4M4Z1NG 7H1NGS W17H NUMB3R 7HE0RY

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Quail Hollow Middle School

This curriculum unit is recommended for:
Middle grade levels (6th – 8th)

Keywords: place value, multiplication, division, decomposition, prime factorization, fractions, factoring, area model, number theory

Teaching Standards: See Appendix1 for teaching standards addressed in this unit.

Synopsis: This unit is designed for students to cultivate an understanding of place value by sparking their interests with enticing number theory problems. Students will appeal to their prior place value knowledge by exploring the basic operation methods that Chinese students use to deepen their understanding of multiplication and place value. Students will understand the interconnectedness of place value within simple to gradually more difficult addition/subtraction and multiplication/division concepts deriving the distributive property by decomposing problems. Students will grasp prior concepts by exploring the reasons behind learning algorithmic methods. Students will collaborate among each other to explore the reasoning behind fun number theory problem solving concepts.

I plan to teach this unit during the coming year in to 108 students in 7th grade math and Math I courses.

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School and Classroom Background

Quail Hollow Middle School (QHMS) comprised of sixth through eighth grade is located in Charlotte, North Carolina. The school's motto is students are on the P.A.T.H. to college focusing on perseverance, achievement, thoughtfulness and health. QHMS contains a diverse population of approximately 950 students with wide range of socio-economic and academic backgrounds. According to 2014-2015 school improvement plan for QHMS, our population identified 40% as White, 32% African American, 13% more than one race, 12% American Indian and 3% Asian including 35% identified as Hispanic ethnicity. Approximately 20% of the population has IEPs or Limited English Proficiency with 6% considered academically gifted.

This year I will begin my fourth year of teaching mathematics at QHMS and will transition to seventh grade after spending three years in the sixth grade. Reflecting on daily math lessons, I strive to improve my teaching strategies. At QHMS for seventh grade this year, we have three levels of math classes. These classes include Math I, which will include our highest level of upcoming seventh graders meeting a strict rubric receiving primarily level fives on EOG tests, an Honors seventh grade math (level 4s) and Standard math. I will teach Math I, highly consistent learners, and Standard seventh grade math, which holds a variety of different levels of learners.

With students learning concepts ranging from basic academic math levels to high academic levels, an array of background knowledge will be used. These students are learning concepts in middle school mathematics curriculum to include ratios and proportions, number systems, algebraic equations, geometric concepts and statistical models where number theory and place value concepts will make intriguing exploration. By utilizing the fundamental mathematical practices of improving problem solving skills, my intention is to create intrigue among students to increase their motivation of learning and loving math.

The school improvement plan will require more rigor, engagement and differentiation. The goal is to attain high growth in proficiency and close the gap between our ethnic and racial categories. Teaching a defined seventh grade and Math I curriculum, I've found that I should teach at a higher level to meet the school improvement plan smart goals. This number theory and place value unit will benefit students by increasing their mathematical process standards whereby increasing their proficiency in conceptual and abstract learning. For the Standard class of seventh grade, an interactive notebook will include their number theory exploratory problems. The Math I course will utilize the Pearson ebook with supplemental eighth grade curriculum within an interactive notebook including this unit.

Many of my standard class students enter my class disliking math and feeling frustrated with concepts that they have not completely understood from prior years. I believe many students fall further behind because they lack questioning ability of the fundamental ideas behind the math they are learning. Consequently, their dislike will impede their growth in math; this unit intends to change that way of thinking and spark a like or love of math.

Rationale

As students move through elementary school, key building block math concepts are introduced. Moving into middle school those same concepts are further explored with richer applications; therefore a firm foundation is necessary. A math classroom provides a dynamic environment for students to acquire interesting math knowledge while developing problem-solving skills that are essential as they progress to high school and beyond. According to David A. Sousa in "How the Brain Learns Mathematics", "Novelty and motivation are also undermined by a mathematics curriculum that focuses mainly on a strict formal approach."[1] My intention of this unit is to provide a fresh approach to increase their motivation for learning math.

By using enriching inquiry based activities; students will be given the opportunity to work together or alone on reaching possible solutions and the rationale associated with them. A few techniques given by the authors in "What Successful Math Teachers Do" for motivating a lesson include "present a challenge", "use recreational math", and "get students actively involved in justifying mathematical curiosities." [2] A math classroom provides students time to share their prior knowledge and intertwining new information to develop their rationale to given problems. Number theory and place value concepts provide that enriching material to challenge students as these concepts relate to their middle school curriculum. Many students enjoy competing with classmates, so adding that flare into math incorporates the process standards thereby making them a natural flow to learning.

Elevating process standard strategies, increasing an overall growth mindset, and providing a number theory unit opportunity are just a few motivating factors for creating this unit. Presenting students with a higher level problem or puzzle can create an environment to persevere and reason quantitatively through the situation. Facilitating students to explore problem reasoning is another driving factor for this unit. Increasing students' growth mindsets is an additional focus of this unit. Carol Dweck, a leading researcher in Psychology states "no matter what your ability is, effort is what ignites that ability and turns it into accomplishment".[3] Equally important is providing students with an opportunity to be exposed to number theory concepts generally encountered within a math club setting or college courses. Keeping process standard strategies, growth mindset, and inspiring opportunities will be key factors for creating a successful unit.

Students will be able to more accurately solve number theory, place value and factorization problems. Creating intriguing unit problems and requiring students to reason abstractly and quantitatively using their new found number theory knowledge will allow students to deepen the relationship of mathematical concepts.

Content Objectives and Strategies

This curriculum unit will incorporate all process standard practices which include making sense of problems, persevering by thinking through abstract problems and reasoning through them. Students will also learn to use models and mathematical tools to accurately derive conclusions based on mathematical structures and proofs. This number theory unit will build primarily from seventh grade math objectives including ratios and proportions, number systems, algebraic equations, geometric concepts and statistics. Amazing Things with Number Theory unit will require a basic prior understanding of place value and number system concepts. North Carolina is reviewing curriculum covered by the Common Core State Standards. A newspaper journalist and former Virginia superintendent of schools sited "problem-solving will require higher order thinking skills along with much more imagination and creativity".[4] My intention is to build higher order problem solving skills set to prepare students for their futures.

I began my unit reflecting on student's concept of multiplication facts. Students begin incorporating concepts of grouping, repeated addition and multiplication throughout their elementary school years with a major focus occurring in the third grade under 3.OA.7 objective. Within this standard, students are pushed to memorize multiplication facts although the concept will continue to be developed. Multiplication and division facts steadily develop throughout concepts using ratios and proportions including standard 7.RP.3 which includes using proportional relationships to solve ratio multistep problems. Students will revisit multiplication in 7.NS.2 as rational numbers and integers are introduced and interpreted extend their knowledge of multiplication and division using operation properties. Extending their rational number multiplication knowledge from sixth grade concepts of 6.RP.3, 6.NS.1, 6.NS.3 and 6.EE.2, students will link their understandings within the seventh grade curriculum. Students' multiplication knowledge is developed as a set of algorithms with little focus on the conceptual understanding; meaning the intertwining of higher level uses involving fractions, integers and algebraic models need to be thoroughly connected in the early years.

Returning back to the basics to assist students with multiplication concepts, we will explore the area model using manipulatives such as Zometools or algebra tiles. Connecting multiplication, place value and area with the area model will provide the overlap to move into higher level concepts of algebra. With repeated exposure to basic concepts, an approach will provide that missing link to their mathematical Venn diagram of knowledge. An area model lends itself nicely to the algebra connection, multi-digit multiplication of whole numbers and decimals and place value importance.

Students will explore the multiplication table for simplifying fractions, multiplication and division of fractions, solving proportions and more ratio number theory concepts. Students were introduced to the previous named objectives in sixth grade math classes, where an in-depth review using number theory concepts will be applied. Compiling those named objectives with number theory concepts; students will analyze the pieces and extend this knowledge. Students will be presented the concept of Chinese multiplication which can be used to further their concept building of multiplication, addition and place value.

Building on students' concept connections of multiplication, they will explore place value concepts with variables. Using base ten place value concepts, students will solve expressions and equations using multiple variables. Allowing students the opportunity to explore complex place value problems will extend their previous knowledge of solving expressions and equations with whole numbers to include rational numbers within the objective of 7.EE.3. Presenting thought provoking concepts, students can develop their algebraic thinking to apply their understanding in rewriting expressions in different forms as 7.EE.2 objective is learned. Students will be able to describe their reasoning of number theory concepts relating to ratios and proportions, number system, algebraic expressions and equations, geometry and probability concepts. Extending students higher level problem solving skills will be the outcome of studying number theory ideas relating to our middle school math objectives.

Classroom Activities

"Thought Provoking Thursdays" will include a brief explanation of the activity selected for each day. This thought provoking activity will allow students to explore given problems using their prior knowledge and strategies with an intriguing problem of the day to persevere through a challenging problem. Upon a brief opener, students will be presented an activity that requires more thought than previously demonstrated.

Thought-Provoking Thursday (TPT) Activity #1: I chose this activity to include within our interactive notebooks as a resource which students could refer to throughout the year. Students are still struggling with multiplication facts; therefore this activity provides a review of repeated addition and multiplication connections. Researching ideas on how to help struggling students grasp the concept of multiplication, I found the works of Brad Fulton, author and educator of Fast Facts and Fractions Too![5] His work encouraged me to revisit the multiplication concept early within the school year.

Students are asked to complete a multiplication table of facts from one through six up to the twelves (See Appendix 2). Ask students to explore how multiplication problems can be derived and solved by knowing the first six arrays of multiplication columns. The importance of the multiplication chart is apparent when discovering the connection of multiples, factors and ratios. Tying the distributive property into these problems allow students to discover the relationship or connectedness of these problems.

Example: Pose this problem: 7 x 6, how many ways can you use the multiplication chart to find the result of 42? Write their suggestions and try to lead them into the following: Can 7 x 6 equal (2x6) + (5x6) = 12 + 30 = 42?
By having and using the first 6 columns of the multiplication chart, students can still be successful in calculating simple multiplication.

For example: Ask students to provide three options for calculating: 9 x 8
Possibilities could include: 3 (3 x 8) = (3x8) + (6x8) = (4 x8) + (5 x 8) or
Probe students understanding with: could it work if we split apart both numbers and why?
9 x 8 = 72
9 = 6 + 3 and 8 = 3 + 5
So (6 + 3) x (3 + 5) = (6 x 3) + (3 x 3) + (6 x 5) + (3 x 5) using the distributive property.
Ask students to explain, (6 x 3) + (3 x 5) why this grouping doesn't work for 9x8?

TPT #2 Activity: Students will begin by reviewing their notes and charts from last TPT activity. I used this activity on the successive day from the previous activity to continue completing their interactive notebook setup. Students will be asked to work with a partner and quickly derive 4 options for solving 8 x 8.

The goal of this activity is for students to use their previous day's complete multiplication chart of 1 through 6 to derive the completion of their next chart. The teacher should lead students to see that addition and multiplication are connected by displaying an example for the sevens column. Students can derive the seven column of data by adding the 3 and 4's columns together or 1 and 6's or 2 and 5's columns. Upon that mini-review of possibilities, students will be asked to complete the 7 through 12 multiplication chart using their prior knowledge given 5 minutes. Remind students to use their knowledge of the distributive property concepts to continue completing their chart.

Students will be asked to compare the twos facts with the fours and eights. How will those correspond to the twelves? What can be expected for the sixteens? A goal of this activity is for students to recognize other comparisons among the threes, sixes and nines. What are some options for completing the elevens multiplication chart? If the concept of following idea isn't verbalized, ask students to provide three options for calculating the multiplication facts for 33, how could they use their two completed charts to assist with this matter? Small sample is shown below:

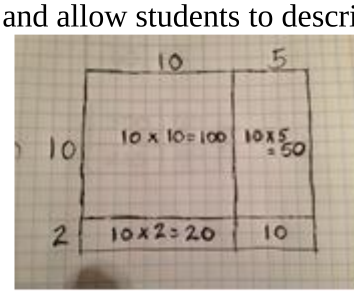
9	11	12	9+11+12 = 32
18	22	24	18+22+24= 64
27	33	36	27+33+36 = 96
36	44	48	36+44+48 = 128
45	55	60	45+55+60 = 160
54	66	72	54+66+72 = 192
63	77	84	63+77+84 = 224
72	88	96	72+88+96= 256

TPT #3: Students will reflect on their understanding of equivalent fractions. Again using their created multiplication chart within their interactive notebooks, students will be asked to locate the numbers 30 and 36 within their chart. As students developed their one through twelves chart, they should find this set of numbers nine times. Where are the two numbers located within the same row or same column on the chart?

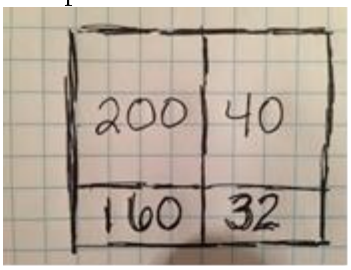
Example #1: The numbers 30 and 36 are on the threes row. Ask them to place their fingers on top of those numbers within the threes row and slide them to the top of the chart to arrive at 10 and 12. What can they describe 10/12 to be compared to the original 30/36? The goal is for students to see that 30/36 is an equivalent fraction to 10/12. Where are 30 and 36 in the same column? What can you derive when you slide your fingers to the right to arrive at 5/6?

Example #2: Ask students to provide equivalent fractions for 27/63, 24/80, 81/45 and 121/66 following the same instructions. Challenge students to find the simplified or lowest possible equivalent fraction. Answers: 3/7, 3/10, 9/5 and 11/6 and more possibilities!

TPT # 4 Area Model: Present a review of the box method or area model of multiplication where students were initially introduced to it in the third grade under common core 3.MD.7 where students relate area to the operations of multiplication and addition. Remind students with an example of the area model for multiplication by writing the following problem on the board and allow students to describe how you (the teacher) arrived at the values in the box.



Following the review of the area model, ask students to think backwards with the example below. What are the possible solutions for this model?



The goal of this activity is urging students to work backwards with their knowledge of place value to derive the original numbers multiplied to achieve the sum of 432. There are several possible answers for this model. A few possible answers are pictured here.

x	20	4
10	200	40
8	160	32

24 x 18 = 432

x	40	8
5	200	40
4	160	32

48 x 9 = 432

x	10	2
20	200	40
16	160	32

36 x 12 = 432

Challenge students with another example of:

x		
	400	40
	160	16

Possible answers: 22 x 28 or 44 x 14.   Could 56 x 11 = (40 +16) x (10 + 1) = 616 be a possible solution?

	30	4
x	2700	360
y	180	24

What are the values of x and y? Solution: x=90 and y=6

TPT #5: Area Model with variables and the distributive property
Students will study expressions, equations and inequalities in their 7th grade curriculum and presenting this model prior to this Expressions and Equations unit. Draw the following chart:

	50	2
40	2000	80
7	350	14

What does this chart show? (40 + 7) x (50 + 2) = 2000 + 350 + 80 + 14 = 2444

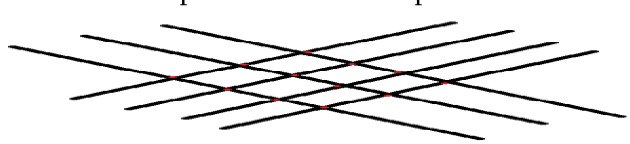
Area Model(Distribution)	C	D
A	AC or A x C	AD or A x D
B	BC or B x C	BD or B x D

What can students conclude from this chart? The goal is the get students to recognize that AC + AD + BC + BD = A(C + D) + B(C + D) = (A + B)(C + D)
Give students some values for a, b, c and d and let them arrive at some distributive property concepts. What is all the variables were negative digit integers? How would that chart compare to a chart with positive multi-digit integers? Encourage students to expand their chart to larger matrix of variables or numbers, such as abc x cde.
Let the variables represent one of the following single digits of 1 through 6, such that the sum of the nine boxes in the area model have the largest possible sum?

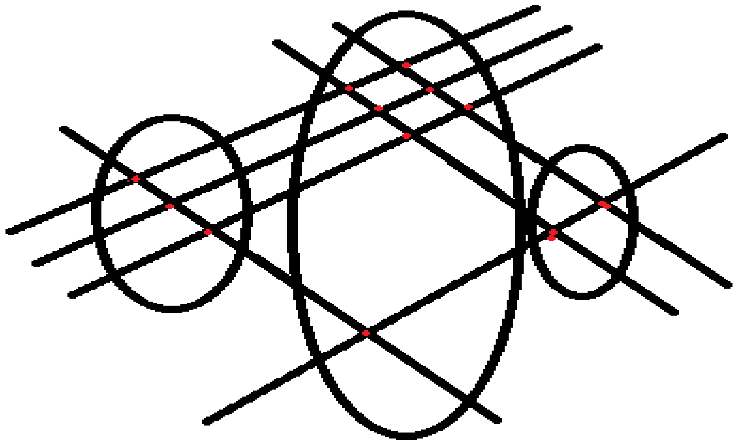
	4	5	6
1	4	5	6
2	8	10	12
3	12	15	18

The sum of the nine boxes equals 4+8+12+5+10+15+6+12+18 = 90, ask the students to find the combination with the largest sum. Using the above configuration, ask students if they discovered that 1+2+3 = 6 and 4+5+6=15 and 15 x 6 = 90. Will students experience the "aha" moment to find that largest arrangement? Answer: Sum should equal 110.

TPT #6 Chinese multiplication
This format of multiplication created a flurry of questions and thoughtful conversations about the understanding of this process, our current multiplication algorithm and how to use them together. With the diagram below, pose the questions: What could they see being represented in this diagram? Can they see numbers are represented and could this relate to multiplication? Students should arrive at the concept of two numbers being displayed as three and four. They should be able to count the points of intersection of the lines, which would represent their multiplied value.



Without a further explanation draw the following diagram (numbers) on the board.



Verbalize to students that the diagram represents multiplication of two 2 digit numbers. What are the numbers represented? Express to students that the diagram represents Chinese multiplication of two 2 digit numbers. If students are having difficulty with arriving at the numbers yet, draw the area model with a, b, c, d, in a 2 by 2 area model. Hopefully students will connect the area model with this diagram as the numbers represented are 31 x 12 = 372. Within each circle, students should count the points where the lines intersect. Notice in box A that the lines intersect 3 times, box B intersects 7 times and box C intersects 2 times, representing their multiplied value of 372. Have students draw the numbers represented with lines of the following problem and solve: 14 x 22. Once they have constructed the diagram, remind them to circle the lines of intersection of each of the three sections.

14 x 22 = 2(100) + 10(10) + 8 = 200 + 100 + 8 = 308

Have students create problems for each other in partners and compare their answers. Why would I present this concept as a thought provoking Thursday idea? Chinese teachers find that "solving one problem with several ways" suggests a higher level of ability with mathematics.[6]

TPT #7: Place Value Game

David Sousa states that "during the adolescent years, the search for novelty becomes more intense" and that a mathematical aptitude involves the transitioning of the new "ability of a student's brain to make right-to-left-hemisphere transitions" with repeated exposures these abilities move from novel ideas to motivated learning.[7]

This game has been modified from a Cathy Skinner lesson, Master Teacher with Betterlesson.com. Choosing this game allows students to review fourth, fifth and sixth grade math standards and future modifications will include seventh grade standards.

Begin the lesson by writing _, _ _ _, _ _ _._ _ _ _on the board. Use these blanks to ask students to name the value of each place represented by the dashes, (ten thousandths, thousandths, hundredths, tenths and ones, tens, hundreds, thousands, ten thousands, hundred thousands, millions).   As students verbally identify the place values, make a chart for students to record their vocabulary in their interactive notebook.

Materials: An online spinner with ten sections to represent the digits zero through nine. A few site options could include wheeldecide.com and gamequarium.com. Students will also need their interactive notebooks and pencils to record their numbers. Projecting a spinner for all students to view on a Smartboard allows visual and verbal clarity of the numerals.

Objective of the PVG: To create the largest and smallest numbers possible with the random digits spun on the spinner. Depending on the grade level, ask students to find either the largest or smallest possible number or both with upper grade level students.

One Rule: Students can't erase or move a number once it is written down. Continue to remind students as the game proceeds so students don't move numbers around.

Depending on the time available, allow students to interact with the online spinner at the Smartboard. Using this model with seventh graders, allow students to assist in spinning the wheel eleven times place their numbers to create both the greatest and smallest numbers. Once students have created their numbers, let students compare their numbers with classmates and explain why they are making the choices that they did. Ask students to share their numbers with the whole group. Ask students to reason quantitatively and critique their reasoning of numbers with place value knowledge.

Extended questions to ask students could include: 1. If the digit in the tens column is moved to the hundreds, how much more is it worth?   Conversely, if the digit in the ones column is moved to the tenths, how would you compare those values?

TPT#8 Place value problems
Credit for problems should be attributed to Harold Reiter and Kjartan Poskitt. For this activity, hoping to spark students' interests with curiosity as to why these selected problems work without much information from the teacher. The objective of this activity is to introduce letter or variables into the place value equations.

Materials needed or available: Calculator or online calculator

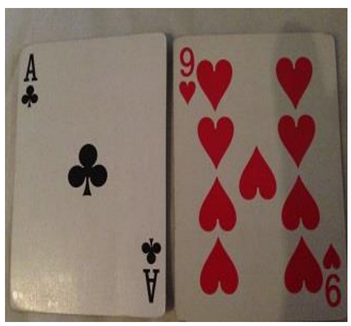
Problem 1: Students will be asked to choose any three digit number such as 783 or 342. Take that number and multiply it by 7 then 11 then 13.
Example: 783 x 7 x 11 x 13 = 783783 = 783 x 1001

What is the result? How does this answer compare to their original number? What would the answer look like if your number was abc?

Problem 2: Students will select any two digit number such as 78 or 34.
Take selected number and multiply it by 3 then 7 then 13 and finally 37.
Example: 78 x 3 x 7 x 13 x 37 = 787878 = 78 x 10101
Are students seeing a pattern? Result for ab?

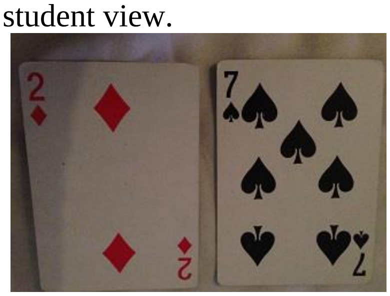
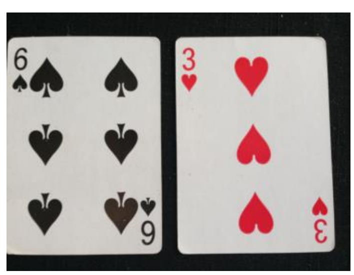
Problem 3: Given abcde, where any letter can represent any positive integer and a doesn't zero. What would abcde need to be multiplied by in order to arrive at abcdeabcde? Answer: 100001 or 11 x 9091

TPT #9 Place value problems (simple) Using base ten representation
Credit for these problems should be attributed to Harold Reiter.
In order to spur the interest of students, pull out a deck of cards. Then ask a student to remove the ten, jack and queen. The remaining cards will represent digits King = 0 and an Ace=1 with the remaining cards representing their principal number. Invite a few students to select four cards for the game setup. For example:

 X 

Teacher will place the cards in the format of ab x cd

Ask students to multiply these numbers 74 x 19 = 1406. If presented with **7a x b9 = 1406**, could you now find the values for (a) and (b). Students should be able to **work backwards** to arrive at a = 4 and b = A or 1. I have selected another example with my cards. Have these cards pre-selected with your own deck of cards but covered from student view.

 X 

This time try **a7 x 6b = 1701**, once again encourage the students to work backwards. Once students have reached their solution allow a student to reveal your cards.

Check for understanding of place value with this problem, if a and b are digits for which:

$$\begin{array}{r} 5a \\ \times\ \underline{b4} \\ 224 \\ \underline{1120} \\ 1344 \end{array}$$

Then a + b = and a x b =

A) 4 and 4 B) 6 and 8 C) 8 and 12 D)12 and 36

Answer: (C) a + b = 6 + 2 = 8

This time have two students each select a card and place it in the ab or cd setup. This will require the teacher to do some quick mental calculations to substitute values and have the students solve for a and b, but the final answer must be a + b. Answers will vary. Example: Student one selects a four card and student two selects a six card.

Write the following on the board:

$$\begin{array}{r} 4a \\ \underline{\times\ b6} \end{array}$$

Teacher will need to select some values for a and b and quickly write the following three lines. Suppose the teacher selects a =3 and b= 9, then the teacher would need to write 258 + 3870. This game or setup can be expanded to three digit numbers.

TPT #10 Place value problems (with letters)

Credit for these problems should be attributed to Harold Reiter. Students are given a problem to work in pairs and use their place value notations to solve as they will need to explain their answer. An online timer will be on the board and students must record their time for finding the solution and for arriving at the proof with place value notation. A prize for the winning pair could include a listen to music pass in class, free homework pass or candy (if allowed by administration).

Problem 1: 9 x abcd = dcba What is the value of abcd where a,b,c, and d represent distinct numbers? Using place value notation, what are some of the assumptions you could conclude for the value of a and d?

Proof: Students should be able to conclude that a= 1 and d=9 since any value larger than 1 for (a) would make the resulting answer 5 digits.

Substituting those values the new equation is 9 x 1bc9 = 9cb1.

Using decimal notation or place value notations, you could express the new equation as
9 x (1000 + 100b + 10c + 9) = 9000 + 100c + 10b + 1

By using the distributive property and combining like terms, the resulting equation would be: 9081 +900b +90c = 9001 + 100c + 10b

(9081 – 9001) + (900b – 10b) = (100c – 90c)

80 + 890b = 10c
Since (c) is a single digit, it can't be greater than 9. However, if c = 9, 80 + 890b = 90, there is no integer solution for b. If c = 8, the 80 + 890b = 80, (b) must be 0.
Therefore, abcd = 1089 as 9 x 1089 = 9801

TPT #11 Credit for these problems should be attributed to Harold Reiter, University of North Carolina at Charlotte Mathematics Professor. Spark interest in students by writing this problem on the whiteboard or Smartboard.

Given a, b, c, and d, let any letter can represent any positive distinct(each represents only 1 number) digit integer where (a) doesn't zero. Find the sum of two four-digit numbers:

    abcd
 + dabc
   7524

What is the value of abcd? Allow students to work with one another or verbalize their thought process. If help is needed, ask probing questions. Example of questions could include: Have you found any parameters for any specific letters or variables, such as (a + d) = 7, so a or d must be less than 7, what are those options?

Not distinct digits		Not possible as (b + a) < 20				
a	**d**	(a + d)	then (d + c)	then (c + b)	then (b + a)	abcd+dabc equals
1	6	7	if d=6, then c = 8	if c = 8 + 1, then b=3	if (3+1+1)=5 then abcd=1386 and dabc=6138	7524
1	5	6	if d=5, then c =9	if c = 9 + 1, then b=2	NP (2+1+1)=4 not 5	
1	4	NP				
a	**d**					
2	5	7	if d=5, then c =9	if c = 9, then b=3	NP (3+1+2) = 6 not 5	
2	4	6	if d=4, then c=0	if c = 0, then b=2	a can't equal b.	
2	3	NP				
a	**d**					
3	4	7	if d=4, then c=0	if c = 0, then b=2	if (2 +3), then abcd=3204 and dabc=4320	7524
3	3	NP				
a	**d**					
4	3	7	if d=3, then c=1	if c = 1, then b =1		
4	2	7	if d=3, then c=2	if c = 2, then b =0	NP (0+4) 4 not 5	
4	1	NP				
a	d					
5	2	7	if d=2, then c =2			
5	1	6	if d=1, then c=3	if c=3, then b=9	if (9+1+5)=15, then abcd=5931 and dabc=1593	7524

Given a, b ,c, and d, let each letter represent any positive distinct digit integer where (a) doesn't zero. Find the sum of two four-digit numbers:

$$\begin{array}{r} abcd \\ +\ \underline{dabc} \\ 6017 \end{array}$$

What is the value of abcd? Encourage students to create their chart or split up the work with one another. Possible solution: abcd = 2743 therefore dabc = 3274, where abcd + dabc = 6017 or 2743 + 3274 = 6017.

TPT #12 Students will build on the skills and the place value thought process the students used from the previous activities. Write the following problem on the board without any prompting and ask students to create a plan with a partner to solve abcd × 4 = dcba, where a, b, c and d are decimal digits. Hopefully students will be able to surmise the possibilities or options for a, b, c, and d. Possible values could include: abcd = 2178 and dcba = 8712

If students haven't exhausted themselves then pose this question. What is the solution for: abcde x 4 = edcba? Will students be able to quickly arrive at abcde = 21978, therefore edcba=87912. Why does this work? What is there hypothesis for abcdef x 4 = fedcba? Solution: abcdef = 219978 and fedcba = 879912. Did they notice in this example that the letters don't have to be distinct integers?

Appendix 1: Implementing Teaching Standards

3.MD.7 Relate area to the operations of multiplication and addition.

6.RP.A.3.A
Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.NS.A.1
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

6.NS.B.3
Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

6.EE.A.2
Write, read, and evaluate expressions in which letters stand for numbers.

7.RP.A.2.A
Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the originCCSS.MATH.CONTENT.7.NS.A.2
Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

7.NS.2
Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

Appendix 2

x	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						

X	7	8	9	10	11	12
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						

Resources

Bogomolny, Alexander. "Arithmetic Articles, Problems, and Puzzles." Arithmetic: Articles and Problems. Accessed May 2, 2015. This resource is loaded with fun ways to present math concepts with puzzles, tricks and games.

Saunders, Bonnie. "Introduction to Number Theory With Applications to Middle School Mathematics and Cryptography." Accessed July 24, 2015. http://homepages.math.uic.edu/~saunders/MTHT467_2012/MTHT467_complete. Included in this paper are excellent examples of cryptographic mathematics and other number theory concepts.

Huang, Jacky. "Teachers from UK Shocked by Chinese Multiplication Table | ChinaHush." ChinaHush RSS. January 23, 2013. Accessed July 10, 2015. http://www.chinahush.com/2013/01/23/teachers-from-uk-shocked-by-chinese-multiplication-table/. I found this article when I was researching why the Chinese culture is so quick with their mathematical abilities.

Fulton, Brad. "Fast Facts and Fractions Too!" Http://domathtogether.com/wp-content/uploads/2013/06/Fast-Facts-and-Fractions-Too.pdf. 2006. Accessed June 6, 2015. Mr. Fulton provided me with a free copy of his article with interesting concepts relating to the multiplication tables.

Poskitt, Kjartan. "The Official MURDEROUS MATHS Site." The Official MURDEROUS MATHS Site. Accessed July 13, 2015. I intend to order some of these books which make math interesting to students with intriguing stories. This website has fun math tricks as well.

Batterson, J. *Competition Math for Middle School*. Alpine, Calif.: Art of Problem Solving, 2010. This book is a great resource for math coaches for competition teams and an excellent resource for quick problem solving warm ups in the classroom.

"Mathematics Standards." | Common Core State Standards Initiative. Accessed October 17, 2015. http://www.corestandards.org/Math/. This resource was used to access specific common core content objectives.

Reiter, Harold. “Place Value Problems” http://math2.uncc.edu/~hbreiter/Mathworks/PlacevalueProblems.pdf. Dr. Reiter has been my largest resource of number theory information for my unit. His depth of knowledge has ignited the love of math which I hope is apparent in this unit. Thank you to Dr. Reiter for providing nights of interesting topics.

[1]Sousa, David A. "Teaching Mathematics to the Adolescent Brain." In How the Brain Learns Mathematics, 127 - 134. Second ed. Thousand Oaks, CA: Corwin Press, 2015.

[2] Posamentier, Alfred S., and Terri L. Williams. What Successful Math Teachers Do, Grades 6-12: 80 Research-based Strategies for the Common Core-aligned Classroom. Second ed. Corwin A SAGE Company, 2013, p. 68.

[3] Dweck, Carol S. 2012. Mindset the new psychology of success. [Kennett Square, PA]: Soundview Executive Book Summaries. http://www.books24x7.com/marc.asp?bookid=45525.

[4] Overstreet, Andy. "Creating School Standards That Make Sense for NC Children." The News & Observer, August 29, 2015, Op-Ed sec. Accessed September 23, 2015. http://www.newsobserver.com/opinion/op-ed/article32670282.html.

[5] Fulton, Brad. "Handouts for Purchase." Practical and Proven Professional Development. 2013. Accessed October 31, 2015. Fast Facts and Fractions Too! http://www.tttpress.com/handouts-for-purchase.html.

[6] Ma, Liping. *Knowing and Teaching Elementary Mathematics Teachers' Understanding of Fundamental Mathematics in China and the United States*. Mahwah, N.J.: Lawrence Erlbaum Associates, 1999. 140.

[7] Sousa, David A., p. 130.