in missing prerequisite skills or knowledge, as needed. -The bridge is designed to be completed within 5-10 minutes and only address the most critical piece(s) of prerequisite knowledge so that most of the instructional time can be spent on grade-level content. *-The bridge is optional and can be omitted if gaps are addressed through the warm-up or student show mastery on the warm-up or pre-test.*
Extension	Differentiation for honors, and IB MYP sections; classes that are at or above grade level	-Each extension activity is a higher-rigor application of the skills, concepts, and knowledge from that day. *-Like the bridge, extension activities are optional for teachers and should be omitted if students have not yet demonstrated proficiency on prerequisite knowledge or formative assessments of daily standards.*
Warm-up (Review Question)	Addressing data from prior units, previous exit tickets, and standards to maintain mastery	-Look for low-rigor questions that students missed due to an easy-to-correct misconception or procedural mistake; the most common pitfall of using a review question to address data is choosing a topic that is too difficult. -Typically, if the mistake can't be corrected with a 2-3 minute explanation (e.g. students using the area formula when calculating arc length as opposed to circumference), the warm-up isn't the ideal place to review it. -I will usually ask similar questions multiple days in a row so that students have the opportunity to re-attempt the question after review.

Chart the Error[37]	Addressing a common misconception or procedural mistake.	-Refer to the guide for when to use, how to prepare, and facilitate chart the error. -Chart the Error Guide

[37] Green-Morris and Laurain. *PD on Chart the Error*.

Appendix 5: Refraction and Wavelength in Context

What is the speed of light in water (n=1.33)? What is the wavelength in water of blue light, with a wavelength of 440 nm in air?[38]

$n_{water} = \frac{c}{v_{water}}$	The refractive index of water, n_{water}, is the inverse proportion of the speed of light in a vacuum, *c*, to its speed in water, v_{water}.
$v_{water} = \frac{c}{n_{water}}$	Solving for v_{water}, the rearranged formula shows that the speed of light in water is the inverse proportion of light's speed in a vacuum, *c*, to the refractive index of water, n_{water}
$v_{water} = \frac{c}{n_{water}}$ $v_{water} = \frac{3.00\times10^{8}\,ms^{-1}}{1.33}$ $v_{water} = 2.26\times10^{8}\,ms^{-1}$	Given: ● The refractive index, $n_{water} = 1.33$, and ● The speed of light in a vacuum, $c = 3.00\times10^{8}\,ms^{-1}$ It can be shown that the velocity of light in water is $2.26\times10^{8}\,ms^{-1}$, approximately ¾ of its speed in a vacuum.
$c = f\cdot\lambda$	For all light (regardless of medium), the speed of light, *c*, is the direct proportion of its frequency, f, and its wavelength λ.
$c = f\cdot\lambda_{air}$ $v_{water} = f\cdot\lambda_{water}$	We previously had shown the speed of light in a vacuum, *c*, to be different from the speed of light in water, v_{water}; however, the same proportional relationship between speed, frequency, and wavelength still applies. Due to the negligible difference between the speed of light in a vacuum and speed of light in air at my current level of precision, I will assume $c = v_{air}$ and $\lambda_{vacuum} = \lambda_{air}$
$n = \frac{c}{v}$ $n_{water} = \frac{c}{v_{water}}$	Recall that the refractive index, *n*, is the proportion of the speed of light in a vacuum, *c*, to the speed of light in a new medium, *v*; our specific medium is water in this example.
$n_{water} = \frac{c}{v_{water}}$ $n_{water} = \frac{f\cdot\lambda_{air}}{f\cdot\lambda_{water}}$ $n_{water} = \frac{\lambda_{air}}{\lambda_{water}}$	Using substitution and the relationship between the speed of light, its frequency, and medium, $c = f\cdot\lambda$, we can create a new proportion (recall $c = f\cdot\lambda_{air}$ and $v_{water} = f\cdot\lambda_{water}$). It can then be shown that the refractive index, *n*, is the proportion of light's wavelengths in both media.
$\lambda_{water} = \frac{\lambda_{air}}{n_{water}}$	Rearranging the previous proportion, we can show that the wavelength of light in water, λ_{water}, is the inverse proportion

[38] Davis, Doug. "PHY 1161C Homework: Chapter 23: Reflection and Refraction of Light."

	of its wavelength in air, λ_{air}, to its refractive index in water, n_{water}.
$\lambda_{water} = \frac{\lambda_{air}}{n_{water}}$ $\lambda_{water} = \frac{440\ nm}{1.33}$ $\lambda_{water} = 330.83\ nm$	Given: ● The refractive index, $n_{water} = 1.33$, and ● The wavelength of blue light in air, $\lambda_{air} = 440\ nm$ ● Differences between the speed (and by extension, wavelength) of light in air and a vacuum at our level of precision is negligible. It can be shown the wavelength of blue light in water is 330.83 nm, approximately ¾ the length of the same blue light in air.

Note that this solution appears counterintuitive and even contradictory to earlier characterizations of electromagnetic radiation at varying wavelengths. 330.83 nm should be a shorter wavelength than is perceptible by the human eye (the visible spectrum is between 400 and 720 nm); while light's color is typically associated with its wavelength (the inverse of frequency), refraction experiments have shown that frequency gives light its color. A blue object dropped into a swimming pool will still appear blue and will not become imperceptible to the human eye by its wavelength (330.83 nm is associated with ultraviolet light).

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