Wealth Inequality

by Jeffrey Barnes, 2017 CTI Fellow
Rocky River High School

This curriculum unit is recommended for:
High School Math III 10th-12th Grade

Keywords: Wealth, Income, Asset, Inequality, Appreciate, exponential functions, compound interest, base, growth/decay factor, transformation(s), rate, principal, increasing/decreasing intervals, x-intercept, y-intercept

Teaching Standards: See Appendix 1 for teaching standards addressed in this unit.

Synopsis: "We get government aid spend it at they stores, puttin' they kids through college We need balance, so we can lease and own Ds in our projects. So I'm askin' Gs to go in their pockets, the racial economic inequality, let's try to solve it." The following lyrics were put forth by the rapper Nas, in the song Nas Album is Done, on DJ Khaleds album Major Key. Working in a school that is predominantly African-American, these lyrics are particularly poignant. Wealth inequality has surpassed being solely defined along racial lines. Economic inequality pervades American society in particular, the world in general. The rich get rich and the poor get poorer is a statement that we often hear. Is there a way to close this ever-increasing gap? How does wealth/income inequality affect the standard of living of those that are affected? How do taxes apply to this situation? How can we use mathematics to address this issue while modeling solutions? All of these are potential questions to be addressed while showing students that mathematics is a tool that when properly used can be used to model and solve problems beyond the use of arithmetic.

I plan to teach this unit during the coming year to 98 students in Math III.

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Introduction

Mathematics, which came from the Greek mathematikos, which was based on mathesis originally meant "mental discipline" or "learning". This mental discipline primarily concerns itself with solving problems through arithmetic, which comes from the Greek arithmos "number". A deeper exploration of the word arithmetic exposes that it means to fit together. Therefore, mathematics primarily concerns itself with the solving of problems through the manipulation of numbers. What of problems that extend beyond mathematics? If these problems can be quantified, can they be solved? One such modern problem is the one of income/wealth inequality.

Did you know that the average family in the top 1 percent of earners makes 40 times more than the average family in the bottom 90 percent of households."Today, an ultra-wealthy group of 400 people in the top 1%, the Forbes 400, now owns an all-time record $2.34 trillion in wealth. Just as staggering, since 1983, this elite group has seen their wealth increase by an average 736%, from $700 million to $5.8 billion, even the least wealthy of the Forbes 400 has exponentially increased their wealth over this time, going from $295 million in 1983 to $1.7 billion in 2015 (an increase of 474%) (Asante-Muhammad 2016, 12)" Families in the top 0.01 percent—the 1 percent of the 1 percent—make, on average, a whopping 198 times more than those in the bottom 90 percent, according to Saez and fellow economist Thomas Piketty's data (20). How can we model and solve this problem with mathematics? How can we view this widening wealth gap from a historical perspective? This concept has been used to make people aware of the ever-widening gap between the haves and the have-nots. Income inequality, as well as the gap in wealth affect the social structure and strata of American society in particular, the world in general. Very few educators discuss the differences between wealth and income with students. Proper schooling helps one to achieve financial success through the acquisition of education that prepares one for college, which in turn provides specialized knowledge that leads to a profession that provides income to raise one's standard of living. Throughout high school and college, there are no courses that are used to teach financial literacy. The proper use of money is often not taught to the masses in public education. The ability to use money as a tool to generate wealth as some secret knowledge for the wealthy 1%. A distinction between wealth and income to impart financial literacy.

The purpose of education is to provide individuals with a level of literacy that to function in society. This type of literacy transcends the common understanding of being able to read and write it also incorporates financial literacy, and mathematical literacy. The lack of mathematical literacy has been referred to as innumeracy, and many have said that as a culture math suffers from a disconnection from everyday affairs. Teaching strategies on finances and discrepancies between income and wealth will help students connect math to everyday life.

The first distinction will be made is the difference between income inequality and wealth inequality. "Wealth, or net worth is the difference between total assets (such as houses, stocks and bonds) and total debt (such as mortgages, car loans, and credit card debt). Keister goes on to state that wealth can produce interest and dividend income that can be used to satisfy consumption needs" (Keister 2000) Income equality refers to the widening gap of annual earnings, whereas wealth inequality refers to unequal distribution of assets. An individual may declare an annual income of $25,000, but have assets worth millions, whereas another individual may have an annual income of $50,000 and have no assets. Who is truly worth more? Research has shown though that closing the income gap will not necessarily address the wealth gap.

"Using income alone to study financial well-being would be sufficient if income and wealth were highly correlated, but there is evidence that they are not. In 1983, the correlation between total household income and household net worth was 0.49; and when asset income was removed from total income, the correlation dropped to 0.26 (Lerman and Miskell, 1988: p. 779). This suggests that many of the truly wealthy have rather low earnings, probably because they are able to support current consumption with income derived from assets. It also suggests that that studies that focus solely on income miss a large part of the story of advantage and disadvantage" (Keister 2000). Taking this position into account. Mathematics will be used to model the wealth gap as well as to offer lessons on how to use income to obtain assets. It has been reported that According to quotes on finances, Albert Einstein stated, " Compound interest is the eighth wonder of the world. He who understands it, earns it ... he who doesn't ... pays it. Compound interest is the most powerful force in the universe. Compound interest is the greatest mathematical discovery of all time, if Einstein made this comment, he realized that the secret of money is to make your money work for you and for an individual not to work for their money. Mathematics is taught and learned in two distinct manners. The first is from a position of mathematics as just the manipulation of numbers, arithmetic, or mathematics as a discipline of reasoning. This type of reasoning is classified as quantitative reasoning. This reasoning requires that one must be able to reason with information that involves mathematical ideas or numbers. A unit on financial literacy and wealth inequality provides an excellent opportunity for students to model and solve real-world problems with mathematics. Fundamentally, mathematics is about problem solving. Problems do not always involve numbers but when a problem can be quantified a student of mathematics should be able to model the situation and then offer concrete ways to solve the problem. In order to fully engage the students with this unit, wealth inequality will be introduced and posed in a mathematical fashion. "Problem posing is both a tool for teaching mathematics through problem solving and an integral part of learning mathematics in that way. For teachers, posing problems and extending them to enrich students' learning are central to teaching mathematics through problem solving. (Schoen 2003)

School Setting

The mission of Rocky River High School is recognized in our school motto: "Learners Today, Leaders Tomorrow" which in turn helps us embody our core beliefs:
- Every student will have a right to a quality education.
- Faculty and staff will have shared responsibility in achievement of school-wide goals.
- All students will be treated with mutual respect and dignity.
- Our school will foster a learning environment that is conducive to academic and personal success.
- Every student will graduate prepared to enter various segments of society and prepared to be a responsible global citizen in the 21st century.

Rocky River High School is located in Mint Hill, NC. Rocky River High School is located in an urban school district and is situated in a small town called Mint Hill, NC. Rocky River is a new school, built in 2010. The school serves students from grade 9 to grade 12 with a current enrollment of approximately 1700 students: 57% African-American, 28% Hispanic, 7% White, and 3% Multi-Racial and 3% Asian. The demographics of our student population has not changed over the last five years.

Rocky River High School serves students living in east Charlotte. A very small percentage of students reside in the Mint Hill, NC community. As a result, most of the students that attend Rocky River are transitioned from their east Charlotte neighborhood to the school. Additionally, there are no after school buses and the Charlotte Area Transit buses do not service the town of Mint Hill.

Curriculum/Goals:

- Math III allows students to pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. Students expand their repertoire of functions to include polynomial, rational, and radical functions. Functions model a variety of real-world situations. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. In order to make sense of problems and solve them, the National Council of Teacher of Mathematics put forth the following higher levels of demands: Require complex and nonalgorithmic thinking-a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships
- Demand self-monitoring or self-regulation of one's own cognitive processes
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

These problem-solving standards are applied and represented through the following curriculum goals and standards:

Rewrite rational expressions, NC.M3.A-APR.6, NC.M3.A-APR.7

Create equations that describe numbers or relationships, NC.M3.A-CED.1, NC.M3.A-CED.2

Interpret the structure of expressions, NC.M3.A-SSE.1a

Understand solving equations as a process of reasoning and explain the reasoning, NC.M3.A-REI.2

Interpret functions that arise in applications in terms of the context, NC.M3.F-IF.4

Analyze functions using different representation, NC.M3.F-IF.7, NC.M3.F-IF.9

Create equations that describe numbers or relationships, NC.M3. A-CED.1, NC.M3. A-CED.2

Understand solving equations as a process of reasoning and explain the reasoning, NC.M3. A-REI.2
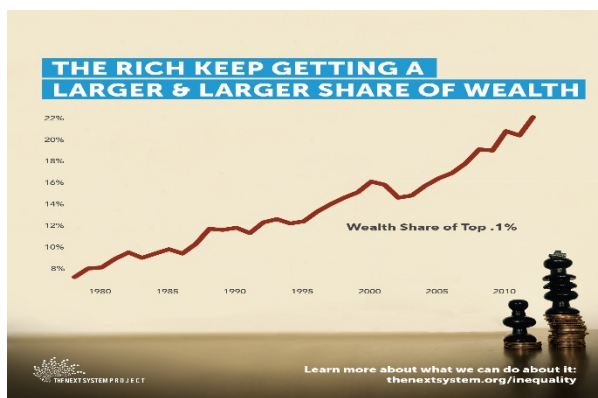
Interpret functions that arise in applications in terms of the context, NC.M3. F-IF.4

Analyze Functions Using Different Representations, NC.M3. F-IF.9

*Build New Functions from Existing Function*s, NC.M3. F-BF.4, NC.M3. F-BF.4a, NC.M3. F-BF.4b, NC.M3. F-BF.4c

Construct and compare linear and exponential models and solve problems, NC.M3. F-LE.4

Background

It has been written that America now has a two-class system. A small predominately, white upper class that welds a disproportionate share of money, power, and political influence and a much larger, minority-heavy (but still mostly white) lower class that is all too frequently subject to the first group's whims. "The intergenerational transfer of wealth occurs when parents pass down wealth, or lack thereof, to their children. Intergenerational transfers play an important role in the continued socioeconomicwealth disparities between whites and minorities" (Kuebler 2013). Kuebler goes on to state that homes are typically the largest asset which is left to heirs and that minorities are less likely to own homes and therefore less likely pass housing wealth to their children. "Oliver and Shapiro (1995) show that at lower income levels, African Americans spend relatively more than whites on housing and transportation rather than income producing assets. However, such spending is a consequence of home mortgage discrimination and the need for many African Americans to purchase reliable cars in order to commute longer distances to work because of because of labor market segregation that locates their jobs longer distances from their homes. Oliver and Shaipiro (1995) also show that at higher income levels, African Americans save more while whites invest more in wealth-generating instruments" (Herring 2016). A truly democratic educational system that concerns itself with social justice will seek to close this gap through the teaching of skills and knowledge that promote equality. Where better to close the wealth gap than mathematics, which concerns itself with the reasoning necessary to solve problems.

Financial Literacy and the acquisition of wealth is not taught in a public school setting. "It's a funny thing that schools teach you how to read and write, how to know history and geography, and how to pass a test-but they never broach the subject of how to get rich" (Cardone 2016). Cardone goes on to state that very few people know how to get money, even fewer know how to keep it, and almost no one knows how to multiply it. The resulting gap between those that know these skills can be linked to the income/wealth gap. Examining the premise of Cardone, it can be extrapolated that individuals must not only be taught how to gain income, they must be taught the necessary skills on how to multiply it through the right investments, which includes savings.

"Wealth, or net worth is the difference between total assets (such as houses, stocks and bonds) and total debt (such as mortgages, car loans, and credit card debt). Keister goes on to state that wealth can produce interest and dividend income that can be used to satisfy consumption needs. In order to fully understand the difference between wealth and income, the following statement can be examined, Last year, middle-class households reaped an income gain of 5.2 percent, the highest level since 2007. Now the bad news: Despite such overdue gains, average American households are barely making more than they did in 1980. Median household incomes have risen just 17 percent (in real dollars) during the past 35 years, lagging far behind GDP growth. Meanwhile, the corporate profits and the average income of the top 1 percent of earners has skyrocketed. Research has shown though that closing the income gap will not necessarily address the wealth gap. "Using income alone to study financial well-being would be sufficient if income and wealth were highly correlated, but there is evidence that they are not. In 1983, the correlation between total household income and household net worth was 0.49; and when asset income was removed from total income, the correlation dropped to 0.26 (Lerman and Miskell, 1988: p. 779). This suggests that many of the truly wealthy have rather low earnings, probably because they are able to support current consumption with income derived from assets. It also suggests that that studies that focus solely on income miss a large part of the story of advantage and disadvantage[11]. "While income is necessary to meet daily expenses, wealth helps families get through lean times and empowers them to climb the economic ladder. Wealth is money in the bank, a first home, a college degree and retirement security-it's the countless opportunities afforded by having savings and investments (Shapiro 2017, 25)Unfortunately, when an overwhelming amount of wealth is concentrated in such few hands, not only do highly unequal societies suffer from significant negative and social and health outcomes, there are also fewer opportunities available for others to get ahead[2].

This statement shows that just increasing income does very little to close the gap between the haves and the have nots. "Why are some people wealthy while others are poor" (DeNarda 2017) "Truly understanding the devastating effects of wealth inequality requires a clear sense of how and why wealth itself matters in the lives of real families, how they acquire it, and what they use it for t doesn't take much education to see that the more income one has, the more money they will have to save and invest. However, closing the income gap does little to close the wealth gap. "Even when people of color get higher-earning jobs, because they are less likely to have good benefits, their paychecks do not translate into similar financial gain as for whites. Liberal and mainstream economic theory assumes that equal opportunity is the path to parity" (Shapiro 2017). How wide is the racial wealth gap, and should schools with a high minority population focus on actively teaching money management skills? Some scholars have said that the average lifespan of the dollar in the African-American community is 6 hours, compared to 28 days in the Asian community and 19 days in the Jewish community. "Since the official start of the recession in 2007, the number of multi-millionaires worldwide has exploded. There are now about 170,000 people worth $30 million or more (up 60%); 37,000 people worth $100 million or more (up 62%); and 1,700 billionaires (up 82%). Since 2008, the U.S. alone has spawned over 1 million new millionaires. Blacks have been in America longer than any other people save the American Indian, yet, of the ten richest African- Americans, there is only one billionaire, Oprah Winfrey, and she is ranked 607th on the Forbes list of wealthiest Americans. ALL ten are in the entertainment field, and their total wealth (7.6 billion) amounts to less than one-tenth that of the richest White man, Bill Gates (78.9 billion). (Reclamation Project 2014). "Even more unfortunate, the extreme rise in overall wealth inequality over the past three decades has only

served to further compound and exacerbate this racial wealth divide. Over that time, the wealthiest 20% of Americans taken 99.4% of all gains in wealth while the bottom 80% have been left to split just 0.6% among themselves. As shocking as this disparity in wealth concentration is, it's even more startling when we realize that today, America's richest 400 individuals-with a collective net worth of $2.34 trillion-now own more wealth than the entire Black population, plus one-third of the Latino population, combined" (Asante-Muhammad 2016, 6). How does the glaring inequality effect society overall?
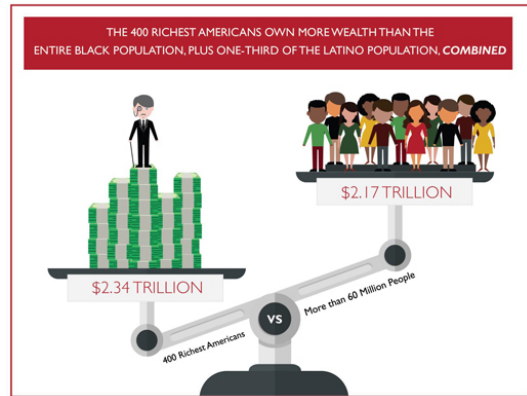
It is interesting that among the 21st century skills are that students must have financial, business and entrepreneurial literacy. Financial literacy is based on these three basic tenets:

- Knowing how to make appropriate personal economic choices
- Understanding the role of the economy in society
- Using entrepreneurial skills to enhance workplace productivity and career options

With this idea in mind, a unit on wealth inequality incorporates a variety of frameworks to increase mathematical knowledge as well as financial literacy. In addition to these skills, the National Council of Teachers of Mathematics advocates that mathematical instruction focus on higher level demands (procedures with connections) and higher-level demands (Doing Mathematics). Incorporating and modeling finances allows for students to perform mathematical procedures with an emphasis on developing deeper levels of mathematical understanding of ideas and concepts.

Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. By incorporating lessons on financial literacy, students are given the opportunity to enter into lessons based on their own understandings and goals. Financial decisions, be they related to asset building or debt management, require the capacity to do calculations, including some complex ones.

In conclusion, when a student asks when will I ever use this, a successful math lesson will incorporate reasoning and problem modeling that will allow students to see real world connections. Providing a working knowledge of how money and the world of finance works has the ability to allow low-income and minority students to gain the necessary skills to begin the process of closing the wealth/income gap. Despite the fact that wealth inequality impacts various facets of life, for the sake of brevity, this unit will focus on exploring habits that are necessary to establish and maintain wealth through the use of asset ownership (housing) taxes, and credit. "A number of studies have documented black-white differences in asset ownership, particularly homeownership[11]. "America's social and fiscal policies regarding homeownership support several economic benefits of owning over renting including (i) lower monthly costs for comparable housing because homeowners do not pay monthly costs which go towards landlords' profit (ii) the income tax tax benefits which allow for mortgage interest and property tax deductions, (iii) forced savings and wealth building created by paying down one's mortgage

which builds home equity. On the other hand, critics argue that the mortgage interest deduction disproportionately helps middle middle and upper income families who can itemize deductions, whereas lower income families may not outlay enough taxes to reap the benefits of deductions hence foregoing this annual benefit of homeownership" (Kuebler 2013).   In addition to home ownership, tax programs have been used to secure and maintain wealth. Ideally at the end of this unit students will be able utilize mathematical modeling to discuss and represent the various issues surrounding wealth inequality.

Financial Background

"Financial Success doesn't happen without a little planning-and planning to win the lottery doesn't count. A child's college tuition. Your retirement. A new car. A first house. A vacation home. That blowout holiday you've been dreaming about for years. All of these big expenses and many more don't just happen by sprinkling a little pixie dust. You have to plan for what you want so that you can map the route to your goal. Certainly, winning the lottery is more convenient, but determining to save the necessary funds over time on your own is more sensible" (Opdyke 2006, 105). In order to maximize money an individual must understand how income can be used to generate wealth as well as understand certain financial principles. According to Asante-Muhammad , it will take over two centuries to close the racial wealth inequality. With that being approximately 10 generations, money skills will need to actively taught in order that they may be taught in all households like other basic skills are. Mathematics provides an excellent vehicle for the teaching of these skills. There are a variety of function and rules that are used to model economic activity. These activities can be used to model the growth of assets as well as predict loss. One such formula/function rule relates to compound interest, $A = P\left(1 + \frac{r}{n}\right)^{nt}$ in addition to this formula there is the following savings plan formula $A = PMT \times \frac{\left[(1+\frac{APR}{n})^{nY}-1\right]}{\left(\frac{APR}{n}\right)}$, it can be argued that the fundamental concept that must be understood to increase wealth are exponential functions modeled in the form of f(x)=ab^x, where a and b are constant values. This form can be used for just about every exponential growth problem. When it comes to finance, different forms of the growth function can be used. For this unit, we will advance to the previous formulas, after demonstrating competency with the following compound interest formula. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

This simple formula can be used to model simple and compound interest which play a significant role in increasing one's wealth. Teachers teach mathematics because they want others to understand it in ways that will contribute to success and satisfaction in school, work, and life. Helping your high school students develop a robust and lasting understanding of functions requires that you understand this mathematics deeply. But what does this mean?" (Cooney 2010). Cooney goes on to state that a cornerstone of high school (and college) is the study of various families of functions. The simplest family of functions that relate to income is the family of linear functions and exponential functions, in addition to making this point, it is further articulated that functions in a family are generally related in more important ways, such as through specific properties that they share and therefore and also through the kinds of situations that they model. For example, linear functions model a constant rate of change, such as income earned in working for a fixed hourly wage (income as a function of hours worked), whereas exponential functions represent a rate of change that is proportional to the value of the function. These types of changes can model the real world financial model such as the accumulation of compound interest, as well as appreciation and depreciation in the value of an object. When these connections are made, they align perfectly with the National Council of teachers of Mathematics Higher level of demands.

Perhaps if money was linked to the teaching of elementary math concepts throughout the lower grades, students would enter secondary school ready to model applicable functions to real world scenarios such as financial investments, developing savings plans, and exploring adult decisions such as financing automobiles and home ownership. The teaching of these concrete skills could potentially help diminish the income inequality which in turn would affect wealth inequality.

Teaching Strategies

Concept Organizer

Students will work individually to complete a circle organizer on the concept of wealth. The students will brainstorm and write down in the circle what wealth means to them. After completing this they will then answer the framework questions: What does it take to become wealthy and How are the wealthy different from others. In addition to completing this individually, a classroom organizer will be used to record student ideas.

Reading

Students will engage in a variety of readings to familiarize themselves with the concept of income/wealth inequality. "Gathering information is the most natural part of the process for a content-area classroom; we want kids to write in order to learn more of our content, right? So they should be encountering new information to share with classmates as well as teacher. Individuals or small teams can each focus on a different aspect of the topic, informing an educating one another, rather than all reciting the same material just to show they read it" (Daniels 2007), In order to demonstrate these skills, students will utilize Cornell Notes, JigSaw reading strategies and concept organizers to help gain deeper understanding of the material that is being taught. Considering the fact that every student has access to a chrome book, the google read &write application will be utilized for students to explore documents in order that they may highlight them as well as annotate them for mathematical concepts and information.

Interactive Strategies

Students will work in dyads for activities such as think-pair-share and collaborative groups for jigsaw activities. In addition to working interpersonally, students will be randomly assigned virtual groups through CANVAS Learning Management System.

Learning Experiences

Unit Essential Vocabulary: Asset, liability, Appreciation(Growth), Depreciation(Decay), Principal (Initial Amount), Interest Rate, Rate of Change, Simple Interest, Compound Interest, APR(Annual Percentage Rate), Investment, Liquidity, Risk, Return, Compound Interest Formula, Natural Base e, inverse functions, logarithms, natural logarithm, Saving Plans Formula (Regular Payments), Capital Gain/Loss, Loans, Installment Loans

DAY 1

Lesson One/Day One: Simple and Compound Interest: The Cost of Banking

Opening Students Compare the rate of change for simple and compound interest and recognize situations in which a quantity grows by a constant percent rate per unit interval. Activating question: Have you ever thought that it is strange that the bank pays you money to hold your money? If the bank pays you to use your money, how do you think bank makes money?
Students will be given the following equations- $a = P(1 + rt), y = mx + b, \ A + P\left(1 + \frac{r}{n}\right)^{nt}, y = ab^x$. Students will predict which formulas produce the greatest amount of change over time, as well as match them with equivalent forms. In addition, the following vocabulary: Principal, Simple Interest and compound interest will be introduced through various relevant financial reading materials.
Student Outcome: Learners will compare linear and exponential models by focusing on how models change over intervals of equal length. Learners will observe from tables and graphs that a function that grows exponentially will eventually exceed a function that grows linearly.
Scenario: Banks originally offered simple interest. Meaning they only paid on the original principal or the amount that was invested. Students will be presented with the following diagram:
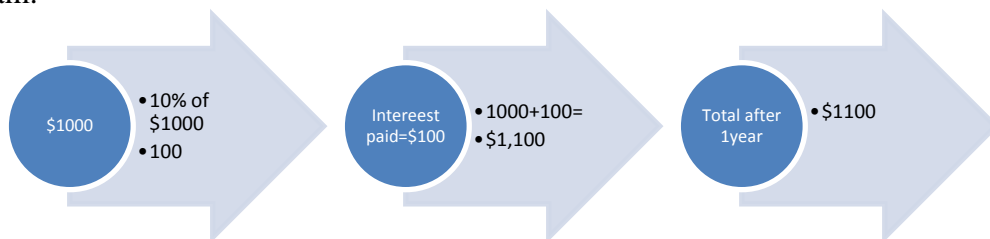
The students will be the given the same model and told instead of one year the investment is now at two years.
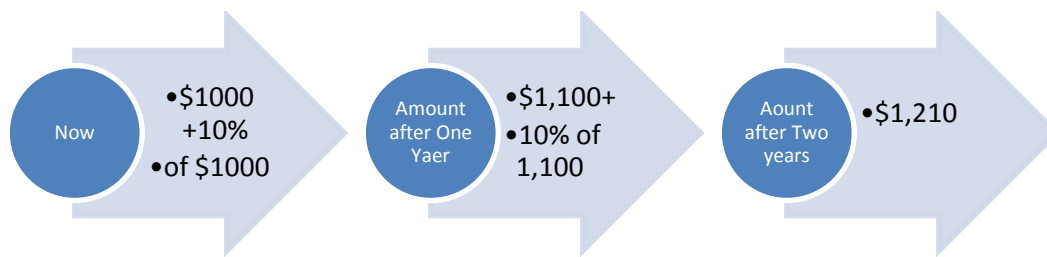
Student Outcomes: After being presented with the preceding diagrams, students will be asked through a series of guiding questions to come up with a formula/general rule to express this relationship. Students will recognize that the general formula I(t)=PRT where I(t) = interest earned after a number of years, P= Principal amount invested or borrowed. And r is the interest rate in decimal form. Students will recognize that this only gives the interest earned and it must be added back to the original amount to get the total earned. After discussing the fact that the bank pays an individual for holding their money, the question will then be raised, does the bank charge you for providing any services? Through guided questioning the students will learn that the bank charges people when they take out a loan. To model this scenario the following problem will be provided: You plan to purchase an Xbox One X. The cost is $500. You borrow money from the bank that charges 4% simple interest per year. How much will you owe if you wait 1 year to pay back the loan? If you wait 2 years? 3 Years? 4 years? 5 years? Construct a table and a formula to model this situation. What do you notice about the pattern? What type of rate does it increase at? Students will then be introduced to the fact that banks found out that they could make more money by compounding interest through the following model:

To scaffold learning, students will then be presented the following table:

Year	1	2	3	4	5
Value	$1,100	$1210	$1331	$1464.10	$1610.51

Through guided practice and discovery students will determine the rate of change and recognize that it is proportional which will lead to the following formula: $FV=PV(1+r)^n$ where FV= future value, PV=Present Value, r=interest rate as an decimal, and n= time in years. At this time a whole class discussion will be held on when it comes to building wealth, one must look at long term goals and decide what type of investments will yield the best return on money invested? Students will then set up and solve the following problem: You have $500 to invest. The bank offers an interest rate compounded annually. How much will you have after 1 year? 2 years? 5 years? 10 years? Students will then be divided into a think-pair-share group and given the following scenario: If you have $200 to invest for ten years, would you rather invest your money in a bank that pays 7% simple interest or in a bank that pays 5% interest compounded annually? Is there anything you could change in the problem that would make you change your answer? As a whole class, pollster will be used to assess student thinking. Students will count of by 1's and two's pair up and solve each component of the problem. Students will report their findings to one another and then another poll will be used to check for understanding.
Lesson Closure: Students will explain the difference between compound and simple interest in their math journals.

DAY 2

Day Two Exponential Growth/Decay The cost of appreciation and depreciation

Student Outcomes: Students will describe and analyze exponential growth/decay models. They will recognize the formulas that model exponential growth and decay; $f(n) = PV(1 \pm r)^n$. Students will recognize that if the growth factor is greater than 1 growth is occurring, if the value is less than 1 loss is occurring.

Explore: Wealth is often generated through investments versus savings. In order to recognize effective tools to establish lasting wealth appropriate investment opportunities must be recognized. Students will be presented with the following definition of wealth: Wealth is determined by taking the total market value of all physical and intangible assets owned, then subtracting all debts. Essentially, wealth is the accumulation of resources. Students will explore the concept of resources and recognize that specific investments represent wealth, whereas other purchases represent debt. Students will be introduced to the concepts of assets and liabilities. One such investment is the purchase of an automobile. This type of investment generates an investment into a resource that costs money while not generating residual income. The students that are served by my school are approaching an age where the y must begin to make decisions about manageable debt. In order to understand how money is a tool that can be used to generate wealth, they must understand depreciating liabilities and appreciating assets. These concepts can be modeled effectively through linear and exponential functions.

Opening Scenario: the average cost of a new car is $33,560 you secure an interest free loan, from your uncle with the options of paying back this car over 3-7 years. How much would your monthly payments be in each scenario. For example, if you chose a three year option at thirty six months, $\frac{33,560}{36}$ = a monthly payment of $932.23. Calculate the rest and determine which option you would choose and why? How much would the monthly payment be under each scenario? . Most families do not have enough cash on hand to purchase a car outright. Instead, your uncle provides you with a 5% down payment. How much does he give you? Now you must go to the bank and ask them for a loan for the remaining amount. How much will you ask the bank for? What you borrow from the bank must be paid back in what is called an installment loan, which can be modeled by the following formula: $PMT = \dfrac{p \times (\frac{apr}{n})}{[1-(1+\frac{APR}{N})^{-nY}]}$

     Once you get to the bank with your down payment, the loan officer approves your principal loan for $31,882. Students will be asked on how this amount was determined. Students will then research various APR rates for 36-72 month loans, compare these options to what they chose from the personal loan options for their uncle and then tell if they changed their mind and if so why? The following guided problem will be given to students a credit union auto loan rate for 36 months is currently an APR of 1.5%. Students will calculate the monthly payments without interest first. APR means annual percentage rate which means that you will pay .125 interest per month. $.125 \left(\frac{31882}{36}\right) + 885.62 = \$996.32$. Assume that this interest rate applies for every loan scenario. Find your monthly paymnets and how much you will end up paying the bank.

     Once you drive a car off the lot it loses 11% of its' value. Express how much value the car still has as a percentage. What is the percentage of value that was loss? A car loses 15-25%

of its value over five years. If the loss was to occur yearly, how could it be modeled with an equation? Set up an equation and model this loss for 15%, 20%, and 25%.

Guided practice:

Number years of having the car	Future value after t years	15% depreciation of current car value	Present Car value minus the 11% depreciation
0	29,868	4480.20	25387.80
1	25387.80	3808.17	21579.63
2	21579.63	3236.94	18342.70
3	18342.70	2751.410	15591.30
4	15591.30	2338.70	13252.60
5	13252.60	1987.89	11264.70

The following equation will be modeled: $f(n) = 29,868(1 - .15)^n$, Students will work under teacher direction to complete the table.

Whole class discussion, is there any point where the balance on the car is greater than what the car is worth. If you are looking to maximize savings, should you purchase a new car or used car.
Introduction to appreciation: Students will explore how home ownership is an asset and can be used to transfer wealth throughout generations. Whereas a car loses value, a home increases in value. It is estimated that a home can appreciate in value 3-5% per year. Assume the cost of a new home is 200,000 If you purchased a home for 200,000 in 2010 with an appreciation rate of 3% what would your home be worth now. $f(n) = 200,000(1.03)^n$ can be used to model this equation. What would the value of your home be now? Suppose this trend continued and you have children. In 20 years what could you sale your home for?

Students will be given the following equations- $a = P(1 + rt), y = mx + b, A + P\left(1 + \frac{r}{n}\right)^{nt}, y = b^x$. Students will predict which formulas produce the greatest amount of change over time, as well as match them with equivalent forms. In addition, the following vocabulary: principal, Simple Interest, and compound interest will be introduced through various relevant financial reading materials.

Most individuals cannot purchase a car with cash outright and have to take a loan out from a bank. The amount borrowed is considered the principal and interest must be paid on it. Various lenders provide different interest rates based on how long the loan is for. Students will work independently on the lending tree website to research interest rates. After finding the rates for each yearly scenario and determining a down payment amount, students will revisit which payment scenario is the best. Students will be introduced to the principal of installment loans and use the formula to calculate monthly payments. $PMT = \frac{p \times (\frac{apr}{n})}{[1-(1+\frac{APR}{N})^{-nY}]}$. Students will be given the scenario that they have a 12,000 loan with a 12% intererest rate or 1% per month. at the end of the month 1% x 12,000= 120. Students will be told that when paying a loan, if only interest is paid, the principal will never be addressed. Students will be given the following

model and shown that when purchasing a large item that requires a loan, paying more than what is necessary will pay the loan down faster by addressing the principal.

End of	Prior Principal	Interest on Prior Principal	Payment Toward Principal	Total Payment	New Principal
Month 1	$12,000	1% x 12000=120	2000	2120	10,000
Month 2	$10,000	.01 x 10,000=100	2000	2100	8000
Month 3	8000	.01 x 8000=80	2000	2080	6000
Month 4	6000	,01 x 6000=60	2000	2060	4000
Month 5	4000	.01 x4000=40	2000	2040	2000
Month 6	2000	.01 x 2000=20	2000	2020	0

Students will then contrast this method with using this formula, $= \frac{p \times (\frac{apr}{n})}{[1-(1+\frac{APR}{N})^{-nY}]}$, where PMT= regular payment amount, P= starting loan principal, APR= Annual percentage rate, n= number of payment periods per year, Y=loan term in years. The following values will then be used to calculate monthly amounts $PMT = \frac{\$1200 \times (\frac{0.12}{12})}{[1-(1+\frac{.12}{112})^{-12 \times .5}]}$ It will be pointed out to students that the decimal 0.5 is used because we want the loan paid off in six months. $\frac{\$1200 \times (0.01)}{[1-(1+0.01)^{-6}]} =$

$\frac{\$12}{1-0.942045235} = \$207.06$. Students will be instructed that even though installment loans such a s automobile loans and mortgages have a schedule set by the lender, they can determine how long they want to pay and set their own schedule using the preceding formula.

The most popular type of installment loan is the mortgage, which is the loan one receives for purchasing a home. Whereas a car is considered an item that loses value. A home gains value over time. "Mortgage interest generally are lower than interest rates on other types of loans because your home itself serves as a payment guarantee"[3] . Both of these purchases require a down payment. An automobile is considered a liability whereas a home is considered an asset. Consider the following scenario, for the next five years it is estimated that homes will increase in value by 3.7%. If you own a home that is currently worth $235,000 how much will it be worth in five years? How can you use the exponential formula to help you solve this problem? Students will be led into setting up the following equation: $f(v) = Pv(235,000 + 1.037)^n$. Students will then apply the payment formula to calculate the following scenarios: payment on a car with a $15,000 principal and apr rate of 2% for 36 months and payment on a house with a principal of $90,000, with an annual percentage rate of 6% and a 30 year mortgage.

Exit ticket: You receive a gift of $3000. What do you consider a better investment, to use this as principal to purchase a car or principal to purchase a home? Students will also be asked to write on how the appreciation of value on a home contributes to wealth whereas the depreciation of an automobile takes away from wealth.

DAY 3

Lesson Three/Day Three Investing for wealth and Developing a Savings Plan

"For most people, a more realistic way to save is by depositing smaller amounts on a regular basis. For example, you might put $50 a month into savings. Such long-term savings plans are so popular that many have special names-and some even get special tax treatment" (Bennett 2011, 227). *This will be explored in a future lesson.

Student Objectives/Outcomes: Students will recognize that in order to reach long term financial goals they have to identify objectives and manage their money effectively.

Bell Ringer

Most people do not have a large sum of money to pay the principal on an automobile or home, so saving a small sum each pay period is a more viable option. Suppose you open an account with $100. The annual percentage rate is 12% divided equally over the year. Complete the table up until the sixth month. What pattern do you notice? Are there any formulas that can be constructed from the table?

End of Month	Prior Balance	Interest on prior Balance Monthly rate of 1%	End-of-Month Deposit	New Balance
1	0	0	100	100
2	100	1%	100	201.00
3	201	1%		
4	303.01			
5	406.04			
6	510.10			

Students will be asked how far can this table be extended and is there a limit that can be placed on it. Students ideally will see that this table can be extended indefinitely and will be introduced to the following formula:$A = PMT \times \dfrac{[(1+\frac{APR}{N})^{NY}-1]}{(\frac{APR}{N})}$ where A= accumulated savings plan balance PMT= regular deposit amount, APR=annual percentage rate as a decimal, n=number of payment periods per year Y= number of years.

Activities

Students will identify personal financial goals such as a college they wish to attend, a home they would like to purchase, and determine the amount that they would have to save monthly to reach their goal. Students will also research and find the best financial institution to place their money in. This lesson will serve as a mini assessment to test for comprehension as well as application of skills and knowledge learned.

DAY 4: Modeling the progressive tax system

Lesson Introduction: "I like paying taxes. With them I build civilization." - Justice Oliver Wendell Holmes (Bennett 2011, 269).

Opening Activity: Students will complete a concept organizer on taxes. They will write everything that they know about taxes and then answer the question: What types of taxes are there and are they fair?

Lesson Introduction: The tax system of the United States is progressive, meaning that people with a higher taxable income pay at a higher tax rate (Bennett 2011, 268). This system is known as a marginal tax rate which assigns a different amount of tax rates to different income ranges. The marginal tax rate system can be effectively modeled with piecewise functions. Students will be introduced to the following table and provided the information that each bracket is associated with a marginal tax rate.

Understanding Piecewise Functions through the Marginal Tax Rate

A piecewise defined that is defined differently for different values of x The following examples will be provided, the following pay 10% if their income represented, by x falls into the following categories single filers $x \leq 9275$ married joint filers $x \leq \$18{,}550$ and head of household is $x \leq 13{,}250$.

2016 Tax Bracket

Rate	Single Filers	Married Joint Filers	Head of Household Filers
10%	0-$9275	$0 to $18,550	$0 to $13,250
15%	$9275-37650	$18,550 to $75,300	$13,250 to $50,400
25%	$37,650 to $91,150	$75,300 to $151,900	$50,400 to $130,150
28%	$91,150 to $190,150	$151,900 to $231,450	$130,150 to $210,800
33%	$190,150 to $413,350	$231,450 to $413,350	$210,800 to $413,350
35%	$413,350 to $415,050	$413,350 to $466,950	$413,350 to $441,000
38%	$415,050+	$466,950+	$441,000+

Student Outcomes: Students will apply and understand set builder notation through the use of marginal tax rates; students will apply and understand the meaning of the graph of y=f(x), that $\{(x,y)\} \,/\, x \in D \text{ and } y = f(x)$. Students will create tables and graphs of functions and interpret key features such as increasing intervals.

Accessing prior Knowledge: Students will discuss the previous tax schedule, the role that domain and range play in linear functions. In addition, students will examine the following tax schedule and discuss the various linear functions that can be made and model them. As guided practice students will discuss the following questions.

1. Write a function f_x that takes in x, the total number of dollars a single filer earned in year 2016and outputs the amount of total taxes they will earn if owe if they earn $9,275 or less. f(x)=0.1x, if $x < \$9{,}275$
2. Write a function $f_2(x)$ that takes in x, the total number of dollars a single filer earned in the year 2016 and outputs the amount of total taxes they will pay if they earn between $9,275 and 37,650. $f(x) = 927.5 + x$ if $9275 < x \le 37650$.

   For independent practice students will compute taxes with the following scenarios: Gene is single and had a taxable income of $35,400. Sarah and John are married filing jointly with a taxable income of $87,500. Chris is married filing separately with a taxable income of $127,300. For the next stage of guided instruction students will be provided with a redacted copy of tax form. Particular focus will be placed on tax credits and deductions. Lowering taxes is the holy grail of taxpayers. "The possibilities of lowering your taxes are enormous and depend entirely on your activities for the year" (Opdyke 2006, 178)Students will be asked what they know about the Child Tax credit. The following question will be asked; What's better a credit or deduction? Students will be led through guided discussion on what it means to deduct versus giving someone credit. Students will recognize that "A credit is a dollar-for-dollar subtraction from your total federal tax bill, whereas a deduction lowers the taxes you owe by a certain percentage, that percentage being whatever marginal tax rate you fall into" (Opdyke 2006, 184)This will be exemplified by this direct teaching Tax deduction is $100 x 25% tax rate=$25 in savings, whereas a Tax of $100 is $100 in tax savings.

DAY 5

Are all taxes equal?

Throughout the history of the income tax, capital gains have been taxed at lower rates than ordinary income. Since 2003, qualified dividends have also been taxed at the lower rates. Defenders of the tax preference argue that lower tax rates for capital gains and dividends offset the taxes that have already been paid at the corporate level. Some also claim that lower tax rates for capital gains spur growth, encourage risk-taking and entrepreneurship, offset the effects of inflation, and prevent "lock-in" (the disincentive to sell assets). Critics, for their part, complain that the lower tax rate disproportionately benefits the wealthy and encourages tax sheltering schemes. (Tax Policy Center 2016).

Outcomes: students will revisit appreciating assets and exponential growth to understand capital gains.

Opening Activity: Remember your home that you own that was purchased for $235,000 five years ago. It increased in value by 3% every year since 2012. What is its value now. $f(v) = 235,000(1.03)^5$ Students will be instructed to solve the problem. Once they get the answer of $272,429, they will be asked what type of profit did you make on the home? The students will be asked via pollster If you sale the home and receive the money, should it be considered income? Based off of student responses, the concept of capital gains will be introduced. "Not all income is created equal, at least not in the eyes of the tax collector! In particular dividends (on stocks) and capital gains-profits from the sale of stock or other property-get special tax treatment. Capital gains are divided into two sub categories. Short-term capital gains are profits on items sold within 12 months of their purchase, and long-term capital gains are profits on items held for more than 12 months before being sold". (Bennett 2011, 273) According to the Motley
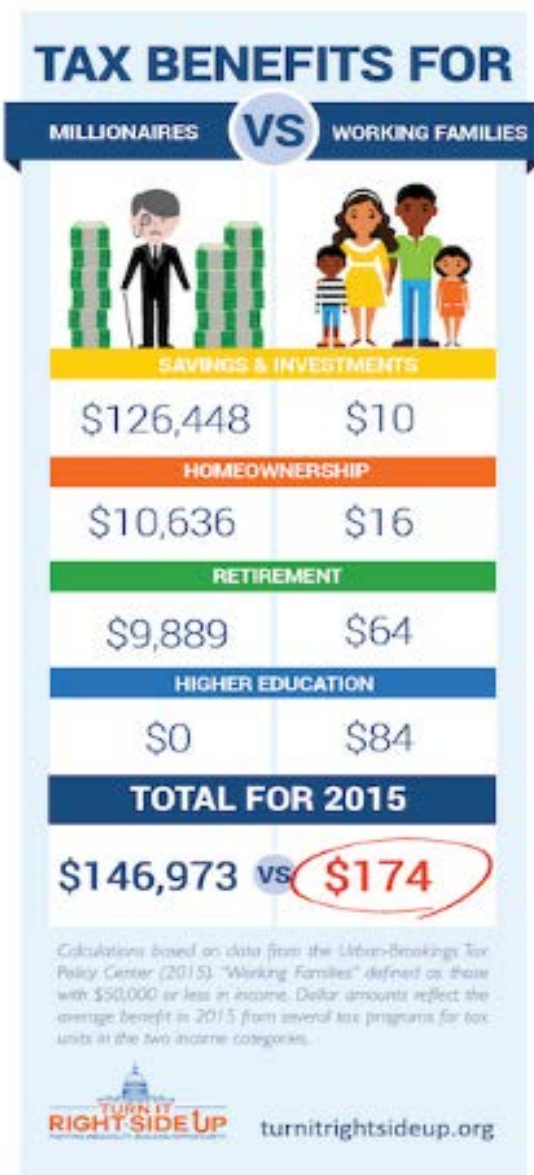
Fool website taxes on long-term capital gains were 20% with most people paying as little as 15%. Bennet goes on to say that often capital gains from the sale of homes are often tax exempt. If this is the case, compare how much taxes someone would pay if they earned 37,429 from income versus you earning the money from selling your home. "As anyone earning a paycheck has no doubt noticed, taxes shrink your income every payday. Collectively, these deductions are the "payroll taxes" siphoned from all workers' paychecks to fund federal programs such as Social Security and Medicare. The tax system takes a pay-as-you-earn approach, deducting throughout the course of the year proportional pieces of your overall tax obligation" (Opdyke 2006, 168-169) In the past twenty years, the federal government has spent more than $ 8 trillion through tax programs to help families build long-term wealth by helping them to save for retirement, purchase a home, start a business or access higher education. Since 1994, the federal government's massive wealth-building spending has more than tripled, going from a little over $200 billion to $660 billion in 2015. Unfortunately, the result of this spending today is that the typical millionaire receives about $145,000 in public tax benefits to grow their wealth , while working families get a grand total of $174 on average (Asante-Muhammad 2016, 17) "It's quite possible that no one fully understands federal income taxes. The complete tax code consists of thousands of pages of detailed regulations (Bennett 2011, 266). Out of these thousands of regulations, what is it that the wealthy have learned that has allowed them to maximize tax benefits, while minimizing payments. "In 2015, a couple with a combined income of $200,000, with no investments or other income, would pay a composite 22% tax rate, assuming they did not use

deductions to reduce their liability or other tricks to hide their wages. Their income would yield $43,000 or so in tax revenues for the federal government. A family who did nothing other than invest its wealth in stocks or property, receiving $200,000 in returns, would owe significantly less. Their $200,000 gain would be taxed at the lower capital gains rate, with the first $75,000 tax free, a 15% tax rate on earnings up to $464,000, and a 20% tax rate on earnings above that figure. The second family would be taxed at a blended rate of 9%, yielding the government about $18,000 in tax returns. The difference between paychecks and profits from wealth is stark: $43,000 in taxes compared to $18,000, a tax rate of 22% versus 9%. What's more, amassing $200,000 from wealth alone would require that the family have at least $4 million in financial wealth to begin with (Shapiro 2017, 154-155). The tax system of the United States is progressive, meaning that people with a higher taxable income pay at a higher tax rate (Bennett 2011, 268).This system is known as a marginal tax rate which assigns a different amount of tax rates to different income ranges. The marginal tax rate system can be effectively modeled with piecewise functions.

Appendix I: Implementing Teaching Standards for North Carolina Standard Couse of Study

Make sense of problems and persevere in solving them.
Reason abstractly and quantitatively.
Construct viable arguments and critique the reasoning of others.
Model with mathematics.
Use appropriate tools strategically.
Attend to precision.
Look for and make use of structure.
Look for and express regularity in repeated reasoning.

Create equations that describe numbers or relationships:NC.M3. A-CED.1, NC.M3. A-CED.2
Understand solving equations as a process of reasoning and explain the reasoning, NC.M3. A-REI.2
Interpret functions that arise in applications in terms of the context, NC.M3. F-IF.4
*Analyze Functions Using Different Representations*NC.M3. F-IF.9
*Build New Functions from Existing Function*s
NC.M3. F-BF.4, NC.M3. F-BF.4a, NC.M3. F-BF.4b, NC.M3. F-BF.4c
Rewrite rational expressions, NC.M3.A-APR.6, NC.M3.A-APR.7
Create equations that describe numbers or relationships, NC.M3.A-CED.1, NC.M3.A-CED.2
Interpret the structure of expressions, NC.M3.A-SSE.1a
Understand solving equations as a process of reasoning and explain the reasoning, NC.M3.A-REI.2
Interpret functions that arise in applications in terms of the context, NC.M3.F-IF.4
Analyze functions using different representation, NC.M3.F-IF.7, NC.M3.F-IF.9
Construct and compare linear and exponential models and solve problems, NC.M3. F-LE.4

Appendix II: Teacher Resources

Institute for Policy Studies . n.d. Wealth Inequality in the United States. Accessed september 17, 2017. https://inequality.org/facts/wealth-inequality/. Government White Paper which explores the social impact of wealth Inequality.

Cardone, Grant. 2016. The Millionare Booklet. Grant Cardone Enterprises. Resource which provides insight on gaining wealth while working.

Fitz, Nicholas. 2015. Economic Inequality: It's Far Worse Than You Think. March 31. Accessed November 4, 2017. https://www.scientificamerican.com/article/economic-inequality-it-s-far-worse-than-you-think/. Explores how pervasive wealth inequality is.

Gilson, David & Rios,Edwin. 2016. 11 Charts That Show Income Inequality Isn't Getting Better Anytime Soon. December 22. Accessed September 17, 2017. http://www.motherjones.com/politics/2016/12/america-income-inequality-wealth-net-worth-charts/. Provides a visual representation of wealth inequality.

Jones, Mother. 2012. Reid Cramer. February 13. Accessed September 19, 2017. http://www.motherjones.com/politics/2012/02/its-wealth-gap-stupid/. Explores the difference between income inequality and wealth inequality.

Kelley, Michael B. 2013. "Wealth Inequality Is MUCH Worse Than You Realize." Business Insider. March 5. Accessed September 19, 2017. http://www.businessinsider.com/inequality-is-worse-than-you-think-2013-3. Provides an anlaysis of the social ramifications of wealth inequality.

Kroll, Luisa. 2017. Forbes 400 2017: Meet The Richest People In America. October 17. Accessed November 4, 2017. https://www.forbes.com/sites/luisakroll/2017/10/17/forbes-400-2017-americas-richest-people-bill-gates-jeff-bezos-mark-zuckerberg-donald-trump/#cebf3145ed53. Provides an overview of the wealthiest Americans.

Muhammad, Michael Z. 2016. A Never Ending Wealth Gap Between Blacks And Whites In America? August 18. Accessed September 2016, 2017. https://www.finalcall.com/artman/publish/Business_amp_Money_12/article_10 Discusses the historical context of the racial aspect of wealth inequality.

Appendix III: Classroom Resources

1. Chromebooks
2. Calculators
3. desmos
4. wealth inequality video
5. Walch workbooks
6. Document Camera
7. Smart Board
8. Internet Access
9. Chart paper

Appendix IV: Student Resources

1. Google Chromebooks
2. Calculators
3. Internet
4. Google Classroom
5. Canvas
6. Pencil
7. Paper
8. Chart Paper
9. Graph Paper
10. Journal
11. Desmos
12. Guided Notes
13. Cornell Note Template
14. Concept Organizer

Bibliography

Institute for Policy Studies . n.d. *Wealth Inequality in the United States.* Accessed September 17, 2017. https://inequality.org/facts/wealth-inequality/.

Asante-Muhammad, Dedrick, Collins, Chuck, Hoxie, Josh & Nieves, Emanuel. 2016. "The Ever-Growing Gap Without Change, African-American and Latino Families Won't Match White Wealth for Centuries." *Prosperity Now.* August. Accessed November 4, 2017. https://prosperitynow.org/resources/ever-growing-gap-without-change-african-american-and-latino-families-wont-match-white.

Bennett, Jeffrey, & Briggs, William. 2011. *Using & Understanding Mathematics A Quantitative Reasoning Approach.* Boston: Pearson.

Cardone, Grant. 2016. *The Millionare Booklet.* Grant Cardone Enterprises.

Cooney, Thomas J., Beckmann Sybilla. 2010. *Developing Essential Undrstanding of Functions Grades 9-12.* Reston: National Council of Teacher of Mathematics.

Daniels, Harvey, Zemelman, Steven, & Steineke, Nancy. 2007. *Content-Area Writing Every Teacher's Guide.* Portsmouth: Heinemann.

DeNarda, Mariacristina, & Fella, Giulio. 2017. "Saving and Wealth Inequality." *Review of Economic Dynamics* 280-300.

Fitz, Nicholas. 2015. *Economic Inequality: It's Far Worse Than You Think.* March 31. Accessed November 4, 2017. https://www.scientificamerican.com/article/economic-inequality-it-s-far-worse-than-you-think/.

Gilson, David & Rios,Edwin. 2016. *11 Charts That Show Income Inequality Isn't Getting Better Anytime Soon.* December 22. Accessed September 17, 2017. http://www.motherjones.com/politics/2016/12/america-income-inequality-wealth-net-worth-charts/.

Herring, Cedric & Henderson, Loren. 2016. "Wealth Inequality in Black and White: Cultural and Structural Sources of the Racial Wealth Gap." *Race & Social Problems* 4-17.

Jones, Mother. 2012. *Reid Cramer.* February 13. Accessed September 19, 2017. http://www.motherjones.com/politics/2012/02/its-wealth-gap-stupid/.

Keister, Lisa A. 2000. "Race and Wealth Inequality: The Impact of Racial Differences in Asset Ownership on the Distribution of Household Wealth." *Social Science Research* 477-502.

Kelley, Michael B. 2013. "Wealth Inequality Is MUCH Worse Than You Realize." *Business Insider.* March 5. Accessed September 19, 2017. http://www.businessinsider.com/inequality-is-worse-than-you-think-2013-3.

Kroll, Luisa. 2017. *Forbes 400 2017: Meet The Richest People In America.* October 17. Accessed November 4, 2017. https://www.forbes.com/sites/luisakroll/2017/10/17/forbes-400-

2017-americas-richest-people-bill-gates-jeff-bezos-mark-zuckerberg-donald-trump/#cebf3145ed53.

Kuebler, Meghan. 2013. "Closing the Wealth Gap: A review of Racial and Ethnic Inequalities in Homeownership." *Sociology Compass* 670-685.

Muhammad, Michael Z. 2016. *A Never Ending Wealth Gap Between Blacks And Whites In America?* August 18. Accessed September 2016, 2017. https://www.finalcall.com/artman/publish/Business_amp_Money_12/article_103256.shtml.

Opdyke, Jeff D. 2006. *The Wall Street Journal. Complete Personal Finance Guidebook.* New York: Three Rivers Press.

Reclamation Project. 2014. *How White Folks Got So Rich.*

Schoen, Howard L., Randall, Charles I. 2003. *Teaching Mathematics through Problem Solving.* Reston: National Council of Teachers of Mathematics.

Schwartzman, Steven. 1994. *The Words of Mathematics An eymological Dictionary of Mathematical Terms Used in English.* Washington, D.C.: The Mathematical Association of America.

Shapiro, Thomas M. 2017. *Toxic Inequality How America's Wealth Gap Destroys Mobility, Deepens the Racial Divide. and Threatens our Future.* New York: Basic Books.

Siegel, Cary. 2016. *Why didn't they teach me this in school?* North Charelston: Simple Strategic Solutions.

The National Council of Teachers of Mathematics. 2014. *Principles to Actions.* Reston: National Council of Teachers of Mathematics.

White, Gillian B. 2017. "Escaping Poverty Requires Almost 20 Years With Nearly Nothing Going Wrong." *The Atlantic.* April 27. Accessed October 14, 2017. https://www.theatlantic.com/business/archive/2017/04/economic-inequality/524610/