Explanation Creation:
Mathematical Thinking and Reasoning through Multiplication

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This curriculum unit is recommended for 3rd Grade Math English Learners

Keywords: Multiplication, English Learner, Writing in Mathematics, Problem Solving

Teaching Standards: See Appendix 1 for teaching standards addressed in this unit.

Synopsis: This unit focuses on English Learners who are building the foundations of multiplication with integration of mathematical writing. Students are engaged in several activities that support writing skills and reasoning through multiplication. Students participate in meaningful discussions with peers and hands-on activities that spark curiosity. Lessons are designed to meet the needs of English Learners using appropriate strategies and scaffolds. The goal is to improve mathematical written expression by integrating writing throughout the unit, specifically focusing on explanatory and creative writing. Students will enjoy learning multiplication while building conceptual knowledge and strengthening mathematical discourse through writing and discussion.

I plan to teach this unit during the coming year to 15-20 students in 3rd grade math, specifically English Learners.

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Introduction

Rationale

My vision of an engaging math class consists of curious learners who are solving problems, using manipulatives, working together and figuring out answers. There tends to be several hands-on activities, students actively asking questions, and students rotating from group to group. This sounds wonderful, but what is generally missing when it comes to math? Writing! From classroom to classroom, I have noticed that writing is limited, a struggle, and tough to teach. Our students at times battle expressing themselves when solving problems, let alone writing it down. We want our students to be able to reason mathematically, write how they solved a problem, argue their way through a problem[1] or analyze a problem using appropriate vocabulary. With some students, this may be an easy task, however with others this can become a challenge, specifically English Learners.

In my experience when students put their mathematical thoughts on paper, I actually see what students are thinking, how they reason and represent situations, and how they support their ideas. It is exciting to see when students can explain or show their thinking with a pencil. Unfortunately, we do not observe this consistently. Possibly due to the weakness of writing across the board in all content areas. When collaborating and discussing with other teachers, facilitators and principals from across the district, writing is voiced as a challenge for our students. We experience a common issue with weak written expression. Why is writing so challenging for students? One reason may be that students do not know how to express their mathematical thinking on paper because writing is uncomfortable and difficult. They are used to verbally responding and having academic conversations, but transferring thoughts on paper is challenging. Students try to find the correct answer, however when responding to the "how" and "why" is where there is disconnection.

The NCTM 2000 Standards emphasize the importance of communication. During mathematics class, students have to explore, engage, interact, and reason on a day-to-day basis, which involves collaborating with one another. Writing is one way students can communicate. Mathematical writing can enhance students' ability to convey mathematical ideas.[2] When focusing on writing as a form of communication, teachers should expose different types and formats whether it be the ability to respond and reason through words or pictures.

As educators, we know the benefits of writing. It is our responsibility to incorporate creative ways to integrate it through content areas, especially math. If students receive instruction on the "how" and get many opportunities to write expressively, we anticipate that we will see improvement in critical thinking skills, reasoning skills, and communication skills. The purpose of creating this curriculum is to give teachers suggestions and strategies to support students with their communication and reasoning mathematically, focusing on English Learners. In *Types of and Purposes for Elementary Mathematical Writing: Task Force*

Recommendations, Casa et al. (2016) states that, "providing a framework for the types of and purposes for mathematical writing enables elementary teachers to leverage writing for students' learning of mathematics, coaches to support teachers with its implementation, assessment developers to design and score such items, and curriculum developers to provide more opportunities for students to engage in this practice".[3] With this in mind, being intentional with mathematical writing will set our students up for success. Knowing the supports and scaffolds used to guide students along to productive writing will give teachers the tools and confidence to teach it.

Demographics/School Information

The demographics for Governor's Village STEM Academy Upper Campus are as follows: 67.2% African American, 14.5% Hispanic, 5.1% White, 9.7% Asian, 2.4% 2 or more races and 1.2% American Indian. There are 1,063 students. We have 98.8% free and reduced lunch. Within our student population, we have students who are Exceptional Child, Talented Development and English Learner certified, and students who are in Multi-Tiered Systems of Support (MTSS). Our instruction is intentional based on the needs of all learners through whole group and small group instruction, as well as a full push-in model provided by the English Learner teachers and Exceptional Child teachers.

Governor's Village STEM Academy is a partial magnet school, pre-kindergarten through 8th grade. Partial magnet means that it is partly a neighborhood school, and magnet which a lottery selects students at random. In addition, we teach all general education subject areas including English Language Arts, Mathematics, Science, Social Studies, and various elective areas. We are also an AVID school, which is a system that focuses on preparing our learners to be college and career ready utilizing research-based strategies.

Alejandra Garcia, Principal of Governor's Village has high expectations for all students. Her goal is to provide equity for all students by focusing on the whole child, and emphasize the importance of core instruction so that sub-groups such as English Learners will succeed. The English Learner teacher role is strictly a push-in model, as well as co-teaching with specific teachers. Research suggests that, "it is no longer acceptable to have a little program here and a pull-out or push-in intervention there for many of these students. As the No Child Left Behind is reshaped and federal policies look toward the research-based instruction that embraces diversity rather than pigeonholes it, schools are likely to have better opportunities to encompass a more holistic approach to education that reaches more children".[4] Mrs. Garcia advocates for English Learners so they receive effective core instruction, all throughout the school day.

Emerging English Learners (ELs) within the mainstream classroom is only going to benefit them because they stay in their classroom environment. At times, I see EL students feel secluded, embarrassed, and isolated when they are pulled out and away from their peers. Major drawbacks from the pullout model that ESL/bilingual teachers have to work with multiple grade level students in the same group in order to accommodate students. This is not giving ELs best practice when it comes to their learning. ELs miss important core content and peer interaction with English speakers. This can lead them to falling behind across content areas, as well as not picking up English at a sufficient pace.[5] I support ELs, but I also work with all students so they receive maximized differentiated instruction. Mrs. Garcia has successfully grown the EL

population with this model at other schools she has lead. She intends to make a difference in ours at GVSA.

Unit Goals

My goal is for teachers to utilize this unit when working with English Learners for third grade multiplication. Through this unit, students will have plenty of writing experiences including the use of words, pictures, explanations and symbols to represent their thinking. I wrote the unit to steer away from memorizing facts, and focus on the conceptual knowledge of multiplication using manipulatives, drawings, words and symbols to assist their thinking through problem-based learning. Lessons give students opportunities for exploration, discussion and written expression. The unit integrates scaffolding techniques and strategies to best support English Learner students.

Through this unit, I share ways to support students in representing their mathematical thinking specifically when solving word problems that involve multiplication. As students write while problem solving, I believe they will be able to deepen ideas. This unit intends to help educators like myself with writing implementation within classrooms. Learning how to engage students with writing in mathematics is going to influence critical thinking skills needed for mathematical reasoning for all ranges of learners, specifically English Learners. It also assists students across content areas with improving their writing skills that will influence self-confidence. Acquiring new styles, techniques and specific purposes of writing from this seminar helps enhance teaching content standards, activities, and improve teaching practices to maximize instruction within the classroom. Writing looks different from student to student therefore; providing different ways of written expression will be beneficial.

The curriculum unit will impact classrooms in various ways. It will help teachers be strategic in planning lessons that reflect the math standards and mathematical practices, including ways to engage English Learners in mathematical discourse. The goal is to be intentional with integrating written explanations within math problems of all areas. I will also strive to meet the needs of all learners by including strategies to help teachers scaffold and differentiate the various writing lessons. The curriculum unit has teachers hold high expectations of all students within the classroom so they can meet their full potential.

The unit concentrates on vocabulary and categorizes content words based on a tier system. Vocabulary falls under three different tiers, depending on the level of complexity. Tier one words are high frequency words that are used on a daily basis. They are present in our everyday language. Tier two words are cross-curricular words that are less familiar, and are helpful when it comes to comprehending concepts. Tier three words are content-specific, and directly relate to concepts needed to learn. They are low-frequency words that are not used in everyday language.[6] When teaching new mathematical concepts, academic vocabulary will play a big role in writing, especially when it comes to power verbs such as demonstrating, analyzing, interpreting, illustrating, etc.

Not only do I want to reach students academically, but I also want to provide practical tools for teachers to help students develop self-confidence in their writing skills. I notice that

many of my students feel badly because they do not know what to write or why the words they have selected are incorrect. They are so afraid of failure, and I believe we need to change that as educators. I want to set my students up for success so that they can utilize their writing techniques in other content areas.

Content Research

Four Types of Writing

Writing in Mathematics seminar investigated how students communicate their thinking through writing in different ways. We researched different ideas on how students generate thoughts and ideas when learning new mathematical concepts, and how to assist students with written expression. We explored different styles of writing that students use when reasoning through mathematical ideas, and how to shift their explanations to a deeper, analytical stage of their thought process. It is important to learn about the different types so that the Curriculum Unit is effective. It is also necessary to consider the audience. Whom is the student writing for? Who will read their responses? This can influence students' mathematical writing.[7]

There are four types of mathematical writing, which support different purposes.[8] The first type of writing is quite common, known as Exploratory. It gives students opportunities to make sense of a problem, situation or a variety of ideas. Exploratory is "low stakes" writing since students are their own audience, and generally used during discussions and exploration.[9] Students use words, pictures, and symbols to represent thinking when working out a problem. Exploratory is appropriate when starting a new mathematical idea, brainstorm thoughts, or clear up unfamiliar ideas.[10] Generally, students serve as their "own audience" but may share their writing with classmates through collaboration and discussion.

The second type of writing is Informative/Explanatory writing. The goal is for students to describe and/or explain ideas that connect to concepts.[11] Students give information in regards to their mathematical reasoning. The teacher and classmates are often the audience for this type of writing. When writing to describe, students can clarify ideas through observations and representations, which will allow for deeper analysis of a mathematical idea. Students also explain the steps taken to solve a problem. It is imperative for students to use specific mathematical language and vocabulary. This will indicate student understanding on a concept. It will help teachers determine if students appropriately use vocabulary through conceptual understanding and/or application. When students use this writing type to explain, they specify strategies utilized when reasoning out a problem.[12]

The third type of writing is Argumentative. Students analyze or construct an argument, which must be clear and concise.[13] The goal is to convince the reader why he or she is correct and to get the reader to agree with the argument. The reader must understand the argument presented in order to determine if the claim is true or false. Students use argumentative writing when they explain how they know something is correct or incorrect.[14] Students may take a work sample by analyzing it and identifying the misconception, and review each other's work using examples or non-examples.[15] Argumentative writing will strengthen their own mathematical ideas by dissecting the problem itself and recognizing other thoughts.

The fourth type is Mathematically Creative writing. Students expand on their thinking by generating original ideas, problems and solutions. They broaden their thinking by looking at problems from multiple perspectives and ideas.[16] Students can be flexible in their thinking, and opens up opportunities for connecting ideas within mathematical concepts or in real life situations.

The table that explains a variety of forms of writing, as well as audience types. These ideas closely associate with the different types of writing and give students an opportunity to reason mathematically in interesting and creative ways.[17] Students engage in a variety of forms and audiences, which can enhance interest in writing in mathematics.

(Table modified from *Types of and Purposes for Elementary Mathematical Writing: Task Force Recommendations)*

Forms of Writing	Audiences
• Apps • Books created by students • Charts/graphs • Concept map • Diagrams • Drawings • Graphic Organizers • Essay • Informational texts • Journal entries • Jots on sticky notes • Lists • Multimedia presentations • Notes • Photos • Quick writes • Short answer responses • Sentence starters/stems • Tables • Word Problems	• Athletes • Book characters • Celebrities • Community members • Fictional characters • Neighbors • Older/younger peers • Other staff members at school • Parents • Professors at Colleges • Researchers • Relatives • Self

Teachers should take into consideration that modifications of writing types are appropriate when meeting the needs of English Learners.[18] Some examples include a word bank, pre-teaching vocabulary, preparing sentence starters, and providing visuals. We must meet learners where they are to offer support. A balance is necessary to set our students up for success. As students make progress, teachers adjust modifications to support independence. We need to honor the level of writing we receive at the beginning to build stamina and confidence, and celebrate the small improvements we notice over time to keep the momentum going of successful writing.

The Support of Classroom Discussions in Math

Classroom conversations are extremely important when it comes to collaboration. Students can interact in many ways with one another and their teachers through communication. It can be formal or informal, structured, argumentative, talking a group or with a partner. What matters is for teachers to use the right tools to support effective classroom talk, which will eventually strengthen writing. An example of tools such as this are called "talk moves".[19] These strategies support math talk and help with mathematical conversation and thinking.[20] There is implementation of talk moves within the curriculum unit because I believe mathematical conversations support successful writing. Teachers have choice to determine which talk moves are appropriate depending on the nature of the lesson and group of students.

As educators, we must be intentional when creating a culture where students can comfortably interact. Interaction is important because it sets students up for success with communication, gives them an opportunity to build on ideas, and helps reinforce a sense of community with their peers. Interaction can also help support writing because students can gather ideas from one another, which strengthens written expression. Our students must feel comfortable and safe with conversation, and feel valued when talking. Ground rules or norms must be set in place, emphasizing respect and courteous talk.[21] Discussion norms are valuable when the class creates them together, and placed on an anchor chart for the teacher to refer to on a day-to-day basis. Students are accountable for withholding these norms and expectations. If a student violates a norm, teachers must address the issue immediately. When thinking about equity among our students, teachers must hold high expectations so all learners can participate. By doing this, teachers should consider that every student is listening to the speaker, the listener hears what the speaker says, and all students have an opportunity to speak.[22] Keep in mind that conversations can and will support our weak writers. If there are rich academic conversations, we will see that come alive in written expression.

In the book, *Classroom Discussions: Using Math Talk to Help Students Learn,* Chapin et al. (2009) suggest five "talk moves" to help engage in conversation and discussion. Research shows that they help students make progress in engaging in mathematical thinking.[23] Talk moves are a great tool to use throughout any lesson delivered because it supports discussion in the classroom, and holds students accountable in an equitable manner. This is important for our English Learners who tend to be hesitant on sharing out in front of their peers. The talk moves help our ELs step out of their comfort zone.

The first talk move is "revoicing".[24] This is when the teacher restates what a student has said into question form to clarify thoughts and reasoning. Since the goal is to improve mathematical reasoning, it is important to tap into students who may be unclear in their explanation. Reasoning and analysis of a concept may not always relate to verbal expression, and we must allow students opportunities to clarify their thoughts.[25] This strategy is also a great way to make sure other students in the class actually understand an idea if it seems unclear. This is especially helpful for our English Learners because it can refine and clarify thoughts that may be confusing or unsettling.

The second talk move is "repeating".[26] Students restate a classmate's thought or idea discussed. There are many advantages to this talk move such as holding students accountable for listening, and allowing the class to hear the statement repeated. This move also supports our

English Learners if they are the speaker or listener. When this is consistent within the classroom, students will begin to recognize that others are paying attention to responses shared, which will motivate them to contribute with effort.[27]

The next talk move is "Reasoning".[28] Students get the opportunity to agree or disagree with one another's claim. Although this is a simple way to get students talking, it is still the teacher's responsibility to lead discussion with purpose and respect. Teachers can probe students with follow up questions to support the discussion in order to guide, clarify or connect one thought to another. The purpose of this move is to allow students to be precise in their own analysis of a concept by relating their thought process to someone else's reasoning.[29] ELs benefit from this talk move because they have to think through a variety of claims, and be explicit with their own opinions or thoughts. ELs use critical thinking skills as they may be translating their native language to English, which reflects on their personal experiences. Their peers can benefit from hearing different perspectives regarding a concept or topic.

The fourth talk move is "Adding on"[30] which is when students can extend a thought that another student's contribution. Students often struggle with this talk move at first because they will want to "add on" to what their partner says, but their reasoning may be unrelated to what was originally said. Teachers need to pay attention to student responses so that it is effective. Modeling is helpful so students can hear what an appropriate "add on" sounds like. When critiquing an academic conversation involving "adding on", it is important to be mindful when giving feedback. A strong classroom culture is crucial on how students react to criticism. When students feel safe sharing out thoughts, they will have the confidence to add meaningful comments. This move takes practice, but can be very effective over time.[31] To make this particular talk move accessible to English Learners, sentence starters will help support this conversation. It is important to hear their reasoning, and model how to connect their idea to another utilizing the sentence starter. It is helpful to have the student repeat the "add on" idea at first to scaffold appropriate discussion.

The final talk move is "Waiting"[32]. This strategy influences students to gather their thoughts. Thinking varies among people, and teachers need to respectfully honor thought process and reasoning. Research states that wait time is valuable within classroom discussion.[33] Wait time can be used when a teacher poses a question whole group, or even when he or she calls on a student. The teacher should wait at least five seconds for students, which takes patience. Even though educators feel pressed for time, many benefits will come out of wait time. Students engage in discussion and are less likely to give up. Since math can be challenging and complicated, giving students wait time to develop ideas is beneficial.[34] For English Learners, wait time is helpful because it may take them more time to digest the question and fully understand the context of it. A suggestion is to pair the EL student with a peer, and discuss what the question is asking for clarification or to hear it another time before the given wait time. Once the English Learner has confidence, the outcome of his or her response will be constructive.

With the five talk moves in mind, teachers need to expose students in different formats of communicating. Switch up interactions so that students engage in class discussion. Some examples are whole group discussion, small group discussion and partner talk. The formats support mathematical learning and allow talk moves to be successful. Students will be more involved and more likely to participate. Students get multiple opportunities to interact using

different approaches to speak when they are comfortable and uncomfortable. Participation gives them practice to increase their communication skills within discussion, specifically areas of weakness. Whole group discussion gives all students time and opportunity to contribute their participation, which at times may be apprehensive, especially with English Learners. If students are shy, talking in small groups or with a partner may help them feel more comfortable sharing within a large group setting.[35]

Teachers get to know their students, and adapt a mathematical learning environment to meet the needs of all learners. Promoting a positive classroom culture where it is safe to speak without feeling uncomfortable is the first step. Implementing consistent talk moves to encourage academic conversations within whole group, small group and partner talk will influence our students in a positive way, specifically their writing skills. When students can orally communicate, they will be more assertive when it comes to written expression.[36] One of the weaknesses of an English Learner tends to be writing. Through intentional academic conversations, writing scaffolds within the lesson, and intentional peer support as needed, ELs will excel. This unit will help English Learners learn mathematical concepts of multiplication and problem solving, through integration of different writing types, including helpful strategies and scaffolds that are easy to implement.

Talk Frames

Madelyn Willams and Tutita Casa are the authors of *Connecting Class Talk with Individual Student Writing*, and they share a strategy to help with mathematical writing called a "talk frame". According to the article, "The talk frame is an interactive graphic organizer that tracks the development of ideas on the board as students reason through a significant mathematical question".[37] It serves as a visual tool used to document classroom discussion. The purpose is to join conversation and student experiences as a whole group with individual writing.[38] Williams, one of the authors of *Connecting Class Talk with Individual Student Writing,* recommends using this talk frame tool to assess students as a class based on discussion participation, and then individually based on the students' written responses.[39] Since the talk frame is visible for the whole class to see, it creates a platform for writing if the student chooses to use it.

The talk frame works with a hands-on activity, so students can come up with ideas and concepts using manipulatives first. The teacher will revisit the question, and allow students to rephrase the question in their own words.[40] Based on their discoveries through hands-on experience, students will reason through various ideas a class, and the teacher will fill out the graphic organizer.[41] All ideas under the section "Talk Idea" come from students. The talk ideas form together as a summary under the "We Understand" section, using student words.[42] Based on the article, students were able to utilize the talk frame, but also expand using their own ideas. I plan to use this strategy within the unit to assist students with written expression. My goal is to help students strengthen writing using this as a scaffold, but also encourage original explanations. I anticipate students to learn from each other to create their own explanation to the question with help from the talk frame.

Supporting English Learners in Mathematical Discussion

There are many ways to engage our English Learners within discussion. Judit Moschkovich says that it is essential to focus on the mathematical content, how students contribute to discussion, and the different ways of collaborating through conversation based on diverse perspectives.[43] Although learning vocabulary and comprehension skills within mathematics is important, teachers should emphasize mathematical communication to enhance understanding.[44] Another perspective from Moschikovich is the focus in participation in oral and written practices, specifically explaining a variety of processes, reciting conjectures, demonstrating conclusions and opinions are required of our English Learners.[45] These skills are critical for ELs when learning mathematical content.

Within her research, Moschikovich uses "discourse" as a standpoint in how English Learners engage in mathematical discussion. She explains that, "Participating in classroom mathematical discourse practices can be understood in general as talking and acting in the ways that mathematically competent people talk and act when talking about mathematics in a classroom, and involves much more that the use of technical language".[46] Students should be using a variety of types of discourse including explanation, proving, making comparisons, and descriptions within math content discussions. The NCTM standards (1989) and various mathematical classroom research stated by Moschikovich suggest[47]:

- Model and engage discussion
- Support conjectures and explanations including evidence
- Focus on the process not only the product
- Compare ideas including explanations and solutions
- Encourage mathematical arguments
- Allow students to repeat each other's words through paraphrasing
- Encourage students to share various methods through activities

Another unit goal is to support discourse through meaningful dialogue considering the ideas above, in addition to language scaffolds and strategies. It is necessary to concentrate on the mathematical content when it comes to the discussion, and the ways listed above will assist it. Furthermore, using talk moves[48], talk frames[49], sentence stems, word banks, vocabulary, visuals, and facilitating instruction will be present. The objective is to build conversation so students can reinforce mathematical discourse within written expression, as well as learning the English language.

Instructional Implementation

Teaching Strategies

Charlotte Mecklenburg Schools' Learning Language and Acquisition Department provides resources for teachers across the district. They created the PACE Tool Kit, which is a collection of strategies to use with students throughout different parts of the lesson[50]. They are purposeful for all ranges of learners, specifically focusing on ELs (English Learners). Teaching strategies within this unit are intentional to help English Learners reach their anticipated potential. The goal is to use activities that are student centered and engaging. Within the lessons, English Learner strategies focus on the four domains of the language: reading, writing, speaking, and listening.

Various strategies learned throughout the seminar are specific to help all learners with written expression in mathematics. The following strategies are best practices to enhance learning, and support English Learners to succeed with mathematical writing.

1. **Sentence Starters**

Sentence starters are useful to assist ELs with all four language domains. Sentence starters scaffold mathematical discourse and ELs can practice speaking, reading and writing in complete sentences. Based on my experience in the classroom, when using sentence starters consistently, students are able to transfer complete sentences into their writing naturally when using them verbally. Sentence starters also assist students in writing complete sentences. They can help ELs communicate what they know about content or a concept. I plan to use sentence starters in all lessons due to the success I have using them during instruction.

2. **Word Banks**

Word banks assist English Learners with vocabulary words as options to use within their writing and speaking. It helps reinforce words used in class, and supports them as they are constructing written responses. Students need to be encouraged to use word banks when they are available. A suggestion is to post specific words on the classroom wall s students can continue to refer to them throughout the school year.

3. **Visuals**

Visuals are helpful for our English Learners because it builds their capacity to understand a concept. It encourages students to make a connection to what they see to what they read, hear, or learn. Research also states, "Using images (or visuals) can enhance student motivation to engage in speaking activities".[51] I also believe this supports writing based on observation in my own classroom. Visuals can support all students, as they naturally engage the learner through excitement and curiosity.

4. **Vocabulary**

Each lesson lists tiered vocabulary. Research suggests that teachers use a variety of vocabulary in mathematical language, and to explicitly teach if needed. It is important to be intentional with selecting appropriate words, and hold students accountable for using the words in their language with their peers during instruction. Students will be encouraged to use vocabulary in writing, so the more exposure of the words, the better.

Personal Academic Command of English (P.A.C.E.) Tool Kit Strategies

1. **Novel Ideas Only**

"Novel Ideas Only" is a strategy that comes from a training offered by Charlotte Mecklenburg Schools called Quality Teaching for English Learners, also known as QTEL. "Novel Ideas Only" prepares the learner for a particular concept or topic.[52] It takes place prior to the lesson that allows students to generate ideas they know about words or phrases, specifically the phrase "groups of things" within the unit. Before introducing multiplication, this will give the teacher a good idea of how they think of groups of objects, which transitions into the basics of multiplication. This encourages students to collaborate, speak, and listen to one another, which holds students accountable.[53]

2. **Four Voices**

"Four Voices" is another QTEL strategy that allows students to practice fluency when reading a word problem. Students get to focus on different parts of the word problem, and use discussion questions to guide their thinking before solving. Students have an opportunity to get deeper understanding of the context of the word problem, determine the action, and solve appropriately.[54]

3. **Create, Exchange, Assess**

"Create, Exchange, Assess" is a QTEL strategy that involves applying what students learned by creating a mathematical problem that another group tries to solve. The group who creates the problem assesses their classmates' solution. This strategy is great for English Learners because it focuses on all four language domains.

Lesson Framework

To design the lessons, I used the "Launch, Explore, Discuss" framework.[55] Using this framework last year, I noticed that my students began to take ownership of their learning, and I released responsibility more often than anticipated. I acted more as a facilitator, and was pleasantly surprised with the student generated ideas. Based on the success I had last year, I used this framework within the unit.

The purpose of this framework is to give students an opportunity to problem solve by exploring mathematical tasks with productive struggle.[56] The lesson begins by launching the topic in an engaging way by introducing the problem, then allowing students to problem solve on a concept they may not be familiar with, followed by discussion where direct instruction takes place based on their outcomes.

The "launch" section is a brief part of the lesson that draws the students in through engagement, building background through visuals, posing questions, or opportunities to have a quick low stake conversation to activate prior knowledge. The teacher communicates expectations by explaining required task.[57] Teachers provide necessary materials to students to complete the given assignment, and make sure they know what to do.

The "explore" part of the lesson is where students use problem-based learning through productive struggle.[58] Students collaborate with one another or complete a problem independently using manipulatives. Students construct a mathematical representation of how they solve the problem using words, pictures, and/or symbols. Teachers use this time to ask guiding questions as students engage with the task. Last year as a team, we came up with a variety of questions to ask during this section of the lesson. We created questions by predicting how students might solve the given problem. We considered misconceptions, in addition to possible strategies that students produce. We prepared ourselves with ways to address misconceptions in a manner that all possible solutions were valued and used in a meaningful manner. In this unit, intentional questions are present within the lessons to engage while students are exploring. They intend to support students when learning mathematical concepts and strategies. When focusing on English learners, this will support their learning as they are exploring challenging tasks.

The "discuss" portion of the lesson is where students shared out various strategies, representations and thought processes of how they solved the problem. It is student-led, while the teacher facilitates the discussion.[59] Based on experience last year, I was strategic in selecting the strategies shared from students. I selected students with the least efficient way to share first, and we worked our way to the most efficient strategy. Students reveal their mathematical representations with the class; explain verbally how they solved the problem. If there was a strategy not shared, I introduced it at the end of the lesson. Options for additional activities or small group would follow to extend learning.

I feel strongly that English Learners benefit from this framework. It pushes them to work with their peers, gives them responsibility to take ownership of their learning, and it's challenging. In addition, students have sufficient time to digest content. This is particularly important for ELs since many of them may be translating ideas in their minds, and students can ask for clarification if they initially do not understand something. The outcome is positive as productive struggle helps develop conceptual knowledge of mathematical understanding.[60]

Unit Design

The unit is designed for five consecutive days beginning with a pre-test, followed by 3 lessons, and ending with a post-test.

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Pre-Test	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Word Problem Detective Day	Post-Test

Assessment

When implementing the unit, the first piece to take note is the assessment. I use a pre- and post-test to assess students on their understanding. Students are assessed on the content standards of multiplication, but specifically on creative and explanatory writing. The pre/post-test gives serves as an assessment to show progress and growth from the beginning of the unit. It also provides data to see growth specifically in explanatory and creative writing. The goal is to see student progress based on lessons taught in the unit that includes hands-on activities and several writing opportunities. There are scoring instructions included on the pre/post-test document.

Looking at the assessment before teaching the unit will allow teachers to see student expectations, such as what students need to know and do. This will help drive instruction appropriately. In The Big Ideas of UbB *(Understanding by Design),* Grant P. Wiggins and Jay McTighe state, "The most successful teaching begins, therefore, with clarity about desired learning outcomes *and* about the evidence that will show that learning has occurred". [61] Analyzing the assessment and completing an exemplar gives teachers the student perspective on the "desired outcome". What strategies are best to meet student needs? What type of writing is

appropriate? What kind of visual representations are applicable? These are all valid questions when thinking about instructional delivery to meet learning outcomes.

Each lesson has some type of formative assessment that serves as an exit ticket. The exit tickets provide valuable information based on student progress. The purpose is for the teacher to use that data to drive small group instruction if applicable. This would be a great opportunity to re-teach or guide students into a deeper dive into the content, or get feedback on writing. The teacher can also enrich learning by providing students opportunities to practice.

Math Word Wall

A math word wall supports students with vocabulary. The unit suggests having one posted as new vocabulary words are taught to students. It involves students because they can draw a visual representation for each word to honor student thinking. Each lesson has specific directions on which words to teach, and suggestions for students to get involved.

Languages Targets

I included Language Targets for each lesson so it is specific to what English Learners need to accomplish with language. It helps teachers know the expectation of the lesson from a language perspective. Charlotte Mecklenburg EL Services suggest selecting a language function, a content stem, and at least one support when creating language targets. Language targets relate to the four domains of the language, but also include verbs for student expectations for a specific lesson using supports to help them accomplish the goal.

Activities

1. **Creative Moment**

   At the end of each lesson, students get a "Creative Moment" to generate additional ideas based on the lesson activities. There is no criteria for this activity; however, the only expectation is for students to participate. It is a time where students have an opportunity to be creative in their mathematical writing. It can be new ideas, opinions, reflections, or thoughts in a student's home language. It is chance for students to express themselves based on what they have learned through written expression.

2. **Mathematical Writing Choice board**

   The mathematical choice board serves as an activity for students to complete during workstations. Each activity is challenging, and gets students to reason and write through various types of writing. Even if a student struggles with writing, the choice board provides the teacher valuable information about where a student is, and where a student needs to go. Students will select at least three activities to complete in order to get credit. Each activity has sentence starters or a word bank to support. It offers students an alternative activity to complete while the teacher can pull a small group, homework, or however the teacher choses to use it.

Lessons

Lesson 1

Essential Question: What objects come in groups?

Brief Lesson Description: This lesson is designed to spark the interest of students relating to the foundations of multiplication. Its purpose is to activate prior knowledge about groups of objects, and the number of groups. Lesson 2 will get into conceptual knowledge of multiplication.

Prerequisite Skills:

- Repeated addition
- Concept of building arrays, and representing them based on rows and columns

Language Targets:

- Students will identify objects in several groups through collaborative group work and activities.

North Carolina State Standards: (See Appendix I for a description of the standard)

- NC.3.OA.1
- NC.3.OA.3

Vocabulary:

- **Tier 2:** equal groups, groups of objects

Materials	
Copies I Need: (highlighted throughout lesson)	**Other:**
● Lesson 1 Launch: Novel Ideas Only Prompt ● Lesson 1 Discuss: Exit Ticket	● index cards ● Lesson 1 Launch: Groups of Objects Visual ● chart paper ● markers ● math journals

Discussion Talk Moves:

Which talk moves will you use throughout this lesson?

- ❑ Revoicing
- ❑ Repeating
- ❑ Reasoning
- ❑ Adding on
- ❑ Waiting

Step-by-Step Process of the Lesson

Step 1: Launch (This portion of the lesson is longer than a typical "launch". The modifications target English Learners)

Task: Novel Ideas Only (*Adapted from PACE Tool Kit)*

1. Pass out Lesson 1: Launch Novel Ideas Only Prompt for each student.
2. Students may work in pairs or a group.
3. Explain to students that they will write a word or phrase that reminds them of the phrase, "groups of objects/things" with their group.
4. Each person will take a turn by reading the sentence frame with his or her idea. For example, if the idea is a pack of cards, the student will say, "Groups of objects/things remind me of a pack of cards" Once a person shares, everyone will verbally repeat the idea, and then write it down.
5. After three minutes of the group member sharing, draw a line under the last item in the list.
6. Everyone stands, and each group needs a spokesperson.
7. First group- The spokesperson reads the prompt, and whatever ideas the team has written down.
8. While spokesperson reads, the rest of the class…
    a. Listens and marks off any ideas that are the same to their group's ideas.
    b. If the group presents a novel (new) idea, you add it under the line drawn on your paper.
9. The next group's spokesperson will only share out "novel ideas only" to the class. Ideas already shared by other groups cannot be repeated. The rest of the class continue to…
    a. Listen and mark off
    b. If everything is marked off, the whole group sits, continuing to listen for Novel Ideas Only.

Step 2: Explore- What are different things that come in groups? (*Adapted from NC Tools for Teachers)*

1. Pass out an index card to each student with a number on it. It must have the numbers 2, 3,4,5,6,7,8,9 or 10. Once students receive the card, they must go find their group or partner.
2. As a team, the group must list out as many things as they can that come in groups of their number they were given on the index card For example, if one group received a card with 12 on it, they could say a dozen donuts, or a dozen bagels. 12 is an example so students have to do the thinking on their own with the other numbers.
3. Students will list their ideas on chart paper, which will be around the room when finished.
4. Suggest the following ideas to support struggling groups: things in nature, packages of food or snacks, sports teams and scoring, money, and geometric shapes.

Guiding questions as students are working:

- What things do we use in the classroom that come in groups?
- When are people found in equal groups? (Sports teams, multiple births, rollercoaster seats, car seats, etc.)
- What body parts can be found in groups?
- What kinds of things come in packages?

5. Allow each group to present their ideas with the whole group. Use sentence stems to support students:
   - Things that come in groups of _____ are...
   - As we brainstormed together, we thought of...
   - _____come in equal groups of_____

Step 3: Discuss

1. Pose the following questions, and allow students to discuss as a group:
   - Which groups do you think were the easiest to find ideas? Why?
   - Which groups do you think were the most difficult? Why?
2. Allow students to share with the whole group their thoughts. Allow students to explain their thinking on both questions.
3. Lesson 1 Discuss: Exit Ticket- Allow students to select one of the writing prompts below. This is in the form of an exit ticket. Encourage students to use the sentence stems as well as the word bank.

   Writing Prompt:

   Which groups do you think were the easiest to find ideas? Explain your thinking, and provide an example using words and pictures.

   - I think groups of _____ were the easiest to find ideas because…
   - For example…
   - My picture shows…

   Which groups do you think were the most difficult? Explain your thinking, and provide an example using words and pictures.

   - I think groups of _____ were the most difficult to find ideas because…
   - For example…
   - My picture shows…

   Word bank to support students: groups, items, objects, common, frequent, often, rare, uncommon, infrequent, unusual

   Assessment:

   Use Lesson 1 Discuss: Exit Ticket to assess written expression. Use this to drive small group instruction as needed.

Step 4: Closing

1. **Creative Moment**: Give students 3-5 minutes to write in their math journals using the following prompt:

   "Write about other ideas you have based off today's activities. You can use words and/or pictures."

2. End the lesson with a closing of the topic. Tell students will be discovering different ways to count multiple groups of things. Review the following vocabulary words, and put them up on your Mathematic word wall: **groups of objects, equal groups.** Select a student draw a visual to post with the words. (Example: two dozen doughnuts that would feed the class)

Lesson 2

Essential Question: How can I find the total number of objects in a group?

Brief Lesson Description: Students will build their knowledge of the foundation of multiplication by exploring with different number objects in groups using manipulatives. Students will practice mathematical writing through various activities.

Prerequisite Skills:

- Repeated addition
- Concept of building arrays, and representing them based on rows and columns
- How to represent arrays using addition equations

Language Targets:

- Students will calculate the total number of objects in several groups using repeated addition, skip counting and/or visual representations.
- Students will construct an array that represents a multiplication problem using snap cubes.

North Carolina State Standards: (See Appendix I for a description of the standard)

- NC.3.OA.1
- NC.3.OA.3

Vocabulary: (Consistently refer to the vocabulary word wall throughout the lesson.)

- **Tier 2:** columns, rows, total, strategy, equal groups, groups of objects, pair
- **Tier 3:** multiplication, factor, product

Materials	
Copies I Need:(highlighted throughout lesson)	**Other:**
• Lesson 2 Launch: Writing Prompt • Lesson 2 Explore: Word Problem • Lesson 2 Discuss: Sentence Frame Sheet • Lesson 2 Discuss: Exit Ticket	• Snap cubes • 1 egg, 1 stick of gum, 1 playing card from a deck, 1 shoe, and 1 glove or mitten for each cooperative group • Math journals

Discussion Talk Moves:

Which talk moves will you use throughout this lesson?

- ❑ Revoicing
- ❑ Repeating
- ❑ Reasoning
- ❑ Adding on
- ❑ Waiting

Step-by-Step Process of the Lesson

Step 1: Launch- What do these things have in common? *(Adapted from NC Tools For Teachers)*

1. Bring in the following items for students to observe: 1 egg, 1 stick of gum, 1 playing card from a deck, 1 shoe, and 1 glove or mitten. Provide each group with the materials so this can be a hands-on activity.

   Ask: What these items have in common? Give them time to think without answering so that each student has time to consider how the items are alike. Students should have prior knowledge from the previous lesson.

2. Have students turn and talk with a partner to share what they noticed.
3. Allow students to respond until someone suggests the idea that these items are found in groups. Eggs are normally sold by the dozen, gum in packs, shoes in pairs, and gloves in pairs.
4. Have students respond to Lesson 2 Launch: Writing Prompt below. Have students use their academic word wall words from yesterday. **(groups of objects, equal groups)** See Appendix II for writing prompt sheet for students.

   What do these items have in common? How do you know?

   - These items have in common…
   - The items are similar because...
   - I know because…

Step 2: Explore- Multiplication Word Problem

1. Present the following word multiplication problem to students as a whole group.

   Ask:

   - What do you know about the word problem?
   - What are you asked to find out?
2. Students may work in pairs, or independently. Allow students to use snap cubes to solve the problem. Students complete this task on the Lesson 2 Explore: Word problem sheet.

Word Problem:

*Daniel was out shopping for his family. He wanted to get each family member a **pair** of socks. He has 8 people in his family. How many **total** socks did he purchase at the store? Use snap cubes to solve the problem. Show your work.*

Guiding questions as students are working:

- What is Daniel doing?
- How many is in a pair?
- How many family members is he shopping for?
- How can you represent the socks using snap cubes?
- How many rows do you have? How many columns?
- How can you represent the socks in a picture?
- What is an equation that can represent your work?

Step 3: Discuss *(Adapted from NC Tools for Teachers)*

1. Gather students back together as a class. Allow students to share their strategies with one another. Encourage students to model how they solved the problem. Be intentional with the strategies you select to be shared, and order them from least efficient to highest efficient if possible. Students can see how strategies build on one another.
2. Introduce the Lesson 2 Discuss: Sentence Frame Sheet to students.

Draw or trace **8 pairs** of socks then write the following sentence frames on the board:

Here are 8 <u>pairs of socks</u>.

There are 2 <u>socks</u> in each <u>pair.</u>

There are _____ socks altogether.

3. Give students a copy of Lesson 2 Discuss: Sentence Frame Sheet. Have them draw a picture on the space above the sentence frame. Then have them show their work for solving the problem.
4. Ask some of them to share their solution and their strategy for solving. Most will solve by skip counting or adding. If no one suggests counting each sock separately, ask them if that would be a useful strategy. Ask them if they could get the correct solution by counting each sock.

5. Ask:
    - How could these pictures be represented by each of the strategies?

      (Write the equation for counting each finger separately (1 + 1 + 1 + 1… = 16) then write the equation 2 + 2 + 2 + 2 + 2+ 2+ 2+ 2= 16.)

    - How these equations are alike, and how they are different?
    - Do they represent the same problem?
6. Ask:
    - How many 2's do you see in the equation?
    - Where do you find that number of groups in the picture and in the sentence frame?
    - Which representation helps us think of the picture as groups of things?
    - Why we might want to think in terms of groups of socks, rather than each separate sock?
7. Ask:
    - How could these pictures be represented with skip counting?

    Write 2, 4, 6, 8, 10, 12, 14, 16. Ask students how many times they skip counted to find the answer.

    Ask:

    - Where is this represented in the picture and the sentence frames?
    - How could we show this on a hundreds board?
    - How is the skip counting strategy related to the addition strategy?

    Students should recognize that both strategies involve counting 8 groups of 2.

8. Tell them that we can represent 8 groups of 2 as 8 x 2 = 16. Write this equation under the other representations on the sentence stem sheet. Ask students to explain to a partner where the number of groups is represented in the equation, and then the number in each group. Point out that there are 8 groups of socks, with 2 socks in each pair, and 16 represents the total number of socks. 8 and 2 are called factors, and 16 represents the total, called the product.
9. Students will complete Lesson 2 Discuss: Exit Ticket.

Assessment:

Lesson 2 Discuss: Exit Ticket will serve as the assessment. Use the data to inform instruction for small group instruction. The section of the exit ticket allows students to create their own multiplication word problem. Since they have not had direct instruction or exposure, they might struggle in this area. It will provide valuable information on where they are at as creative mathematical writers, which can help with lesson 4.

Step 4: Closing

1. **Creative Moment**: Give students 3-5 minutes to write in their math journals using the following prompt:

   "Write about other ideas you have based off today's activities. You can use words and/or pictures."
2. End the lesson with a closing of the topic. Tell students we will continue to dive into multiplication situations, and doing more work with creative writing. Review the following vocabulary words, and put them up on your Mathematic word wall: **factor, product, total, and pair.** Select a student draw a visual to post with the words. (Example: 4 lady bugs with 6 legs equal 24 legs, pair of socks)

Lesson 3

Essential Question: How can I find the total number of objects in a group?

Brief Lesson Description: Students will build their knowledge of the foundation of multiplication by exploring with different number of objects in groups using manipulatives relating to a given word problem.

Prerequisite Skills:

- Repeated addition
- Concept of building arrays, and representing them based on rows and columns
- How to represent arrays using addition equations

Language Targets:

- Students will calculate the total number of objects in several groups using repeated addition, skip counting and/or visual representations.
- Students will deconstruct a word problem by using graphic organizers to support their reasoning.

North Carolina State Standards: (See Appendix I for a description of the standard)

- NC.3.OA.1
- NC.3.OA.3

Vocabulary: (Consistently refer to the vocabulary word wall throughout the lesson.)

- **Tier 2:** columns, rows, total, strategy, equal groups, groups of objects, pair
- **Tier 3:** multiplication, factor, product

Materials	
Copies I Need: (highlighted throughout lesson)	**Other:**
● Lesson 3 Explore: Word Problem ● Lesson 3 Explore: Gem Graphic Organizer	● Lesson 3 Launch: What Do You See? ● Lesson 3 Discuss: Talk Frame Slide ● Whiteboards ● Dry erase markers ● Snap cubes ● Highlighter ● math journal

Discussion Talk Moves:

Which talk moves will you use throughout this lesson?

- ❑ Revoicing
- ❑ Repeating
- ❑ Reasoning
- ❑ Adding on
- ❑ Waiting

Step-by-Step Process of the Lesson

Step 1: Launch- What Do You See? *(Adapted from NC Tools for Teachers)*

1. Display the images in "Lesson 3 Launch: What Do You See?" one at a time.
2. For each image, show the image for 2 seconds then remove it.
3. Tell students to draw a picture of what they think they saw on a whiteboard with a dry erase marker.
4. Show the image again for 5 seconds and have students edit their drawing.
5. Ask students to describe the picture with others at their table.
6. Bring the class together and ask:

    - How many dots do you see?
    - How did you find the number of dots?
    - Did anyone see groups of dots?
    - What are some equations we can write for this picture?

Step 2: Explore- Four Voices as Word Problem Detectives *(Adapted from PACE Tool Kit)*

1. Students in groups of 4 will read the word problem that have differentiated sections with plain text, bold font, italics, and underlining. Use the Lesson 3 Explore: Word Problem student worksheet.
2. Students will have a set of questions to think about while reading the word problem.
3. Direct students to read their assigned parts in their small groups.
4. Have them highlight their part they are responsible for reading.
5. Allow time for students to discuss the questions after they complete reading.
6. Student will complete the word problem using the Lesson 3 Explore: Gem Graphic Organizer.
7. Allow students to use snap cubes to solve the problem.
8. They will complete 4 sections with their group. They will rewrite the word problem in the center, write the question on the top, write an equation with the unknown on the left, and provide a visual representation to the right.
9. Do not have them explain their findings on the bottom part. This will be done during the discussion.

Step 3: Discuss- Talk Frame *(Adapted from Connecting Class Talk with Individual Student Writing)*

The talk frame assists with discussion so students can explain how they solved the problem. You can use the slide provided, or create one on the whiteboard. Make sure the talk frame is visible to all students.

1. Show students Lesson 3 Discuss: Talk Frame Slide.
2. Present the word problem, and have them paraphrase the question. Write it in the "Think!" box.
3. Focus on the question, and write down strategies in words presented by students whether they are correct or incorrect. Write strategies under the "Talk Idea" section of the slide.
4. Once ideas and strategies are discussed, summarize student ideas in the "We Understand!" section.
5. In the bottom part of the Gem Graphic Organizer, have students explain how they solved the problem in their own words. They can refer to the talk frame for help. They complete this activity independently.
6. Post the following sentence starters for help:
    - I solved the problem by…
    - First… Then…
    - The strategy I used was…
    - I drew a picture of…
    - This helped me because...

    Assessment:

    Use Lesson 3 Discuss: Gem Graphic Organizer as their exit ticket. Specifically focus on the bottom portion, which is their independent explanatory writing piece. This will give valuable information on how the Talk Frame supports student written expression.

Step 4: Closing

1. **Creative Moment**: Give students 3-5 minutes to write in their math journals using the following prompt:

   "Write about other ideas you have based off today's activities. You can use words and/or pictures."
2. Wrap up the lesson by shouting out groups of students for excellent collaborative work. Explain to students that the next lesson will focus on creative writing with their own word problem. Tell them to start thinking of multiplication situations that they experience in everyday life, and be prepared to use share/use these ideas in the next lesson. Review the following vocabulary words, and put it up on your Mathematic word wall: **multiplication, strategy.** Select a few Gem Graphic Organizers from the lesson to post on the wall next to the word. Select ones that use different strategies.

Lesson 4

Essential Question: How can I find the total number of objects in a group?

Brief Lesson Description: Students will build their knowledge of the foundation of multiplication by creating a multiplication word problem with a partner.

Prerequisite Skills:

- Repeated addition
- Concept of building arrays, and representing them based on rows and columns
- How to represent arrays using addition equations
- Conceptual understanding of the foundation of multiplication
- Concept of arrays: rows and columns
- How to analyze a word problem

Language Targets:

- Students will calculate the total number of objects in several groups using repeated addition, skip counting and/or visual representations.
- Students will design a word problem using the Gem Graphic Organizer

North Carolina State Standards: (See Appendix I for a description of the standard)

- NC.3.OA.1
- NC.3.OA.3

Vocabulary: (Consistently refer to the vocabulary word wall throughout the lesson.)

- **Tier 2:** columns, rows, total, strategy, equal groups, groups of objects, pair
- **Tier 3:** multiplication, factor, product

Materials	
Copies I Need: (highlighted throughout lesson)	**Other:**
● Lesson 4 Explore: Word Problem Graphic Organizer ● Lesson 4 Explore: Gem Graphic Organizer	● sticky notes ● anchor chart paper ● markers ● snap cubes ● math journals

Discussion Talk Moves:

Which talk moves will you use throughout this lesson?

- ❑ Revoicing
- ❑ Repeating
- ❑ Reasoning
- ❑ Adding on
- ❑ Waiting

Step-by-Step Process of the Lesson

Step 1: Launch-What are real life examples of multiplication? *(Adapted from NC Tools for Teachers)*

1. Tell students to brainstorm ideas of how multiplication is used in real life. Give them plenty of time to come up with ideas. Have them jot down ideas on a sticky note.
2. Students will share out ideas in a "Mix and Mingle" format. Play music, and have students walk around with their sticky note. When the music stops, have them find a partner or group to share ideas. Start the music, and repeat. Students are encouraged to find new group members to share.

   Possible ideas: total legs of dogs at the dog park, buying packs of 6 cookies for a class size of 20

3. Gather students back as a whole group, and generate a list of ideas on an anchor chart. Keep this visible throughout the lesson to support ideas for the word problem they will create in the next part of the lesson.

4. Explain to students that they will be creating their very own multiplication word problem. Students can work in pairs or independently.
5. Give students Lesson 4 Explore: Word Problem Graphic Organizer. Explain they will use this to help organize their thoughts for the word problem they will create.

Step 2: Explore- Create, Exchange, Assess *(Adapted from the PACE Tool Kit)*

1. Students will work independently or with a pair on their graphic organizers. Encourage students to use the class made anchor chart for ideas. Allow students to use snap cubes to work out their problems.

   Ask:

   - Who is in your story problem?
   - What is your story problem about?
   - Where does your story take place?
   - What objects are involved?
   - How many groups of objects are there?
   - How many objects in each group?
   - How many total objects?
   - How can you represent your problem using an equation with the unknown?

Step 3: Discuss- Create, Exchange, Assess continued...

1. Place students into groups, and allow students to share with each other. Students can use this time to give each other feedback on their problems. Students may make edits to their work to make it stronger.

   Post the following sentence starters to help guide discussion:

Glows	Grows
• I like how you created… • The best part of your work is… • I enjoyed_____ because… • This part is creative because...	• One suggestion would be… • I think you should add… • I strongly suggest… • Can you clarify...?

2. Gather as a whole group, and have students may share out their ideas with peer suggestions.

   Ask:

   - What worked with the peer edit time?
   - How was this process helpful?
   - What were some ah-ha moments to help your word problem improve?
   - How was the graphic organizer helpful?

3. Students will use their word problem to fill out each component of Lesson 4 Discuss: Gem Graphic Organizer. This will serve as their exemplar/answer key for the "Word Problem Detective Day" that will occur the next day.

Assessment:

Collect the Gem graphic organizers as an exit ticket to assess on how well students understood their own word problem, and the steps to solve. Pay attention to their explanation to assess their written expression. Use data to for small group instruction

Step 4: Closing

1. **Creative Moment**: Give students 3-5 minutes to write in their math journals using the following prompt:
   "Write about other ideas you have based off today's activities. You can use words and/or pictures."
2. Share with students that mathematical writing has been a focus during this unit and compliment their hard work. There will be an opportunity during "Word Problem Detective Day" to try out the different word problems the class has created for more practice. Review the math word wall as a closing activity.

Word Problem Detective Day

Essential Question: How can I find the total number of objects in a group?

Brief Activity Description: Students will build their knowledge of the foundation of multiplication by solving multiplication word problems created by their peers.

Prerequisite Skills:

- Repeated addition
- Concept of building arrays, and representing them based on rows and columns
- How to represent arrays using addition equations
- Conceptual understanding of the foundation of multiplication
- Concept of arrays: rows and columns
- How to analyze a word problem

Language Targets:

- Students will calculate the total number of objects in several groups using repeated addition, skip counting and/or visual representations.

North Carolina State Standards: (See Appendix I for a description of the standard)

- NC.3.OA.1
- NC.3.OA.3

Vocabulary: (Consistently refer to the vocabulary word wall throughout the lesson.)

- **Tier 2:** columns, rows, total, strategy, equal groups, groups of objects, pair
- **Tier 3:** multiplication, factor, product

Materials	
Copies I Need: (highlighted in lesson)	**Other:**
• Lesson 4 Explore: Gem Graphic Organizer (copied front/back)	• markers • construction paper

Step-by-Step Process of the Activity

1. Have students rewrite their word problems from lesson 4 on a half sheet of construction paper.
2. Group students with a partner or pair.
3. Students will exchange problems on the construction paper, and solve them by using Lesson 4 Discuss: Gem Graphic Organizer.
4. The other groups will grade the work according to their exemplar created from lesson 4.
5. Repeat activity with a different group.
6. Students will have 2 more opportunities to practice solve word problems while applying their mathematical thinking through written expression.

Closing

1. **Creative Moment**: Give students 3-5 minutes to write in their math journals using the following prompt:

   "Write about other ideas you have based off today's activities. You can use words and/or pictures.

Appendix 1: Teaching Standards

North Carolina Course of Study 3rd Grade Standards

Represent and solve problems involving multiplication and division.

NC.3.OA.1 For products of whole numbers with two factors up to and including 10:

• Interpret the factors as representing the number of equal groups and the number of objects in each group.

• Illustrate and explain strategies including arrays, repeated addition, decomposing a factor, and applying the commutative and associative properties.

NC.3.OA.3 Represent, interpret, and solve one-step problems involving multiplication and division.

• Solve multiplication word problems with factors up to and including 10. Represent the problem using arrays, pictures, and/or equations with a symbol for the unknown number to represent the problem.

• Solve division word problems with a divisor and quotient up to and including 10. Represent the problem using arrays, pictures, repeated subtraction and/or equations with a symbol for the unknown number to represent the problem.

Appendix 2: Lesson 1 Materials

Lesson 1

Lesson 1 Launch: Groups of Objects Visual

Lesson 1 Launch: Novel Ideas Only Prompt

Lesson 1 Discuss: Exit Ticket

Appendix 3: Lesson 2 Materials

Lesson 2 Launch: Writing Prompt

Lesson 2 Explore: Word Problem

Lesson 2 Discuss: Sentence Frame

Lesson 2 Discuss: Exit Ticket

Appendix 4: Lesson 3 Materials

Lesson 3 Launch: What Do You See?

Lesson 3 Explore: Word Problem

Lesson 3 Explore: Gem Graphic Organizer

Lesson 3 Discuss: Talk Frame Slide

Appendix 5: Lesson 4 Materials

Lesson 4 Explore: Word Problem Graphic Organizer

Lesson 4 Explore: Gem Graphic Organizer

Appendix 6: Additional Materials

Formal Assessment

Pre/Post Test

"Word Problem Detective Day"

Lesson 4 Explore: Gem Graphic Organizer

Choice Board

Mathematical Writing Choice Board

Student Resources

Wibbly Wobbily Pobble Arrays!

- This website is a fun and engaging game for students to practice multiplication.

"Each Orange Has 8 Slices" Author: Paul Giganti, Illustrator: Donald Crews

- This is a great book for students to read with a partner. The Sentence Stem worksheet in Lesson 2 works well with this book. It relates to multiplication by incorporating real world application.

Multiplication Thinking Blocks

- Thinking blocks are a great way to break down word problems visually. Students build a model based on a given word problem. This website serves as a great station activity to students.

Teacher Resources

Novel Ideas Only Video

Four Voices Video

Teachers can use the videos above to see a model of both strategies in the unit. The videos serve as an example how to implement this activity with students but in different subject areas.

Talk Frame Example

The Talk Frame comes from a very helpful article that explains how to use it in depth. The authors use this talk from in a first grade class, focusing on Symmetry.

PACE Tool Kit

The PACE Tool Kit is provided by the Learning and Language Acquisition Department of Charlotte Mecklenburg Schools. This resource contains best practices to engage an English Learner. The strategies used in this unit come from this tool kit, and have additional strategies that are easy to implement in the classroom.

Language Target Guide

This resource is helpful for teachers to design language targets within lessons. It provides the structure of the language targets, and several examples of language functions, content stems, and supports.

Endnotes

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[4] Margarita Calderon, *Teaching Reading & Comprehension to English Learners K-5* (Bloomington, IN: Solution Tree Press, 2011), 9.
[5] Calderon, *Teaching Reading & Comprehension to English Learners K-5,* 13.
[6] Lazel, "PreK-5 Common Core Academic Vocabulary-Learning A." (2017).
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- The book listed above is a great resource because it provides English Learner teachers and content teachers suggestions of meeting the needs of ELs for reading and comprehension. It provides strategies and instructional practices. The book goes in depth breaking literacy apart, and explains how ELs learn best based on research.

Casa, T. M., J. M. Firmender, J. Cahill, F. Cardetti, J. M. Choppin, J. Cohen, and R. Zawodniak. "Types of and purposes for elementary mathematical writing: Task force recommendations." (2016). Retrieved from http://Mathwriting.education.uconn.edu

- This resource explains the four different types of writing used in mathematics, including purposes and audiences. It is helpful for teachers because it gives ideas how to implement writing in the math classroom, and provides examples of student work.

Chapin, Suzanne H., Catherine O'Connor, Mary Catherine O'Connor, and Nancy Canavan Anderson. *Classroom discussions: Using math talk to help students learn, Grades K-6*. Math Solutions, 2009.

- The authors of this book break down "talk moves" and provides examples of what it looks like in the classroom. The different talk moves are thoroughly explained, and gives teachers tools to promote classroom talk.

Common Core State Standards Initiative. Common Core State Standards for Mathematics. Washington, DC: NGA Center and ImCCSSO, 2010. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.

- The website lists the Common Core State Standards, Mathematical Practices, and provides examples of mathematical situations relating to the standards. It explains each standard for each topic and grade level. It teaches the reader how to read and interpret each standard.

Ferlazzo, Larry, and Katie Hull Sypnieski. *The ELL Teacher's Toolbox: Hundreds of Practical Ideas to Support Your Students*. John Wiley & Sons, 2018.

- This resource provides helpful English Learner strategies. All teachers can benefit from this resource because the strategies are practical and engaging.

Firmender, Janine M., Tutita M. Casa, and Madelynn W. Colonnese. "Write On." *Teaching Children Mathematics* 24, no. 2 (2017): 84. doi:10.5951/teacchilmath.24.2.0084.

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Lazel. “PreK-5 Common Core Academic Vocabulary - Learning A.” Learning A-Z.Lazel Inc., June 1, 2017. https://www.learninga-z.com/site/what-we-do/standards/common-core/academic-vocabulary#AcademicVocab.

- This quick article breaks down tiered vocabulary, and the importance of it in the classroom. It is helpful for teachers because it shows how to categorize words using the tier system.

Moschkovich, Judit. "Supporting the participation of English language learners in mathematical discussions." *For the learning of mathematics* 19, no. 1 (1999): 11-19.

- This article specifically targets English Learners. It suggests ways to support mathematical discussion. Moschkovich provides examples from real classrooms, students and teachers and shares interesting research.

North Carolina Department of Instruction (2018) 3rd Grade Unpacking Document. Retrieved from http://www.ncpublicschools.org/docs/curriculum/mathematics/scos/current/3rd-unpacking.pdf

- The unpacking document breaks down 3rd grade math standards by unpacking each standard. Teachers use this document to focus on coherence of the standard, pacing, and look at examples to determine what students need to know and do.

NC2ML (2017). Launch-Explore-Discuss Lesson Framework. Research and Practice Briefs. North Carolina Collaborative for Mathematics Learning. Greensboro, NC. Retrieved from http://nc2ml.org/ .

- This article explains the Launch, Explore, Discuss Framework, and how teachers can use it for instructional design and lessons. It explains the purpose of the framework and the benefits.

"Third Grade." Tools 4 NC Teachers | Math Science Partnership Grant Website. Retrieved from https://tools4ncteachers.com/third-grade/.

- This website has lessons created for teachers using the Launch, Explore, Discuss model. They are directly aligned with North Carolina Course of Study Standards for third grade.

Wiggins, Grant P., and Jay McTighe. *The understanding by design guide to creating high-quality units*. ASCD, 2011.

- The big ideas of UbD suggests ways to construct an effective unit. It explains how to work backwards when creating a unit, starting with the assessment.

Williams, Madelyn M., and Tutita M. Casa. "Connecting class talk with individual student writing." *Teaching Children's Mathematics* 18, no. 5 (2012): 314-321.

- This article explains the “talk frame” in depth, and shares research behind it in a first grade classroom. Readers understand the purpose of the “talk frame”, and learn how to implement it in the classroom. This article is highly recommended because teachers can benefit from the example, and try it in their own classroom.