



The Hot Hand and Other Ways to Relate Math in Sports

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This curriculum unit is recommended for:
Mathematics Grade 7

Keywords: Sports, Probability, Integers, Rational Numbers, Exponents, Hot Hand, NASCAR, Golf, Football

Teaching Standards: See [Appendix 1](#) for teaching standards addressed in this unit. (Insert a hyperlink to Appendix 1 where you've stated your unit's main standards.)

Synopsis: In this unit math teachers will be provided information related to the specific topics they are to cover such as statistics and probability, algebra concepts related to the use of formulas in real life scenarios as well as the rules of exponents and operations with integers. The concept of the hot hand phenomenon will be covered and then the idea of supporting or arguing against will be an exercise undertaken by students using the math skills they have learned to support their position. Using exponents to determine success in baseball and how that has changed the way the business of baseball is conducted is explored in the unit. Comparing great runners like Usain Bolt, Jesse Owens and others is possible with the use of mathematics by converting units and creating models. In the unit students will learn about speed and NASCAR and how important it is to hang onto the lug nuts during a pit stop. The games of golf and football will be looked at in a very different way that allows for students to use and learn mathematical operations with integers. The unit itself provides numerous ideas to teachers about how to use sports to explore mathematics.

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The Hot Hand and Other Ways to Relate Math in Sports

Michael Pillsbury

Background

I currently am in my tenth year of working in education. This year brought a change to my career. I was promoted to be the secondary math specialist for the Charlotte Mecklenburg School System. I left my classroom of nine years and I am now responsible for forty two middle schools and over four hundred teachers. When I started on this unit I was under the impression that I would still be in the classroom teaching seventh grade mathematics, as I had for the previous nine years. This unit is written for a seventh grade mathematics classroom and is aligned with the Common Core State Standards.

This will be my last unit with the Charlotte Teachers Institute as, by definition, the Institute is for teachers and I am now in a position of support and guidance for math teachers across all middle grades. The institute model is that it be controlled and guided by classroom teachers. It has been, without a doubt the best professional development that I have ever experienced.

The middle years of education offer unique challenges to teachers. Students are experiencing adolescent development and their bodies are changing as much as their minds and views on life. In mathematics, moving students from “concrete” thinking to “abstract” is critical to the success of each individual student. Given the topics of their math curriculum, students cannot physically handle or touch things in order to see the answers. They have to stretch their mind beyond what they see in the classroom and envision the outside world and the applications of mathematics in the world without actually being there.

I am keeping with the theme of the seminar of math in sports by using sports as a way to learn mathematics. By this I do not mean just learn how to do math but to use sports to explain mathematics in a way that students can understand and therefore be more proficient. Making a connection to real world situations is the very heart of Common Core. I believe that students learn more effectively and retain this knowledge by making the connections from the classroom to the real world.

Rationale

I want to use sports to address three areas of the seventh grade common core standards. I feel that these areas will combine rather nicely to make an interesting unit and provide students with a better understanding of statistics, probability and the use of problem solving (formulas). The current CCSS (Common Core State Standards) for statistics and

probability address the use of central tendency, comparing different sets of data, determining the probability that an event or compound events will occur, and recognizing representative samples from populations and drawing inferences from the population. I also want to use sports to solve the very common rate, time and distance problem that is ever prevalent in mathematics assessments, in some form or another. The understanding of percent's fits perfectly with sports and even sporting goods!

Students often inquire as to why they need to learn the mathematics being taught. They fail to see the real life connections. Using sports allows the educator to tap into a topic that most students are familiar with. A point of note that may be made is about whether girls will support this as something both exciting and interesting. Female athletes are very plentiful and I have found in my previous forays into sports with mathematics that girls do find it interesting to learn about sports through mathematics. It allows them to have a kind of sports literacy. The reason that they often do not like sports is because they do not understand what is going on within the game or the terminology. The teacher should keep this in mind when using sports with mathematics instruction. It may be necessary to explain what a "down" is in football or what a free throw is.

Statistics and Probability

Common Core State Standards

The way I would like to start my unit is with an understanding of statistics and probability. North Carolina follows the Common Core State Standards and the seventh grade standards that fit well include 7.SP.2 in which students will collect sample data and then make generalizations about a population based on the collected data. Students will also address 7.SP.3 in which students will compare two data sets of information and build on their understanding of mean, median, inter-quartile range and Mean Absolute Deviation from 6th grade. The main concept is to find the variability of the data. Standard 7.SP.4 requires students to use measures of central tendency to draw informal comparisons about the two populations. Understanding the probability of an event and expressing its likelihood is standard 7.SP.5. The standards 7.SP.6 and 7.SP.7 are very similar and require an experiment be carried out to determine the relative frequency based on the actual experiment.

The first standard being addressed (7.SP.2) will require students to take surveys and then record their results. The richness of this is that students can be exposed to bias and the concept of random sampling. I would not want to take a sample of middle school students and ask them about their experiences with car insurance companies. In keeping with the sports in math concept I would encourage students to take surveys related to sports.

Students should be allowed to choose their own survey questions and then decide whom they want to survey. When the survey is complete then they should draw conclusions based on the survey results. They must support these results with the data they have collected.

The next standard (7.SP.3) requires some data be available. Options abound with this from the teacher’s perspective. The students could collect their own data or have it provided to them. If students collect their own data it is imperative that they know what they need. The concept is to collect data about two different items. The data should be similar in nature. A good example might be weights or heights or scores of two different teams or groups.

If I have the following data from two different sports:

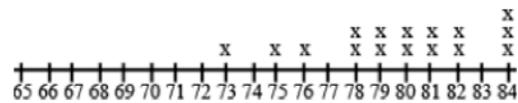
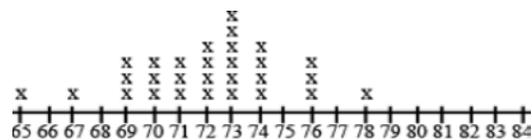
Basketball Team – Height of Players in inches for 2010 Season

75, 73, 76, 78, 79, 78, 79, 81, 80, 82, 81, 84, 82, 84, 80, 84

Soccer Team – Height of Players in inches for 2010

73, 73, 73, 72, 69, 76, 72, 73, 74, 70, 65, 71, 74, 76, 70, 72, 71, 74, 71, 74, 73, 67, 70, 72, 69, 78, 73, 76, 69

The objective is to compare the heights of players on a basketball team to those on a soccer team. The way this is done is by comparing their measures of variability. If a dot plot is constructed:



Observing the dot plots reveals that there is an overlap between the two data sets. There do not appear to be any extreme outliers. Students should have understood from

sixth grade that finding the mean and mean absolute deviation (MAD) would be the best way to compare these two sets of data.

The mean height of the basketball players is 79.75 inches while the mean height of the soccer players is 72.07 inches. Their mean heights vary 7.68 inches. The next step is to determine the mean absolute deviation or (MAD). The procedure to do is to take the absolute value of the difference between the actual number and the mean number. Once this value is determined then the mean of the differences determines the mean absolute deviation (MAD).

Example I have two test scores of 85 and 93. The mean score is 89. The first test score differs from 89 by 4 and the second differs by 4. I add these numbers to get 8 and then divide by 2 to get 4. This gives me a mean absolute deviation (MAD) of 4. The tables (Fig 1 and Fig 2) in the appendix show how the MAD is determined for the basketball and soccer players.

The MAD for the soccer players is 2.14 inches and 2.53 inches for the basketball players. To determine the measure of variability the difference in mean heights is divided by the MAD.

$$(7.68 \div 2.53 = 3.04; 7.68 \div 2.14 = 3.59)$$

This results in a measure of variability of 3.04 for the basketball players and 3.59 for the soccer players. The conclusion drawn is that there is slightly more variability in the height of the soccer players.

Standard 7.SP.4 calls for students to draw informal inference from the measure of central tendency for two different sets of data. Students would need to determine the best measures of central tendency (mean, median, variability MAD and the inter quartile range) to draw these conclusions from.

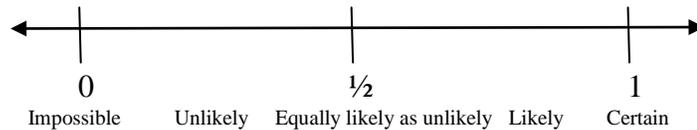
An example of this would be based on the following data:

Company A: 1.2 million, 242,000, 265,500, 140,000, 281,000, 265,000, 211,000
Company B: 5 million, 154,000, 250,000, 250,000, 200,000, 160,000, 190,000

Since both companies have extreme outliers then comparing medians would be the best approach. The outliers skew the mean and thus utilizing the variability MAD would not be the best approach.

Standard 7.SP.5 introduces students to the concept of probability. The best way to begin talking about probability with students is by discussing percents. Most students will be familiar with the concept of percents (ratio of a number to 100). In their mind they

should grasp, perhaps through examples of rain prediction that is a probability is close to 100% then it is almost certainly going to happen. In the number line below it shows from zero to one. Students have to understand that 1 is 100%. This can be demonstrated by setting up a simple ratio of $100/100 = 1$.



I would encourage teachers to allow students to explain the number line and how it would relate to percents. Start with saying there is a 25% chance of rain. Students should be able to explain that rain is unlikely.

The last two areas that would be addressed involve the development of a model where experiments can be conducted and based on the results of the experiments the idea of relative frequency can be determined.

I would then want students to understand the concept of theoretical probability and compare that with experimental probability.

Probability can also be used to determine the chance that something will happen. For example if I want the probability of getting two heads in a row if I flip a coin twice how do I do it. The probability of getting a head on a coin is one out of two. What does this mean? It means that there are two outcomes possible and one of those is a head. This is critically important to relate to students as teachers often just get caught up in the numbers and the calculations to get an answer. It makes sense then that if I flip a coin once I have a $\frac{1}{2}$ chance to get a head and then flipping it again gives me another $\frac{1}{2}$ chance. Teachers often tell students to just multiply and get $\frac{1}{4}$ so we have our answer. This is correct however we should break this out and explain first why we multiply and second what $\frac{1}{4}$ means.

First Flip	Second Flip	Possible Outcomes
H	H	HH
H	T	HT
T	T	TT
T	H	TH

In the above table we have all the possible outcomes that could be obtained when flipping coins. There are a total of four possible outcomes and one of those outcomes is HH, which is what we wanted.

Why did we multiply? If we looked at a coin flipped three times would we have six outcomes?

First Flip	Second Flip	Third Flip	Possible Outcomes
H	H	H	HHH
H	T	T	HTT
H	H	T	HHT
H	T	H	HTH
T	T	T	TTT
T	H	T	THT
T	T	H	TTH
T	H	H	TTH

We would get eight outcomes. Students should investigate this and discover why this works. In the above table a good question might be what the probability I would get at least is two heads in I flipped a coin three times. The answer is $\frac{4}{8}$. I think it is important to leave the initial answer as just that $\frac{4}{8}$ so students can accurately say that there are 4 possibilities out of 8 total outcomes. This could then be reduced to $\frac{1}{2}$ with discussion about this.

Algebra

I want students to understand how to take real world scenarios and be able to solve real world problems. I have listed this under the umbrella of Algebra but what I want to incorporate involves so many areas such as operations with integers and real numbers as well as solving ratios and proportional problems.

Solving rate, time and distance problems presents us with one of the most well-known math formulas ($RT=D$). This formula means that the rate multiplied by the amount of time will result in the distance covered or travelled. This seems like common sense on the surface but can be a little more complicated than it may seem. Teachers often make the mistake of presenting the formula to students and then having them “plug and play.” This is a term that is modern but means that they simply find an item in the word problem and substitute into a formula and then solve for the missing variable in the formula.

With $RT=D$ the rate should be a unit rate. Unit rates are rates per one of something. An example of this would be miles per gallon or price per pound or dollars per hour. Note here that the denominator is a single unit while the numerator does not have to be. In a $RT=D$ the denominator unit should be the same as the time unit. If I had a rate of 5 feet per second then the time would have to be seconds. Here again is a rich opportunity for teachers or a teachable moment, as many call it. If I had a problem where I was given a rate of fifty miles per hour and a time of 6 hours have a student explain the math of why our answer would be three hundred miles. Let us look at what happens mathematically.

First Example $\frac{3}{5}(5)$ can be solved by reducing prior to carrying out the multiplication. $\frac{3}{5}(5) = 3(1)$ which equals three. The fives reduced down to a value of one. In the second example $\frac{3}{x}(x)$ can also be reduced to $3(1)$ which equals three. The x's reduce to be one. In the third example $\frac{3}{x}(4x)$ the x's here also reduce to have a value of one which leaves us with an answer of twelve. Students and sadly some teachers will say that they "cancel" out. I highly recommend that this word be avoided and that the idea that they reduce to a value of one be understood. This applies to the RT=D formula as an explanation of why the distance comes out to be understood as a unit of measure rather than a ratio of two things. Of course rate could be broken down to unit of length/unit of time multiplied by the unit of time which equals a unit of length. If we assign variables to this scenario where we let l= unit of length and t= unit of time then we would have a formula that looked something like this $\frac{l}{t}(t) = total\ length$. Note how the t's would reduce to be one and thus leave only the length as a unit. I think as math teachers we often just "do the math" or show how to do the math and we fail to really understand what is happening. We explain about coefficients and variables but students often do not see how this relationship actually applies to units. A unit acts similar to a variable when solving problems. This concept holds true when converting units of measures. If I look at Usain Bolt who ran a 9.58 sec 100 meter dash and I wanted to know what his average miles per hour rate was then I would need to convert.

$$\frac{100meters}{9.58\ sec} \times \frac{3.28\ feet}{1\ meter} \times \frac{1\ mile}{5,280\ feet} \times \frac{3600\ sec}{1\ hour}$$

In the above formula we can see where the units act similar to variables and when converting we actually reduce them down (cancel out) leave us with miles per (over) hours. We can then reduce this down to a unit rate. We get 1,180,800 miles per 50,582.4 hours. This reduces to 23.34 miles per hour!

Exponents

This can also be tied directly to rules or laws of exponents. Connections should be made whenever you are teaching mathematics. Students must see that what they are learning today will be a base or platform for learning in the future. Math should be understood not done. In rules of exponents I often ask my students what is the value of a number raised to the zero power. Most know it to have a value of one but I do get an occasional zero. The key here is the next question. Why does something raised to the zero power have a value of one?

If I multiply to numbers with the same base students will often know that they simply add the exponents together. This is a rule they are taught or what I call a trick as they often do not understand why to do it they just know to do it. $4^2 \cdot 4^2 = 4^4$ can be rewritten as: $4 \cdot 4 \cdot 4 \cdot 4 = 4^4$ Students can see rather easily why they add the exponents when a problem is rewritten. They should also understand that this same rationale applies to variables. They can substitute an x for the 4 and get x^4 as an answer.

Students often forget what to do when numbers are being divided and they have the same base. Some will guess and say subtract. That of course is correct but make sure that they understand why this is true as well. In this example I will use variables.

$$\frac{x^3}{x^2}$$

This could be rewritten as:

$$\frac{x \cdot x \cdot x}{x \cdot x}$$

The x's divide to

$$\frac{x \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x}}$$

Leave us with an answer of x

What happens if we have the same number of x's in the numerator as in the Denominator then we will always have a value of one. This is exactly what happens when we cancel out our units during conversions.

Exponents figure prominently in baseball and basketball. In our recent history math has been discovered a s a way to predict or determine success in sports. In the movie "Moneyball" the Oakland A's General Manager Billy Beane selected players based off mathematical formulas for success. This approach was revolutionary and very controversial at the time. It has since proven to be very effective and the Boston Red Sox, winners of three of the last ten World Series, are proof. Bill James developed a formula:

$$\text{Win} = \frac{\text{runs scored}^2}{\text{runs scored}^2 + \text{runs allowed}^2} = \frac{1}{1 + \left(\frac{\text{runs allowed}}{\text{runs scored}}\right)^2}$$

Where *Win* is the winning ratio generated by the formula. The expected number of wins would be the expected winning ratio multiplied by the number of games played.¹ For instance if a team scored 10 runs but allowed 4 to be scored against them then the ratio would be .86 and if we played 5 games we could expect to win 4.3 games or round to 4.

It does make sense that a team could win 4 out of 5 games if they scored 10 runs to every 4 scored by the opponent overall. If we played 100 games though now we have only won 86 games out of 100. In a typical baseball season teams play 162 games so a team could expect to win 139 games if they scored 10 runs to every 4 their opponent scored. This formula proved accurate for the 2002 Yankees who scored 897 runs and gave up 697 they should have won 101 games but instead won 103 games. The formula has been tweaked but with seventh graders I would suggest staying with the original formula.

Operations with Integers

What is an integer? Most students think it is a negative number. The fault here falls directly on the teachers' shoulders. In order to understand numbers I find that using the number line is the best way. Students are exposed to a number line very early in their education but teachers often stray away and it is a golden opportunity for learning.

Teaching math is not about teaching one idea and then another and then another. I think that you get my point here. Teachers have to show the connection of mathematics. Start with the first numbers that students learn. It may seem rudimentary with middle school students but it shows the connections. If we were to look at the history of numbers I would imagine we might find this to be a very interesting relationship.

Starting with the numbers we count on our pudgy little fingers we call these counting or natural numbers. Writing these numbers on a number line will be the first step. The next set of numbers would be whole numbers and we just write in a zero. Pointing out that students do not start counting with zero on their fingers can lead to some interesting conversations about the number zero. Now we add in negative numbers since we are heading in that direction on the number line. This makes up the set of integers. Point out that we are just building off the previous set of numbers. Many of my colleagues like to use Venn diagrams to support this but be careful when introducing these as it can get very confusing to students. What numbers are we missing? The numbers that are between the numbers we have written in. Using a ruler here makes great sense as it has lines marked on it to show that there are indeed numbers between the numbers we have written. Here we have a great teachable moment. The idea of mixed numbers and fractions with a value greater than one (improper fractions) is often misunderstood by students. These numbers fall into the realm of rational numbers.

Showing students why numbers are rational is very important. Teachers often just say that rational numbers are numbers that terminate or repeat and that irrational numbers never terminate and never repeat. Great definitions for vocabulary purposes but very weak for math purposes. I believe students should know exactly what a rational number is. The next best level of explanation is that a rational number is the ratio of two integers. This can confuse students to think that rational numbers are only fractions. This of course

is not true as all integers, whole and natural numbers are rational numbers. We are building from these numbers.

In my first year of teaching I made the mistake of trying to just use a calculator to show that $1/7$ was a rational number. What I had forgotten was that the calculator rounds the last number on the screen. I then instructed my students to do division as long as they could on $1/7$. This proved to be full of mistakes as well because many students are very weak at long division and asking them to go on and on proved a battle. I did discover from my colleague Dr. Harold Reiter (University of North Carolina at Charlotte) that repeated multiplication could also be used.

Since we use the base ten number system we simply multiply by ten and keep the whole numbers as our next decimal place value working down from $1/10^{\text{th}}$. The fractional remainder is then multiplied by ten and the process is repeated over and over again.

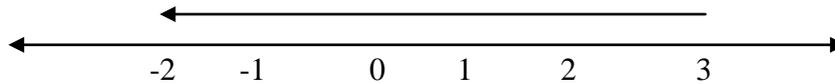
$$\begin{array}{l} 1/7(10)= 10/7 \text{ or } \{1\} \text{ and } 3/7 \\ 3/7(10)= 30/7 \text{ or } \{4\} \text{ and } 2/7 \\ 2/7(10)= 20/7 \text{ or } \{2\} \text{ and } 6/7 \\ 6/7(10)= 60/7 \text{ or } \{8\} \text{ and } 4/7 \\ 4/7(10)= 40/7 \text{ or } \{5\} \text{ and } 5/7 \\ 5/7(10)= 50/7 \text{ or } \{7\} \text{ and } 1/7 \\ 1/7(10)= 10/7 \text{ or } 1 \frac{3}{10} \text{ (begins to repeat)} \end{array} \qquad 0.\overline{142857}$$

The next concept of numbers involves the numbers between the rational numbers. My friend Dr. Tim Chartier (Davidson College) does a mime performance in which he takes an imaginary segment of rope or string and cuts it to get a smaller section. He then cuts this and cuts again and again and again, repeating this with the facial expressions of an exhausted and frustrated person. What is showing in this performance is that there are an infinite number of numbers, or in his show amount of string, contained in one section. These numbers that continually get smaller and smaller are irrational. Demonstrating to students this concept through performance is brilliant. The only way to make these smaller and smaller numbers is through the use of irrational numbers. Teachers often will tell students that the number pi is irrational as it goes on forever and never repeats. I would recommend this as well as the roots of numbers that are not a perfect square, cubes etc. The idea that these numbers exist and where they exist is important.

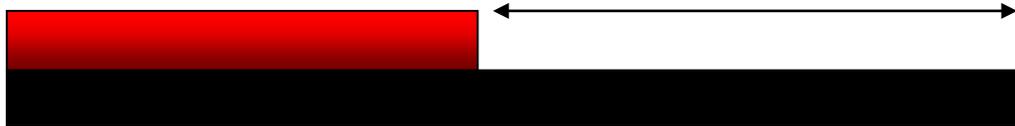
Operations with integers can be shown in numerous ways and teachers will argue that their way is better than another. I prefer to use what works for me and it is based on Singapore math. When first introducing adding with integers many teachers will use red and black chip manipulative to first explain the concept of a zero pair and then actually perform the operations. The concept of zero pairs is interesting at first because if we understand this than we can discuss absolute value. I inject this concept here because I

feel again that these are teachable points and they fit well. If I add a negative one and a positive one I will get zero. If I add any number and its opposite I get zero. The question to be asked here is why? If we explain that absolute value is the distance from zero then 5 and -5 are the same distance from zero. There is no difference in their distance or zero difference in their distance. This should be a light bulb going off moment. This is such a deep concept because we are talking about differences while doing addition. When we actually do addition of $-3+5$ we are finding the difference between their distances from zero. The same can be said for $3+5$ but here we adding the distances from zero for each because neither is a negative number.

What happens when you have $-3-5$? Since both are on the same side of zero we simply add their distances from zero to get -8 . In this situation be careful to explain that this is not an absolute value situation. What if we have $3-5$? I think the use of a number line is important here



Using bars is a method that I have always preferred as well. Let's consider $-32+80$



This addresses the zero pair idea with the chips and moves us closer to a number line concept.

When multiplying or dividing integers I prefer to use the concept of money. I give an example of $3(-5)$ as if I owed 3 people five dollars apiece. How much do I owe in total? I would owe a total of \$15 which should be written as -15 . If those same people owed me five dollars apiece $-3(-5)$ then I would be up or in the positive \$15. If I owed a total of 20 dollars to 4 people $-20/4$ then I owed each person five dollars or -5 . If I was owed \$20 by a total of 4 people $-20/-4$ then each one would owe me five dollars.

Activities

Hot or Not?

A question I would like to pose to my students will begin with a video. The video will be of Michael Jordan hitting 6 straight three pointers in the 1992 NBA Championships against the Portland Trailblazers.

What I want to have students discuss and then support is there such a thing as being hot or on fire? For the 1992 season Jordan shot 27% from 3 point range. In the playoffs he went 17-44 meaning he hit six straight and then went 11 for 38 so for the whole series he shot 38.6%. I would like students to understand that this is only over 22 games not the 82 for the whole season. In that game alone he went 0-4 after making 6 straight the next night he went 0-4 the night after that 0-1 and then 2-6, 2-4 and 2-3. By investigating this I hope to get students to understand probability a little more. I will also try to investigate scenarios that Jordan or their favorite players might have to do in order to increase their probability. We can then investigate percent of increase and decrease.

We mention Michael Jordan as a blast from the past but my favorite player, currently playing, is Steph Curry with the Golden State Warriors. Steph went to college at Davidson College and I owned season tickets. Watching him was a real pleasure. I have mixed feelings about being hot or not. Last year in the NBA Steph scored 54 points in a game on 2/27/13. During that game he went an astounding 11 for 13 from beyond the 3 point line. In the games following that he went 3 for 11, 3 for 9, 2 for 5 and 2 for 5. He shot an amazing 85% for that one game then 27%, 33%, 40% and 40%. His season average for the 2012 and 2013 season was 45%.

In this activity you could share these examples or look up some examples of your own. The important thing is not to provide your own opinion to sway students in any particular direction. Studies have been done on this phenomenon. In a study done by Gilovich, Vallone and Tversky² they found that the probability of success on any given shot was essentially independent of previous shots. They also related that a player was more likely to take the teams next shot (.25 to .20) after making a shot.³ What studies did find was that players were more likely to take the next shot and often because of their perceived hotness take a shot from a greater distance or more difficult shot.⁴ In addition to the player's own confidence the defenders were more likely to guard the hot player more intensely since they believe in the hot hand phenomenon or their coaches do.⁵

Teachers should have students explore this and using numbers support their opinion. I would suggest that you have them work in groups of no more than three. They can investigate this and support it with a presentation to the class. By support I would suggest the use of data. What is accomplished for the students is that they are able to determine

percent's, use them in real life situations and understand that they how they relate to probability can vary. Sample size will also be a very key concept. If we just look at one game such as Jordan's NBA playoff game or Steph Curry's 54 point game is this an accurate portrayal of their overall ability? Were they hot or not? Did they do better or worse before or after the hot streak? How does the hot streak contribute to their overall 3 point percentage? These are just a few of the discussion questions that should be addressed by your students during their work.

Which One?

In this activity I would have students try to determine who has the greater possibility of only making one point in a one and one free throw situation in basketball. I will explain to the students that in college basketball a one and one is awarded to a player who is fouled by a player who commits a seventh team foul that occurs when the player in possession of the ball is not in the act of shooting. The player is allowed to take one free throw shot and if the player makes it they are awarded a second free throw shot. If they miss the first shot then the ball is alive and in play.

My scenario involves a 30% free throw shooter and a 70% free throw shooter. The students will be asked who has the better chance at only making one point in a one and one situation. The worksheet (Figure 3) will be the sheet given to the students.

When students have completed the worksheets the real learning will begin. If they have done their math correctly then they will realize that the probability is the same for any two shooters whose free throw percentage adds up to be 100%. Students should then discuss with the teacher why this is true? The reasons are very opposite as one is a great free throw shooter and the probability they would miss a shot is therefore dramatically reduced. The other is a less efficient free throw shooter and therefore the probability of making a shot is greatly reduced.

Points Scored

I intend to have students use sports statistics to determine MAD (mean absolute deviation). The concept here would be to have students use media sources to gather stats and then determine the MAD and compare them and then draw inferences about the results.

The teacher should suggest such things as number of points scored by basketball players on one team compared to another team on any given night. They could use points scored in a season for a team compared to another team in any sport. Students should compile the data, determine the MAD and then indicate what can be interpreted from this information.

How Far is Time?

I really would like to use NASCAR and track and field for another area that I think students will find interesting. How to use rate, time and distance often perplexes students. This problem compounds itself when the units need to be converted. In track and field converting an event to miles per hour provides interesting challenges. Here we have metric to imperial or English measuring units as well as other conversions that need to occur. In NASCAR speed is relayed to drivers in the form of time. A driver is 1.1 seconds behind not so many miles per hour slower. I want students to contemplate why this is done? We will look at some NASCAR video and visually investigate this as a car enters and exits a corner qualifying visual displays and comparisons to the pole sitter. I also want students to look at qualifying speeds and then think about how much slower a car is at the rear of the pack and how quick he could be lapped. What factors into this?

NASCAR Activity

There is so much that can be done with NASCAR related to mathematics. I will just take a few big ideas and place them together to construct what I will call a super activity. I am inferring that this super activity could be broken up into many smaller activities.

Charlotte Motor Speedway has what they call a “Pole Night.” This is a very common expression in racing and refers to the night when the drivers race on the track all by themselves to see who can obtain the fastest lap time. The pole position (first place) is determined by the driver who makes it around the track the quickest in one lap. They will often give the drivers two laps and take the quickest lap time. The term pole came from horse racing where the fastest horse was placed on the inside closest to the pole.⁶

Jeff Gordon qualified October 10, 2013 with a time of 27.791seconds to complete a lap. The next qualifier was Greg Biffle at 27.841seconds to complete a lap. I would first have students determine how many seconds slower was Biffle than Gordon and then have them determine the difference in speed (miles per hour) of the two cars. They of course would need to know that Charlotte Motor Speedway is a 1.5 mile long track.

In the next problem we will say that a driver *A* was racing in a 500 mile race and he was averaging 150 miles per hour. A competitor, driver *B*, was racing at an average speed of 149.5 miles per hour in the same race. How far behind the driver *A* would driver *B* be at the end of the race? How many seconds or perhaps minutes would he be behind?

A pit stop is what occurs during a race so that the crews can put new tires and gasoline into the race cars so that they can complete the race and have the best chance of winning. Pit stops during green flag conditions can be very costly if done poorly. We could have students determine the amount of time and distance a driver would lose to a competitor if

they had a bad pit stop. This could be something as small as dropping the lug nuts or the car not jacking the car up quickly. Have students make up their own scenarios and give times to these mistakes. What if a dropped lug nut cost three seconds of time on a pit stop?

Track and Field

Usain Bolt ran a 9.63 second 100 meter yard dash at the summer Olympics held in London England in 2012. When talking about this incredible feat with students I am often asked how fast did he run? What a great question for students to answer. Converting units is a very difficult task for many students but converting two units is even more confounding to many.

Video games are very popular on our society, especially among our youth. One of the more popular types of games is when current superstar athletes are compared to an older generations' superstar athlete and they are able to compete in the video game. What if we could do this in the world of math? Could we compare Jesse Owens and Usain Bolt? We have the times that they ran in the 100 meter dash so why not? The interesting thing is to see exactly how much faster athletes have become. In the table (figure 4) we have times recorded from 1896 to 2012. Students could work on converting these times into miles per hour and then distances. I would recommend that you have them create a display of some kind showing where the runners would be when Usain Bolt crossed the finish line. This could be kind of mini project. They should also research who the runners were and display their speed in miles per hour. Perhaps they could go even further and research animals and their top speed and compare that to Usain Bolt.

Integers in Sports

Football

The National Football league is the number one sport in the United States based on attendance and television ratings. It is also one of the few sports where negative numbers occur. Making a mock football game and having students play might help them to understand how to add and even subtract negative numbers. A football field is much like a number line. The field is lined for fifty yards in two different directions. This could be compared to a number line.

The plays that occur in football can result in positive or negative yardage. The factor that could be added is penalties. What if a play results in a 3 yard loss because the running back was tackled as soon as he got the ball in the backfield? This could be understood as a -3 on total yards gained. In football however there are penalties and the team who is the offender loses yards based on the severity of the penalty. In our scenario here there was a 5 yard penalty for a defensive infraction. The ball is moved from its original place, before the play, 5 yards. If the offense was on the 20 yard line, on their

side of the field, then it would be placed at the 25 yard line. The play itself ended at the 17. So we have $20 + (-3) + x = 25$. We really have a net gain of 8 yards because of the penalty, correct? What if a player started at the 20 gained 5 yards but was then penalized five yards. This could be written as $20+5-5-5$ to determine the yard marker the ball is now placed. Have students describe what each number means. The 20 is the original placement of the ball, the runner gained 5 yards, the team lost five yards because the play was negated, the ball was moved back five yards for the penalty.

What happens when a team is on their opponent's side of the field? They are actually gaining yards as they move down the field but the place on the field is going down. For example a play started at the 30 yard line that gains 4 yards ends at the 26 yard line. In this case how could we write an equation to show this when we know we gained 4 yards?

I would encourage you to use my ideas and create scenarios or even better have your class develop scenarios where integers are used. They could even get into multiplying integers by looking at multiple penalties or loss of yards.

Golf

The game of golf is a game where negative is good. I think at first a teacher should explain golf and the terms associated with the sport. Par is the number of times the ball is hit from the tee to the hole. A par 5 hole suggests if the ball is struck 5 times and end up in the hole then this is a zero score. This can be a route to a problem. In golf if I hit it 4 times then my score is a negative 1. Meaning I used one less stroke, or hit of the ball, than was par for the hole. This is good because the lowest overall score wins the match. If I did the math for this scenario $5-4 = 1$. So shouldn't I have a positive 1? We could go in two ways here. I would recommend having a student figure out mathematically how to make this work. One way would be to say that a par 5 is actually a negative 5 and then I add a 1 to the par every time I hit the ball. In this case if I hit it 4 times then I would have a -1 as a score which is in fact how golf is scored. So what would be score if I hit it 6 times?

We could use things to create a sample golf game like shooting paper balls at trashcans either strategically placed around the room or outside. Teachers could provide a mock game situation and have students figure out a score. The richness of this also comes with adding both positive and negative numbers.

Figure 1

Soccer Players ($n = 29$)		
Height (in)	Deviation from Mean (in)	Absolute Deviation (in)
65	-7	7
67	-5	5
69	-3	3
69	-3	3
69	-3	3
70	-2	2
70	-2	2
71	-1	1
71	-1	1
71	-1	1
72	0	0
72	0	0
72	0	0
72	0	0
73	+1	1
73	+1	1
73	+1	1
73	+1	1
73	+1	1
73	+1	1
74	+2	2
74	+2	2
74	+2	2
74	+2	2
76	+4	4
76	+4	4
76	+4	4
78	+6	6
$\Sigma = 2090$		$\Sigma = 62$

Figure 2

Basketball Players (n = 16)		
Height (in)	Deviation from Mean (in)	Absolute Deviation (in)
73	-7	7
75	-5	5
76	-4	4
78	-2	2
78	-2	2
79	-1	1
79	-1	1
80	0	0
80	0	0
81	+1	1
81	+1	1
82	+2	2
82	+2	2
84	+4	4
84	+4	4
84	+4	4
$\Sigma = 1276$		$\Sigma = 40$

Figure 3

Player Free Throw Percentage	0 PTS (Probability)	1 POINT (Probability)	2 POINTS (Probability)
10%			
20%			
30%			
40%			
50%			
60%			
70%			
80%			
90%			
100%			

Bibliography for Teachers

Attali, Yigal. "Perceived Hotness Affects Behavior of Basketball Players and Coaches."

Psychological Science 24, no. 7 (2013): 1151-1156.

This source was used to provide researched information on the hot hand phenomenon.

Gilovich, T. "The Hot Hand In Basketball: On The Misperception Of Random Sequences*1." *Cognitive Psychology* 17, no. 3 (1985): 295-314.

This source was used to provide the scholarly research information when discussing the hot hand phenomenon.

Wikimedia Foundation. "Pole Position." Wikipedia.

http://en.wikipedia.org/wiki/Pole_position#Origin (accessed November 23, 2013).

Used to describe where the term pole position came from.

Wikimedia Foundation. "Pythagorean expectation." Wikipedia.

http://en.wikipedia.org/wiki/Pythagorean_expectation (accessed Reynolds,

Gretchen. "Are 'Hot Hans' in Sports a Real Thing." *NY times (New York)*, May 8, 2013.

Reading Materials for Students

None recommended

Other Materials for Classroom

Use <http://youtu.be/G8OqJqOIdb4> (You Tube Video Link to 1992 Michael Jordan Highlights)

Appendix 1: Implementing Common Core Standards

7.SP.2

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates of predictions.

7.SP.3

Informally address the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a measure of variability.

7.SP.4

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

The way I would like to start my unit is with an understanding of statistics and probability. North Carolina follows the Common Core State Standards and the seventh grade standards that fit well include 7.SP.2 in which students will collect sample data and then make generalizations about a population based on the collected data. Students will also address 7.SP.3 in which students will compare two data sets of information and build on their understanding of mean, median, inter-quartile range and Mean Absolute Deviation from 6th grade. The main concept is to find the variability of the data. Standard 7.SP.4 requires students to use measures of central tendency to draw informal comparisons about the two populations.

¹ (Foundation 2013)

² (Thomas Gilovich 1985)

³ (Attali 2013)

⁴ (Reynolds 2013)

⁵ (Attali 2013)

⁶ (Foundation, Pole Position n.d.)