

## Math- The Write Way

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This curriculum unit is recommended for 4th Grade Mathematics

**Keywords:** Math, Math Talks, Talk Frame, Conceptual Learning, Mathematical Writing, Writing in Math, Math Problem of the Week

**Teaching Standards:** See <u>Appendix 1</u> for teaching standards addressed in this unit.

**Synopsis:** This unit focuses on the ability to write in mathematics versus the ability to write about mathematics. "Math-The Write Way!" is an opportunity for teachers to embrace writing in the math curriculum and enhance the conceptual learning of their students. The unit will focus around 40 weeks of problem solving in the form of a MPOW (Math Problem of the Week). Students will complete one MPOW in the classroom each week with the help of classmates and a modified Talk Frame<sup>1</sup> completed by the class. Students will write in various ways to explain their thinking and add to the thinking of their peers. At home, students will complete a similar MPOW using their own thoughts and ideas, as well as the conceptual understanding they have gained from the class discussion. The intent of this unit is to encourage mathematical writing in the classroom, as well as to prove its effectiveness in enhancing learning outcomes.

I plan to teach this unit during the coming year to 60 students in 4th grade mathematics classrooms.

I give Charlotte Teachers Institute permission to publish my curriculum unit in print and online. I understand that I will be credited as the author of my work.

# **Math- The Write Way**

by Katelyn Gardepe

#### Introduction

As an elementary math teacher, I can wholeheartedly agree with the idea that writing and reading need to be integrated into our curriculum. In some ways, it already is, but the intention is not always explicit. In reading many of the resources provided to us, it seems that including reading and writing in our math class is a no-brainer. It allows students to clarify their thoughts in a concrete way—where they do not have to spend additional time working hard to recall their learned knowledge.

In the past few years, I have spent a lot of time asking my students to talk about the math. With the introduction (or heavy encouragement) of academic conversations in our district, it was an easy way to include the literacy based strategy in our math classrooms. It is obvious that academic conversations or "math talks" allow students to openly express their ideas and mathematical thinking. Students have found a way to respectfully listen and criticize others ideas. This has helped to extend their learning. As a teacher, it has been an obvious way to encourage students who may have or may not have understood the topic that had been taught. By doing this, students who were not so sure of their answers are able to "piggy-back" on others' thinking to solidify their own thinking or give them an initial idea of their own.

While I have spent the time using "math talks" in my classroom, I seldom ask students to write about what they have discussed. Honestly, it just seemed like a step that would take additional time out of my lesson when they had already spoken about what they thought. Though it seemed so obvious when reading it in research, I did not think about the idea that it could further one's understanding of the topic.

## **Demographic**

My school is located in an affluent, suburban neighborhood in South Charlotte. The population of the school is made up of approximately 76% White, 13% Black, 7% Hispanic, and 3% Asian students. About 22% of the students at Selwyn are receiving free and reduced lunch services.

Currently, I teach approximately 62 fourth graders, the majority of which perform above grade level. I departmentalize with one additional teacher and split the class between her and I. She teaches all of our students Literacy and I teach Math, Science and Social Studies. Because our demographic produces such high performing students, we have elected to pull out the students

performing at above the 80th percentile on their state reading exam. The additional students have been placed in a heterogeneous group of all other levels of cognitive ability. While we firmly believe in mixed ability research, we have found it extremely difficult to meet the needs of a student in the 99th percentile, at the same time as a student in the 1st percentile. Our placements have allowed us to find a "medium ground" for our students, while still presenting them with the gift of learning from their peers.

#### Rationale

In my unit, I want the students to continue their work with "Math Talks" and also become more accustomed to the writing aspect. It is my intention to create a series of writing tasks, a Form A and a Form B, that align to the content standards for 4<sup>th</sup> grade math students. In order to help students better understand what is expected of them, "Form A" of each task will take place in class the first week and "Form B" will be completed by the students independently the following week. Form A will pose as an in-class assignment, where teacher and students are exploring ideas together and creating a talk frame to reference while writing. Students will alternate between Form A (group assignment) and Form B (individual assignment) of each task. Both forms will cover the same content standard. This will allow students to use the knowledge gathered from the discussion in class (on Form A) on their work at home for Form B.

I will discuss the topics that are necessary for successful implementation of these prompts prior to releasing students to write. For instance, class discussion of Form A prompts will be a vital part of completing these prompts correctly. This is important for the students if you want them to feel successful in their writing. A student who has no idea how to answer the academic piece of this puzzle will inevitably struggle with getting their thoughts out to their classmates. As a teacher, you will need to model the talking frame, taking notes on it, respecting and valuing other's ideas, and making small changes to already given ideas as we discover more about the topic (from discussion or further exploration of the topic).

With my past classes in mind, I have created this unit understanding that students of all levels WANT to know how to do math and how to do it well. I have also learned that students of high ability are not always able to think conceptually about the math that they, on the same hand, are able to compute without flaw. I wanted to give the opportunity for all learners to deepen their understanding of the fourth grade concepts as I teach, and to feel successful in math. I have created Form A and Form B of each prompt. These forms differ in their immediate questioning, but require the same content to solve. The intention of the different forms of a prompt allow students to gain understanding from their classmates, offer understanding to others, and to work as a team to solve a problem.

To create the prompts for each topic, I have used the "backwards planning" design. I began by identifying what the mastery of each standard looks like. Using a graphic organizer, spoke about in more depth within the unit, I then determined what mastery looks like for students. Next, I worked to create my prompts focused around that mastery and what each prompt will require students to know in order to be successful. When the two aligned, a writing prompt was created! In addition, using the assessments from past school years has helped me to create some prompts that require students to explore a misconception from one student to another.

The unit will cover a small section of the 4<sup>th</sup> grade curriculum for mathematics. It will stretch from Day One of math class, through the first six weeks of school. Originally, I had intended for the unit to cover the entire year. As I thought more about it, and conferred with fellow colleagues, I realized that there was little chance I would keep what I had created, had I not been able to make changes as I go. Not only to the talk frame and writing prompt, but also to the content itself. I wanted to ensure that throughout the year my students have the ability to reflect on misconceptions and deepen their understanding of ideas that were more tough for them. Mapping out an entire year without the flexibility to make changes, just seemed a bit unrealistic. With that said, this unit will give a jumping off point for writing in math, but also give you the flexibility to adapt it to your students and their individual needs.

#### **Content Research**

What is UbD?

Understanding by Design<sup>2</sup> is a purposeful framework for creating instructional plans. When using UbD, teachers reflect on what it is intended to be the end result of their lesson or unit. As ideas form to create the unit, teachers must first look at the assessment or the desired outcome of success. This framework also requires the end result to be considered an example of authentic performance. UbD emphasizes the teacher as the "coach of understanding" rather than merely a means of transferring information from one individual to another. The teacher must be confident in his/ her abilities to produce meaningful knowledge to their students and thus allow them to produce this means of understanding as an authentic response to their instruction. Students must see relationships between the content that is learned and thus produce connections from prior knowledge. In addition, students who "put their hands on things" are expected to "store" their knowledge in their long-term memory versus those who learn via a lecture.

## What is understanding?

What do we mean when we say a child "understands"? How do we determine the difference between those who understand, those who don't, and those who meet us somewhere in the middle? Understanding is quite different from merely knowing or repeating what is taught to you. In many math classes, students are taught procedural knowledge that may be easily recalled and repeated, as needed. However, the depth of true understanding truly lacks in many of these situations. If a student was asked to take that idea and apply it to an unfamiliar or unrehearsed situation, they would not be able to. For example, have you ever had that student who can tell you all of the steps to regrouping across zeros, but has no idea why they are adding a "1" above a place value? If you are borrowing "1", why does the place value go up by 10, but not by 1? Students often learn the procedure to regrouping because it can be easier (not to mention less time consuming) for a teacher to teach the steps rather than break out the base-ten blocks and have them determine that they are actually adding a ten, or a hundred. So how do we prevent this? How do we create true "understanding"?

Teaching for understanding is something that has to be carefully designed. Using the UbD process requires you to be clear about your desired outcome, to yourself and to your students. How will you know that understanding has occurred at the end of the lesson? One answer is creating curriculum that includes your desired outcome and a performance task that will assess these outcomes at the end. These performance tasks should reflect a "real-world" application of the skill that has been taught. This planning backward approach is not new and has been in discussion since 1948³ with three vital stages in curriculum planning:

- 1. Identify Desired Results
- 2. Determine Acceptable Evidence
- 3. Plan Learning Experiences and Instruction Accordingly

Stage 1 requires the teacher to reflect on the goals of the lesson and determine how meaningful understanding can be obtained. Stage 2 asks the creator what a meaningful end-result could look like. All assessments should be easily aligned with the desired results from Stage 1. In Stage 3, the teacher or creator will address the instructional design of the plan. What learning activities will get the students to Stage 1 and 2, and how will their progress be monitored? Can students achieve the goal via the given instruction or will modifications need to be made to supplement the given instruction? In the end, your evaluation of the lesson should include purposeful thinking. If the desired result of my lesson is "x", then the evidence of their ability will be "y", and all of the learning events must produce "x". <sup>4</sup>

## Writing in Math

For a long time, there has been a push for writing across the curriculum, including math. As early as 1989, the National Council of Teachers of Mathematics (NCTM) identified learning to communicate math as a major goal for students. Writing has increased in math classes throughout the years, but the type of writing has varied. Some teachers encourage their students to write math journals where they reflect on how they felt about today's math lesson. (Monroe n.d.) Some ask students to create a step-by-step article about how they solved the work they did that day. While writing in this way is beneficial to our writing standards, it can be argued that it has a good amount less value to the mathematical understanding of our students. Here's where we discuss the difference between writing IN math and writing ABOUT math. <sup>5</sup>

Writing ABOUT math may include writing about the lesson they had that day or the last mathematician they learned about from a read aloud. Writing ABOUT math continues to put literacy at the forefront of the instructional day and encourages growth in that domain. While we are happy to add to the content knowledge of our readers and writers, we are also working to create mathematicians.

Writing IN math differs in that it can support students in developing conceptual understanding of the topic at hand. The intent of writing IN math is that students continue their understanding of the lesson by expressing their knowledge on paper. Writing IN math requires different and additional skills than that of simply writing in literacy.<sup>6</sup> In my experience, students must have a solid understanding of various representations, their uses and their interconnectedness. A large array of math tools must be accessible for students to write in math class. Porter and Masinglia (2000) contend that writing to learn math most benefits the student by requiring them to investigate and consider mathematical concepts and connections while communicating with others. Math writing involves more than just written word; they can include text, numbers, pictures, charts, tables, etc. <sup>7</sup>

#### The Four Types of Writing

The Elementary Mathematical Writing Task Force <sup>8</sup> recommends that there are four types of mathematical writing. They are exploratory, informative/explanatory, argumentative, and mathematically creative.

Exploratory writing allows students to make sense of problems and persevere in solving them. Students will use this type of writing to sort through their thoughts and create possible solutions to the problem at hand. In this type of writing, students might write notes to themselves on the top of the paper to dissect the information before writing.

Informative/Explanatory writing gives students the opportunity to describe and explain mathematical ideas. In this form of writing, students are encouraged to be accurate in their thoughts and use appropriate vocabulary to describe the situation. Students may be asked to compare representations, explain how they have solved a problem, or make comparisons to real-world applications.

Argumentative writing is a space for students to "plead their case". This type of writing clearly exhibits the mathematical practices of constructing viable arguments and critiquing the arguments of others. Students may opt to agree or disagree with the said statement and are encouraged to explain further as to provide explanation for their stance. Not only do students have to justify their own answers, but they may also be asked to justify the response of others. Students should be encouraged not only to disagree with the prompts that are created, but also to agree and share their content knowledge. <sup>9</sup>

Mathematically creative writing allows students to think creatively and document their mathematical ideas far beyond what the question is asking. Students think outside of the box and use connecting ideas to aid their response in writing. Students see past the question at hand and make additional applications, as needed.

## Math Talks 10

Prior to writing in math, it is useful to have productive math talks within the classroom. As students learn how to communicate mathematically aloud, they will soon be able to organize their ideas for a writing piece more clearly. Math talks are a vital part of math class and should be a regular occurrence in your classroom. Prior to setting up math talks though, you should work with your classroom to set up clear expectations for communication among peers. Students need to understand that the classroom will be a safe place to express their thoughts, even if they are incorrect. If students do not feel that the classroom is a safe place they will be less likely to participate and/or gain anything from the experience. Setting clear expectations of respect and open-mindedness, allow for students to embark on mathematical discourse that is valuable for all. <sup>11</sup>

In the book "Classroom Discussions" (2013), Chapin, O'Connor, and Anderson suggest five talk moves that will help you promote productive mathematical talk within your classroom, along with examples.

- 1. Revoicing. ("So you're saying that it's an odd number?")
- 2. Repeating: Asking students to restate someone else's reasoning (Can you repeat what he just said in your own words?")
- 3. Reasoning: Asking students to apply their own reasoning to someone else's ("Do you agree or disagree? Why?")
- 4. Adding On: Prompting students for further participation ("Would someone like to add something more to this?")
- 5. Waiting: Using Wait Time ("Take your time... we'll wait...")

#### Talk Frame<sup>12</sup>

The talk frame (Casa 2013) is a graphic organizer used to help students guide their thinking. It can assist students in taking ideas that they themselves are thinking, or that their classmates have offered up, and put them on paper. Knowing that students can struggle with putting their ideas into words or their words on their paper, the Talk Frame acts as a draft of ideas which continually improves throughout class time. While graphic organizers can be plentiful, the idea of a talk frame is to organize the thoughts of the classroom, appreciate the thoughts of peers, and engage students in active, ever-changing discourse. As a teacher records the ideas, he/she does not make any statements regarding the accuracy of the student ideas. Whether correct or incorrect, if a student shares their thought it will go up on the talk frame. As the discussion continues, it is a hope that the students work as a class to make corrections to any statements that were incorrect and recorded. After completing the class discussion, students may use the talk frame to inform their writing. <sup>13</sup>

## **Teaching Strategies**

During class time, the discussion will be framed as a whole-class and small-group format. We will pose the question – aligned with our teaching standards for the week—and give students an opportunity to explore their ideas together. On Monday, students may have very little idea of how to solve the problem posed to them. We will add the ideas that are brought up to a "talk frame" or graphic organizer used by the class on a weekly basis. This organizer can be used throughout our whole class discussion and then subsequently in the work they do at home for Form B. Throughout the week, students may think of ways that we can alter statements or ideas on our "talk frame"—and we will make these changes. After having a class discussion, students can choose to make small changes to ideas in order to make the ideas we have gathered more accurate. All ideas will always be valued and deemed important. In our classroom, I will work hard to ensure that this time is respected and that the ideas of all students are respected as well. In addition, the following strategies will be used throughout the unit:

*Note-taking*: Students will take notes as they participate and listen to the classroom discourse that is happening around the room. Students will need to refer back to this during their independent writing time. To organize this information, students will utilize a graphic organizer, in this case called a Talk Frame.

Cooperative Learning: Students will work together to create insights into the mathematical concepts that are being discussed. Students will build off of each other's ideas.

Academic Conversation: Students will spend time working together to come up with "big" ideas and create mathematical discourse by agreeing, disagreeing, and adding on to the ideas of their peers.

*Talk Frame*: Students will use the talk frame as a resource for writing in math class. The teacher will complete the talk frame as the students participate in discourse, and work their way towards solutions to the problems given to them.

Reinforcing effort and providing recognition: Throughout the lesson, students will need to feel comfortable expressing their thoughts and ideas in the classroom-- especially if they think it will be wrong. By reinforcing the weight of their effort, the teacher allows them to understand "it is better to try and fail than to never try at all". This is extremely important in this unit, as students who "check out" because of fear they will gain very little.

*Inquiry-based Learning*: Students will work as a class to solve problems. The teacher will direct the class in solving the problem and encouraging them to think deeper about the problem at hand. As a class, students will fill up the Talk Frame with their ideas.

*Small-Group Instruction*: As needed, students will be pulled into small groups for remediation. This may include students who struggle with writing because of learning disabilities, or students who struggle with processing. Using the talk frame, the teacher will walk students through the assignment.

Partnered Instruction: Learning from a peer is so valuable. Throughout this unit, students will work together to solve problems and explore big ideas. Students will use their neighbor to throw ideas off of and to help elaborate on their own ideas. As a team, the partners will work to come up with a solution that meets the consensus of both individuals, but has been obtained through the work of both combined.

## **Instructional Implementation**

Day One: Pre-Assessment

Today will be used as an introduction into the idea of writing in math. As a pre-assessment, students will use the talk frame as a graphic organizer for their thoughts prior to addressing the writing assignment. Prior to handing out the resources, introduce students to the talk frame and explain that the pieces should be used to brainstorm ideas on their topic. Both resources, the talk frame and the writing prompt, will be given to students and used to assess their writing ability prior to beginning the remainder of this unit in class. The rubric provided will be used to assess the students understanding.

Day Two: Introduction of the talk frame (Limit this to the first 5-10 minutes of class.)

Display the talk frame where students can easily view it throughout the week. Give each student a copy of the talk frame to glue into their notebook. (As time moves on, students can either glue the talk frame in their notebook or they can draw their own once they understand the organization of it.) Address today's prompt with the students.

**Think:** Joey has the number 46,098. He says that the 4 in his number has a value of 400 tens. Raquel says that the 4 in his number only has a value of 40 tens. Who is correct? How do you know?

Tell the students that you will take the first 5-10 minutes of class each day this week to work on this prompt. As a group, we will work together to come up with some mathematical ideas to add to our talk frame. The ideas should help the class to determine who is correct and then push us to examine why. Ask the class for initial ideas. (*Note that if students show sufficient knowledge of the prompt and are ready to write today, allow them to. For tomorrow's lesson, create a challenge version of this question to allow students to push further in their thinking.*) Tell them you do not want the answer just yet, but that you are looking more for ideas. If students are confident they know the answer already, push students to identify misconceptions someone else might have and how they would explain it to them. At this time, it would also be beneficial to present students with an idea on the talk frame that is not accurate. Allow them to argue the idea and make justifications on why it is or is not correct. Encourage students to think about how they would explain their reasoning to someone who may not understand the answer right off the bat.

Write ideas in the talk frame, using one box for each idea (correct or incorrect). As students start to see mistakes, allow them to make edits to the comments that have already been shared on the talk frame. After a few ideas have been added to the talk frame, ask students to think more about

today's prompt throughout the day and into the night for homework. We will add more thoughts/connections/mathematical ideas tomorrow.

Day Three: Back to the talk frame

As students enter class, have them write down on a sticky note any ideas they might have regarding the math prompt that have not already been shared. Once students have had the time to do so, have them share their sticky notes with others in their group. Students should talk with one another to do two things:

- 1. Determine whether they agree with the ideas on the sticky note and tell why.
- 2. Decide whether the information shared is already on the talk frame or if it needed to be added.

After giving the students a few minutes to do this, allow groups to share any sticky notes that might need to be added to the talk frame. Add the information as it is given and have students do the same.

Day Four: Writing with the talk frame

Today students will have one last opportunity to determine if information needs to be added to the talk frame, or if our ideas are sufficient enough to answer the prompt with good mathematical reasoning. As a group, decide whether or not the ideas on the talk frame can lead us to our solution. If not, continue to explore mathematical reasoning from students. If so, begin the writing process with students.

To help in creating a good response, have students share the solution to the problem and their reasoning behind where they stand. This will help students to grasp their explanation before trying to put it on paper. Have students number the ideas on the talk frame in order, step-by-step, just the way that many of their classmates just described the solution. Share, as a group, which idea should come first, second, third, etc. Encourage students to think about how we might have organized these ideas on our talk frame initially to help our writing process go smoother.

Day Five: Writing in Math

Use the numbered talk frame from yesterday to guide the beginning of the writing process for students. Begin writing as a class. Talk through the process aloud, leading students to create a good leg to stand on in their explanation. Have students use the time to continue writing and expressing their thoughts on the prompt. Students should complete their writing today and have a

sound solution written down prior to the end of writing time. Pull small groups, as needed, to help them through the writing process.

Tell students that they will complete a similar talk frame and prompt next week, on their own. They should reflect on their thoughts from this week, as well as use the talk frame, to assist them in tackling their new prompt.

## Post-Assessment

After six weeks of writing in math, give the students the post-assessment to determine their growth in writing. Use the rubric provided to assess their progress.

# **Appendix 1: Teaching Standards**

# Common Core Writing Standards

#### W.4.2.a

Introduce a topic clearly and group related information in paragraphs and sections; include formatting (e.g., headings), illustrations, and multimedia when useful to aiding comprehension.

#### W.4.2.b

Develop the topic with facts, definitions, concrete details, quotations, or other information and examples related to the topic.

#### W.4.2.c

Link ideas within categories of information using words and phrases (e.g., another, for example, also, because).

#### W.4.2.d

Use precise language and domain-specific vocabulary to inform about or explain the topic.

#### W.4.2.e

Provide a concluding statement or section related to the information or explanation presented.

# North Carolina Standard Course of Study Math Standards

#### 4.NBT.1

Explain that in a multi-digit whole number, a digit in one place represents 10 times as much as it represents in the place to its right, up to 100,000.

#### 4.NBT.2

Read and write multi-digit whole numbers up to and including 100,000 using numerals, number names, and expanded form.

#### 4.NBT.7

Compare two multi-digit numbers up to and including 100,000 based on the values of the digits in each place, using >, =, and < symbols to record the results of comparisons.

#### 4.NBT.4

Add and subtract multi-digit whole numbers up to and including 100,000 using the standard algorithm with place value understanding.

#### 4.NBT.5

Multiply a whole number of up to three digits by a one-digit whole number, and multiply up to two two-digit numbers with place value understanding using area models, partial products, and the properties of operations. Use models to make connections and develop the algorithm.

#### **Annotated List of Teacher Resources**

- Bossé, Michael J., and Johna Faulconer. "Learning and assessing mathematics through reading and writing." *School Science and Mathematics* 108, no. 1 (2008): 8-19. This resource is a great introduction resource to explaining why writing in math is important and what it can do for your students.
- Casa, T. M., J. M. Firmender, J. Cahill, F. Cardetti, J. M. Choppin, J. Cohen, and R. Zawodniak.

  "Types of and purposes for elementary mathematical writing: Task force
  recommendations." (2016).

  This resource gives definitions for each type of mathematical writing and great examples
  of authentic student work to represent each.
- Chapin, Suzanne H., Catherine O'Connor, Mary Catherine O'Connor, and Nancy Canavan Anderson. *Classroom discussions: Using math talk to help students learn, Grades K-6*. Math Solutions, 2009.

  This resource is a great place to learn about what the classroom discussions in math class should look like and the specific routines that need to be put in place to make this time valuable for everyone.
- National Council of Teachers of Mathematics, ed. *Principles and standards for school mathematics*. Vol. 1. National Council of Teachers of, 2000.

  This article provides an understanding of what it truly means to "reason" in mathematics and how to ensure you get your students there.
- Wiggins, Grant P., and Jay McTighe. *The understanding by design guide to creating high-quality units*. ASCD, 2011.

  This is a great article to gain a better understanding of the backwards planning approach and how you can implement it in your classroom.
- Wilcox, Brad, and Eula Ewing Monroe. "Integrating writing and mathematics." *The Reading Teacher* 64, no. 7 (2011): 521-529.

  This article is a good, brief introduction into why we need to write in math, and also what writing in math has looked like over the last decade.
- Williams, Madelyn M., and Tutita M. Casa. "Connecting class talk with individual student writing." *Teaching Children's Mathematics* 18, no. 5 (2012): 314-321. This article provides great insight into the Talk Frame and how to use it in your classroom.

# Notes

- <sup>1</sup> (Casa 2011) <sup>2</sup> (Wiggins 2011) <sup>3</sup> (Wiggins 2011)
- <sup>4</sup>(Wiggins 2011)
- <sup>5</sup> (Wiggins 2011)
- <sup>6</sup> (Wiggins 2011)
- <sup>7</sup> (Wiggins 2011)

  <sup>8</sup> (K. E. T. Casa 2017)
- <sup>9</sup> (K. E. T. Casa 2017)
- <sup>10</sup>(J. F. T. Casa n.d.)
- 11 (Suzanne Chapin n.d.)
  12 (Casa 2011)
  13 (Casa 2011)

## **Bibliography**

- Bossé, Michael J., and Johna Faulconer. "Learning and assessing mathematics through reading and writing." *School Science and Mathematics* 108, no. 1 (2008): 8-19.
- Casa, T. M., J. M. Firmender, J. Cahill, F. Cardetti, J. M. Choppin, J. Cohen, and R. Zawodniak. "Types of and purposes for elementary mathematical writing: Task force recommendations." (2016).
- Chapin, Suzanne H., Catherine O'Connor, Mary Catherine O'Connor, and Nancy Canavan Anderson. *Classroom discussions: Using math talk to help students learn, Grades K-6*. Math Solutions, 2009.
- National Council of Teachers of Mathematics, ed. *Principles and standards for school mathematics*. Vol. 1. National Council of Teachers of, 2000.
- Porter, Mary K., and Joanna O. Masingila. "Examining the effects of writing on conceptual and procedural knowledge in calculus." *Educational Studies in Mathematics* 42, no. 2 (2000): 165-177.
- Wiggins, Grant P., and Jay McTighe. *The understanding by design guide to creating high-quality units*. ASCD, 2011.
- Wilcox, Brad, and Eula Ewing Monroe. "Integrating writing and mathematics." *The Reading Teacher* 64, no. 7 (2011): 521-529.
- Williams, Madelyn M., and Tutita M. Casa. "Connecting class talk with individual student writing." *Teaching Children's Mathematics* 18, no. 5 (2012): 314-321.