The Art of Trigonometry

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Background

This curriculum unit is intended to help students of Advanced Functions and Modeling (AFM) and/or Honors Pre-Calculus understand and use the trigonometric functions $y = sin(x)$, and $y = cos(x)$, and their associated graphs.

In the North Carolina Standard Course of Study (NCSCOS), AFM is essentially the non-honors, or standard, pre-calculus course. (See Appendix for specific standards implemented.) It covers much more trigonometry than the right triangle trigonometry which is introduced in Geometry and briefly touched upon in standard Algebra 2. In my classes, I generally spend about four weeks of instruction and assessment time covering all of trigonometry. The portion devoted to the graphs of $y = sin(x)$ and $y = cos(x)$ usually lasts about one week.

I teach in Charlotte Mecklenburg Schools (CMS), an urban school district which serves approximately 135,000 students in grades K-12. The high school in which I teach is a large, suburban school located in Cornelius, NC, in the northern part of Mecklenburg County. The school is brand new, having just opened in August 2010, and currently serves about 1500 freshman, sophomore and junior students. The student body consists of 76.6% white students, 11.1% African-American students, 2.2% Asian students, 7.0% Hispanic students, 0.4% American-Indian students and 2.8% Multi-Racial students.[1] District practice is to open new high schools with no seniors, so the AFM classes this year are comprised of all juniors. The school draws from a population of relatively high socioeconomic status, and has a very active and well-funded PTSA. Therefore, resources not normally available through the school system may be procured with the assistance of the PTSA.

The classes I teach are on a 4 x 4 block schedule; all lessons and activities take place during a 90-minute class period. The activities can be completed during a shorter class period, so if your school follows a traditional schedule with 40-50 minute classes, this unit is still applicable. The lessons and activities in this curriculum unit are designed to be taught in any classroom. Desks can be arranged in a traditional rows and columns layout, or in groups for cooperative learning. One of the activities requires all students to have access to computers connected to the Internet, either in the classroom or in a computer lab. My classroom has a projector connected to the computer, and I use Microsoft PowerPoint presentations for many of the lessons, but they can be taught using

overhead transparencies, a blackboard or whiteboard, or a Promethean or Smart Board if available.

Objectives

The primary goal of this unit is to get students to be able to visualize a concept that can be very difficult for them to understand – the trigonometric functions of $y = sin(x)$, and $y = cos(x)$, and their graphs. In the North Carolina high school math curriculum students are usually first exposed to the concept of right triangle trigonometry in Geometry. When they get to AFM or Pre-Calculus, they usually recognize "SOH-CAH-TOA," even if they don't remember what it means or what to do with it. With refresher lessons and practice, most students can successfully apply right triangle trigonometry to "real world" situations to solve for missing distances or angle measures.

When asked to move on from right triangle trigonometry, however, many students hit a roadblock in understanding. Trying to make the connections from right triangles to the unit circle, to angles in standard position, and ultimately to graphs and trigonometric identities causes many students to want to give up. And in an AFM class, many of the students are just trying to get their last math credit to meet high school graduation requirements. They are not interested in pursuing math or hard sciences in their futures; rather they are interested in liberal arts, fine arts, or social sciences. Many believe the two are mutually exclusive, and I often hear the question that is universally dreaded by teachers, "When am I ever going to use this in real life?" So this curriculum unit has a secondary goal of keeping "non-math" students interested in math by showing them some "real-life" applications of mathematics in areas that do hold their interest. If a student who is interested in the visual arts can actually see how changes in certain parameters change functions and their graphs, rather than just getting numbers from a calculator, perhaps their interest will be piqued enough to make them want to understand at a higher level.

Strategies

This curriculum unit starts with refresher lessons on the basic trigonometric ratios in right triangles. The students need to have an essential understanding that in a right triangle, the trigonometric ratios of the acute angles are functions of the size of the *angles* as opposed to the sizes of the *sides* of the triangle. This is a common point of misconception. How can we get students to see that trigonometric ratios are tied to individual angles, not to the entire triangle? We will do small-scale activities using paper triangles and compare trigonometric ratios of congruent angles among triangles of different overall dimensions. Then we will move outdoors and use student-made hypsometers (made with protractors, drinking straws, string, and paper clips) to use trigonometry to find large-scale measures like the height of the goalposts in the football

stadium. Once the students have demonstrated a solid understanding of the basic trigonometric ratios in right triangles, the unit will move on to the next component.

The second component of the curriculum unit introduces students to many new concepts: the unit circle, trigonometric functions for angles in standard position, the Law of Sines and Law of Cosines, radian measure, arc length and area of circular sectors, and, finally, the graphs of the functions $y = sin(x)$ and $y = cos(x)$. Students will correlate the trigonometric functions of angles on the unit circle to the graphs of sine and cosine functions. A class session in one of the school's computer labs will allow students to use one of several good computer applications which can help students see that going around the unit circle to get values of trigonometric functions is cyclical, just as following a sine or cosine curve toward positive or negative infinity is cyclical.[2,3] Once students understand the graphs of the basic sine and cosine functions, they will investigate changes in amplitude and period, phase shifts, and vertical shifts. Groups of students will graph on transparencies trigonometric functions of varying amplitude, and frequency and then they will overlay those transparencies to see how changes in different parameters bring about changes in the graphs. Students will also take advantage of the graphing calculator technology to explore the graphs of families of trigonometric functions. The ultimate goal is to have students understand all the possible manipulations of the basic functions $y = \sin(x)$ and $y = \cos(x)$ which result in the functions taking on the form $y = Asin(kx + c) + h$ and $y = Acos(kx + c) + h$.

The final, "fun" component of the curriculum unit, which can be subsequent to or concurrent with the second component, is the visual art aspect. Students will have opportunities to look at and look for art that has trigonometric functions visually incorporated in it. With an understanding of the basic shape of the sine and cosine curves, students will look for examples in school, at home, on the Internet, and in nature of artistic or natural phenomena that have a sinusoidal shape. They will spend time using various internet resources to "play" with sinusoidal curves and experiment with geometric transformations of those curves, using computer graphics to generate ideas for their original work of art. Then they will be assigned a project which gives them the chance to get creative and create an original work of art, in the medium or mixed media of their choice, which uses or visually represents one or more of the trigonometric functions. The project may also have a research component which will require students to link trigonometry to possible career choices by researching careers that use trigonometry or components of trigonometry.
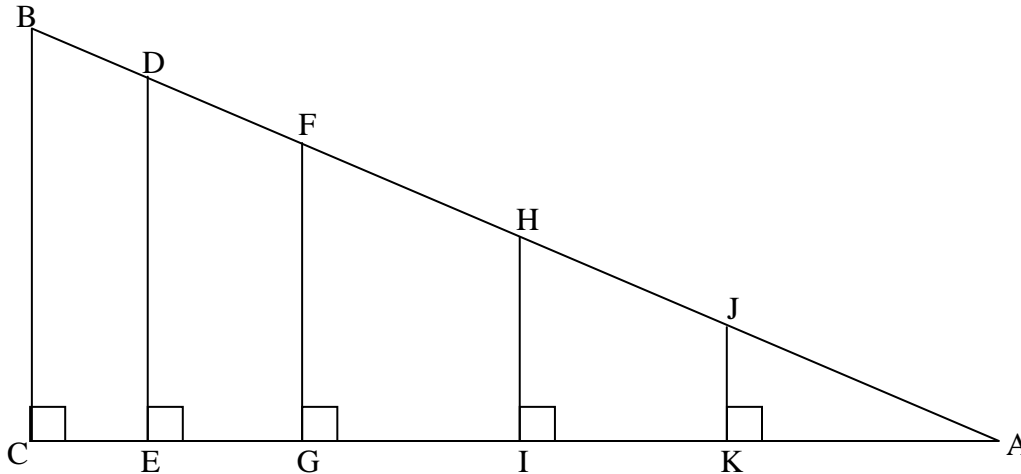
Lesson Plans and Classroom Activities

Lesson 1: Right Triangle Trigonometry Refresher

I find it helps to do a warm-up refresher of identifying the parts of a right triangle. Students often misidentify the "opposite" and "adjacent" sides because they don't understand that the designation of opposite or adjacent is relative to a particular angle, not to the entire triangle. Ask the class, "What makes the hypotenuse the hypotenuse?" and most students will respond with, "It's the longest side." I then explain that it is the longest side *because* it's the hypotenuse, but what defines it as the hypotenuse is that it is opposite the right angle, *not* that it is the longest side.

Using right triangles cut out of paper or cardstock, (or using a worksheet with diagrams like those shown in Figure 1 and Figure 2) have students measure (in cm) the lengths of the legs and the measures in degrees of the acute angles. Using SOH-CAH-TOA, they will record the three basic trigonometric ratios for each angle in each triangle, and then convert or approximate the ratios to decimals. Like fractions, each of the trigonometric ratios must be understood as both a ratio and as a number. The students will compare the numeric values of the different ratios by completing a table similar to Table 1.

Figure 1: Right Triangle Demonstrating Opposite Side, Adjacent Side, and Hypotenuse
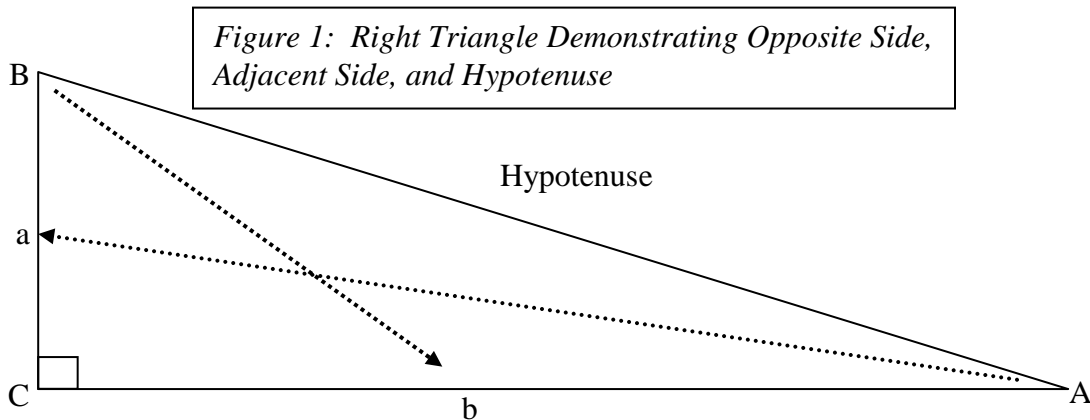
Figure 2: Right Triangle For Measuring Trigonometric Ratios for Acute Angles

Acute Angle Name	Angle Measure (degrees)	Sine Ratio	Sine Numeric Value	Cosine Ratio	Cosine Numeric Value	Tangent Ratio	Tangent Numeric Value
A							
B							
D							
F							
H							
J							

Table 1: Trigonometric Ratios and Numeric Approximations in Right Triangles

The students can do this individually or in small groups. Important concepts for students to grasp from this activity are that the value of a trigonometric ratio does not depend on the measurements taken, rather, the value of the ratio depends on the ratio of the measurements; and that the ratios stay the same for all sizes of similar triangles; and the ratios are each related to the measures of the angles, not strictly to the lengths of the sides.

It helps to also include in this lesson a refresher of "special" right triangles (30°-60°-90° and 45°-45°-90° triangles.) If students have a solid understanding of the trigonometric ratios in these special triangles, the connection to the unit circle will come more quickly. Give students diagrams of each of the special right triangles and have them label the measures of the sides based on the rules they learned in geometry, (for 30°-60°-90°, legs of 1 unit and $\sqrt{3}$ units and hypotenuse of 2 units; for 45°-45°-90°, legs

of 1 unit each and hypotenuse of $\sqrt{2}$ units) and then have them modify those measures to show a hypotenuse of measure 1 unit. The result appears in Figure 3.

Figure 3: Special Right Triangles
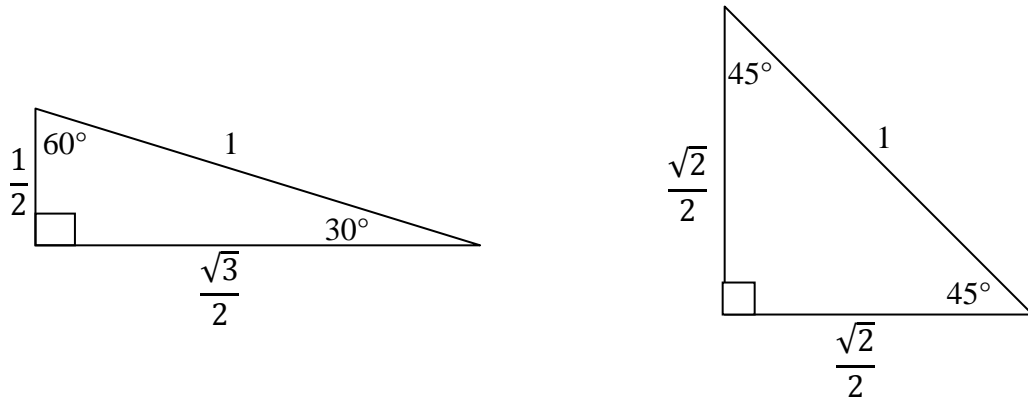
Lesson 2: Radian Measure, the Unit Circle, Trigonometric Functions for an Angle θ on the Unit Circle, and Trigonometric Functions for an Angle θ in Standard Position

This is a two- to three-day lesson which will introduce students to radian measure and the unit circle. The lesson assumes that students have shown mastery of right triangle trigonometry. It begins with the definition of radians. There are several websites with good explanations of the fact that one radian is defined as "the unit of plane angle adopted under the Systeme International d'Unites; equal to the angle at the center of a circle subtended by an arc equal in length to the radius (approximately 57.295 degrees)."[4] The website "Trigonometry Realms" provides a good animation to help the students visualize an angle that subtends an arc of length equal to the length of the radius of the circle.[5] Then, building on their prior knowledge of the circumference of a circle, and the fact that there are 360° in a circle, they are able to derive that 360° = 2π radians.

After defining radian measure, I overlay the special right triangles on the unit circle to show how the sine, cosine and tangent of 30°, 45°, and 60° angles are derived from the x- and y-coordinates of a point on the terminal side of each of those angles on the unit circle. Once students understand where the trigonometric functions of those three common angles come from, have them use the "fill in the unit circle" available at embeddedmath.com and have them complete the x- and y- values for the rest of the unit circle. (See Appendix B for the "fill-in" and completed versions of the unit circle.) Students then derive that for any angle θ on the unit circle, the following trigonometric functions will always apply:

$$sin\theta = y \qquad\qquad csc\theta = \frac{1}{y}$$

$$cos\theta = x \qquad\qquad sec\theta = \frac{1}{x}$$

$$tan\theta = \frac{y}{x} \qquad\qquad cot\theta = \frac{x}{y}$$

After the students demonstrate an understanding of the unit circle and the trigonometric functions for angles on the unit circle, ask them to think about an angle in standard position which has a radius of some length other than one unit. Can they use the same ideas from the unit circle and right triangle trigonometry to derive the values of the trigonometric functions of an angle in standard position if they know the x- and y-values of a point on its terminal side? How is it similar to the unit circle? How is it different?

Students need to see the connection that for an angle θ on the unit circle, we define the hypotenuse of a right triangle as the radius of the circle, which has a given measure of 1 unit, so that $sin\theta = \frac{opposite}{hypotenuse} = \frac{y}{1} = y$, but an angle θ in standard position has a hypotenuse of some measure other than 1 unit and we have to use the Pythagorean Theorem to determine the length of that radius. It helps to have students derive "Pythagorean shortcuts" and write in their notes: $r = \sqrt{x^2 + y^2}$, $x = \sqrt{r^2 - y^2}$, and $y = \sqrt{r^2 - x^2}$. Students can now understand the trigonometric functions of an angle θ in standard position:

$$sin\theta = \frac{y}{r} \qquad\qquad csc\theta = \frac{r}{y}$$

$$cos\theta = \frac{x}{r} \qquad\qquad sec\theta = \frac{r}{x}$$

$$tan\theta = \frac{y}{x} \qquad\qquad cot\theta = \frac{x}{y}$$

Lesson 3: Graphing Sine and Cosine Functions

I find this to be one of the most challenging lessons to teach in AFM. Students need to understand what defines a periodic function, and how to find the period. They need to understand that the scale of the x-axis will be in terms of π rather than in terms of the integers they are used to working with. They don't understand why they should have to refer to the graph of the function to find the sine or cosine of an angle of given measure. They ask questions like, "Why can't we just use the calculator to find that the sine of $-\frac{9\pi}{2} = -1$?" I get the feeling that the ability to read a graph has become a lost art in our society. Students know how to plug the functions into the "$Y=$" button on their graphing

calculators and hit "graph," but they don't know how to interpret these graphs. They don't know how to set the window to get x-axis and y-axis settings that make sense for the graph.

So the first part of this lesson is to have students create a table of values for the function $y = sin(x)$. Working in small groups, assign each group a different range of x-values. For instance, one group will have values $-2\pi \leq x \leq 2\pi$, another group will use $-4\pi \leq x \leq -2\pi$, a third group will use $2\pi \leq x \leq 4\pi$, etc. They use a scale of $\frac{\pi}{2}$, so that each group has nine points to plot. It's best not to allow students to use for this part of the activity. Instead of using the calculator, they must refer to the unit circle to find the values of $sin(x)$ for their given values of x. Each group will complete a table of values similar to the Table 2.

Table 2: Sample Table of Values for y=sin(x)	
X	**Sin(x)**
-2π	0
$-\frac{3\pi}{2}$	1
$-\pi$	0
$-\frac{\pi}{2}$	-1
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

Once each group has completed their table of values, I will bring their attention back to the whiteboard, where I have projected a large grid. A student will come to the board to draw in the x- and y-axes, and then the class should discuss what the scale should be on each axis. "What kind of x-values do we have in our tables of values?" "Are they integers?" "Would it make sense to use integers to mark off our x-axis?" "If not integers, what should we use to mark off our x-axis?" These questions help students to realize that the x-scale should be in terms of π, and that each "tick mark" should be some fraction of π. We agree on either $\frac{\pi}{2}$ or $\frac{\pi}{4}$ as an appropriate x-scale. I then have two

students come to the board to write in the values for the x-axis, one working from the origin in the positive direction, the other working from the origin in the negative direction. When they run out of grid marks from the projected image, I ask the class if we have enough space on our x-axis to show all the points from their tables of values. When some groups realize that their assigned x-values are not shown on the board, they understand that we have to extend the x-axis in both the positive and negative direction beyond the projected image in order to graph all the points they have found.

Next we turn our attention to the y-axis. "What is the smallest y-value?" "What is the largest?" "How much space do you need between 0 and 1, and between 0 and -1?" These questions lead them to choosing an appropriate y-scale. We usually end up with a scale of 0.25, but some classes agree on a scale of 0.1.

Finally, each group sends a representative to the board to plot the points from their assigned table of values. I then connect the dots to show them the shape of the function $y = \sin(x)$. I choose a few x-values that fall in between those that they found for their tables of values (ex., $-\frac{\pi}{4}, \frac{\pi}{8},$) use the calculator to get a decimal approximation of the sine for each of those values, and then plot those points to demonstrate that the points are connected by a curve, not by straight segments.

At this point, it helps to make a graphing calculator connection. What they see on the white board -- a graph that had to be extended beyond the projected image because of limitations of the size of the projection area – is similar to what they are able to see in the window of a graphing calculator. Before our graph made any sense, we had to determine an appropriate scale for each axis, and the size of the projected grid limited how much of the graph we could draw on that grid. The grid that is being projected onto the board is like the window of the graphing calculator. In order to see on the graphing calculator the graph for the x-values that we have extended on the white board beyond the projected image, they will have to change the minimum and maximum x-values in the calculator's viewing window.

Now that students have a strong visual image of what the periodic function $y = \sin(x)$ looks like, I ask one student to come to the board and pick a point, any point on the graph, and draw a vertical line through it. I ask another student to come to the board and draw another vertical line where the pattern has gone through one complete "cycle," i.e. where the y-value has gone up/down to its maximum/minimum value, back down/up to its minimum/maximum value and then reaches a point equal to the y-value of the starting point. I have one more student come up to mark one more "cycle" on the graph. This opens the discussion of period, what it is, and what defines a periodic function. Students can see the pattern repeating itself. I have them find the difference in the x-values between sets of points that have the same y-value.

At this point, students complete a table in their notes, summarizing the properties of the graphs of the two functions. Their table looks like Table 3.

	$y = \sin(x)$	$y = \cos(x)$
Table 3: Properties of the graphs of y = sin(x) and y = cos(x)		
Period	2π	2π
Domain	All real numbers	All real numbers
Range	Real numbers between -1 and 1, inclusive	Real numbers between -1 and 1, inclusive
x-intercepts	πn, where n is an integer	$\frac{\pi}{2} + \pi n$, where n is an integer
y-intercept	0	1
Maximum value	$y = 1$, at $x = \frac{\pi}{2} + 2\pi n$, where n is an integer	$y = 1$, at $x = \pi n$, where n is an even integer
Minimum value	$y = -1$, at $x = \frac{3\pi}{2} + 2\pi n$, where n is an integer	$y = -1$, at $x = \pi n$, where n is an odd integer

The final step in this lesson is to have students graph $y = \sin(x)$ and $y = \cos(x)$ in the same window of the graphing calculator so they can see the similarities and differences between the two functions.

A fun enrichment to this lesson is to have students use an online biorhythm generator (like those located at http://biorhythms.sitefoundry.com/ or at http://www.astrojyoti.com/Biorhythm.htm)[6,7] to graph their own biorhythms. This helps them to understand how to read graphs of sinusoidal functions in a way they find fun and interesting. Because the three major biorhythms have different periods, this can be a good discovery activity to lead in to the next lesson which addresses, among other things, changes in period of the sine and cosine functions.

Lesson 4: Amplitude, Period, and Translations of Sine and Cosine Functions

In this lesson, students learn how to "stretch," "compress," and translate the curves of $y = \sin(x)$ and $y = \cos(x)$.

Students start out by comparing the graphs of $y = \sin(x)$ and $y = A \sin(x)$ or $y = \cos(x)$ and $y = A \cos(x)$. They look at the graphs with different values of A, graphing them on the same set of axes, to discover that the maximum and minimum values of the functions change from $|1|$ to $|A|$. They examine the period of these altered graphs and discover that changes in amplitude do not cause any changes to the period. The functions simply get "taller," or "stretch" upward/downward.

Next, students compare the graphs of $y = \sin(x)$ and $y = \sin(kx)$ or $y = \cos(x)$ and $y = \cos(kx)$. They look at the graphs with different values of k, graphing them on the same set of axes, to discover that the period of these altered graphs changes from 2π to $\frac{2\pi}{k}$ for $k > 0$. The visual effect they see is that of the graph of $y = \sin(x)$ or $y = \cos(x)$ being "condensed" or "squeezed like an accordion."

Finally, students look at phase shift and vertical translations. Students compare the graphs of $y = \sin(x)$ and $y = \sin(x + c)$ or $y = \cos(x)$ and $y = \cos(x + c)$. Again graphing on the same set of axes, students can see that the graphs are shifted c units left for $c > 0$ and c units right for $c < 0$. And comparing on the same set of axes $y = \sin(x)$ and $y = \sin(x) + h$ or $y = \cos(x)$ and $y = \cos(x) + h$ students can see that the graphs are shifted h units upward for $h > 0$ and h units downward for $h < 0$.

To help students visualize these phase shifts and vertical translations, it helps to use pipe cleaners that are bent into the shape of a simple sinusoidal curve to show how the curve is simply "picked up and moved" by changes in c or h, but the basic shape of the curve (with amplitude 1 and period 2π) is not affected at all.

To wrap up this lesson I project onto the white board a grid with the graph of either $y = \sin(x)$ or $y = \cos(x)$. I write on the board an equation of the form $y = A\sin(kx + c) + h$ or $y = A\cos(kx + c) + h$ and have a student, or a small group of students, come to the board to sketch their prediction of how the altered graph will compare to the original graph. Will it be stretched or compressed? Will it be shifted upward or downward? Left or right? What information can justify their prediction? We repeat this process with several different equations with variations in the values of A, k, c, and h until students can easily predict the changes in the graph that will correspond to the changes in the given values.

Unit Project: Creating an Original Work of Trigonometric Art

The culmination of this unit will be a project which requires students to create an original work of art using shapes generated from graphing variations of $y = A\sin(kx + c) + h$ and $y = A\cos(kx + c) + h$. Students may work in the medium or mixed media of their choice, including computer graphics for those who are not inclined to doing "hands on" artwork.

Figure 4: *Yin and Yang*, from *New World Encyclopedia*

To introduce the project, I will present a slide show in class which gives students an overview of art and architecture with trigonometric elements. Two good sites for finding images to create a slide show are http://oolong.co.uk/trig.htm, and http://mathfax.com/nice-trigonometry-photos/. Other images, like the ancient symbol of yin and yang shown in Figure 4[8], and the "Mathematical Paintings of Crockett Johnson" exhibit in the Smithsonian Institution's National Museum of American History, an example of which is shown in Figure 5[9], will round out the slide show.

I will then encourage students to research this further during a session in the school's computer lab or on their own time as a homework assignment. Students will look for examples of art that have elements of sinusoidal curves in them, and share those examples with the class through photos, web site links, or files (Word, PowerPoint, etc.). As a class, we will examine the students' contributions and discuss whether they show sinusoidal curves or not.

Figure 5: *Curve Tangents,* by Crockett Johnson. Smithsonian Institution

As a class activity, we will take a trip around the school to look for any places where we can see sinusoidal curves in the building or, weather permitting, around the school grounds. We will take digital photos of any examples we find. Back in the classroom, students will work with printouts of the examples they found in their individual research and of the photos we took around the school grounds. They will cut out or otherwise highlight the sinusoidal curves they have found in the art.

Choosing one or two of the curves they have selected, students will then be required to write an equation for their curve in the form $y = Asin(kx + c) + h$ and compare that curve to a basic $y = sin(x)$ curve. As an example, I will show students how I was inspired while unwrapping a simple peppermint candy (a starlight mint). I saw in that candy what appeared to be a sinusoidal curve which had been rotated around a central point. I then took a one-period piece of the graph of $y = sin(x)$, copied and pasted it four times, and then rotated each of those iterations around a central point to get the image shown in Figure 6.
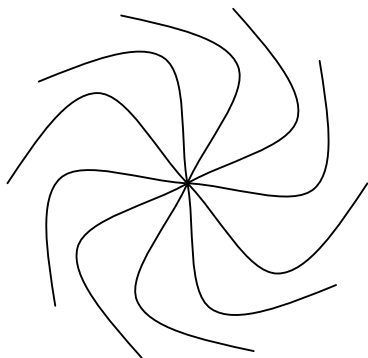
Figure 6: my initial sketch of a starlight mint.

This presents an opportunity to explore the concept of symmetry as it applies to the trigonometric functions. David W. Farmer defines symmetry of a figure as "a rigid motion which leaves the figure unchanged."[10] The two specific types of symmetry we will explore are reflectional symmetry and rotational symmetry. In the case of the starlight mint, the section of the curve of $y = sin(x)$ for $0 \le x \le 2\pi$ appears to have rotational symmetry around the point $(\pi, 0)$, but no reflectional symmetry. When $y = sin(x)$ and $y = 2sin\left(\frac{x}{2}\right)$ are graphed for $-2\pi \le x \le 2\pi$, the resulting image appears to have rotational symmetry around the origin, but still no reflectional symmetry (see Figure 7.)[11] There are infinite possibilities for changing the parameters of the functions, and for graphing families of functions on the same set of axes. But what type of symmetry can students discover in these families of graphs? After reviewing reflectional and rotational symmetry (concepts with which they should already be familiar from previous math courses), allow students class time to "play" with graphs using a graphing calculator. Have them start with a single curve of $y = sin(x)$ or $y = cos(x)$, and discuss the symmetry there, and what would have to be done to create other symmetries. Then challenge the students to try to discover a family of graphs that shows one type of symmetry. Then have them try to find a family of graphs that shows both types. Finally, have them see if they can create an image with no symmetry at all.
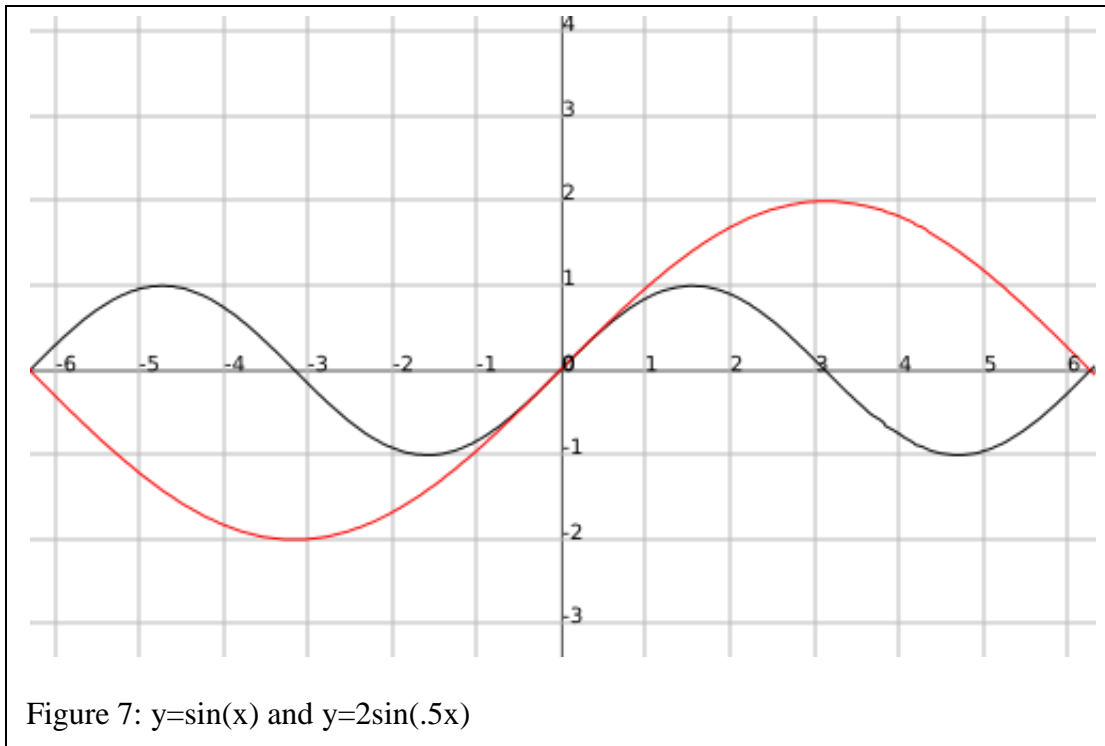
Figure 7: y=sin(x) and y=2sin(.5x)

After exploring sinusoidal curves in art, architecture, and nature, and exploring symmetry with respect to these curves, the final piece of the unit project is to give students the assignment of creating an original work of art. The guidelines should be somewhat flexible to allow as much creativity as possible, but must retain the focus on the mathematics behind the art. Each student's submitted project shall have two components: the mathematical background which includes pencil-and-paper graphs of a set of sinusoidal curves with properly labeled x- and y-axes, and clearly labeled curves; and the original work of art. The mathematical background piece should indicate what aspects of the graphs gave the inspiration for the work of art.

Art that includes right triangles, particularly 30°-60°-90° and 45°-45°-90° triangles will also be acceptable in this project, if the student is able to demonstrate that they understand the mathematical connections between the triangles and the graphs of the sine and cosine functions.

When all the works of art have been completed, they should be displayed in the classroom, or better yet, in one of the school's more public display areas for the entire school community to enjoy and learn about the art of trigonometry.

Works Cited

" Standard Course of Study 2003 (Current) - Mathematics."
> http://math.ncwiseowl.org/curriculum___instruction/standard_course_of_study_
> 2003__current_/ (accessed November 30, 2010). The North Carolina Standard
> Course of Study for Mathematics is the guideline by which all K-12 math
> courses in North Carolina are structured. Teachers much cover all goals and
> objectives for the given course in order to effectively teach the subject. AFM has
> two goals with eight objectives for reaching those goals.

Cockerham, Teresa. Interview by author. Email interview. Cornelius, NC, November 6,
> 2010. Dr. Cockerham is the principal at W.A. Hough High School. I
> Interviewed her to get demographic data about student/faculty population of the
> school to help teachers reading this unit a way of comparing their school
> environment to the environment in which this course is taught. The data came
> from the "School Improvement Plan" document, of which Dr. Cockerham is a
> primary author.

"Biorhythm Generator - improve peak performance by analyzing your emotional,
> intellectual, and physical cycles." Biorhythm Generator - improve peak
> performance by analyzing your emotional, intellectual, and physical cycles.
> http://biorhythms.sitefoundry.com/ (accessed October 3, 2010). This website is
> an example that can be used for an enrichment activity to see an application of
> sinusoidal graphs. It must be noted, and students are informed that biorhythms
> are not "hard" science, and not part of the standard curriculum, but simply an
> interesting way to see what various sinusoidal graphs look like. The website is
> created and maintained by The Site Foundry, who claim to be makers of
> innovative web applications.

"Biorhythms." Vedic astrology by astrologer sptata for personal predictions, rasi forecast,
> panchang, rahukalam, planets, navagraha, online predictions, astrology books,
> ayurveda, herbal medicines, health products, vedic scriptures, remedies, stotras,
> mantra, yantra, t. http://www.astrojyoti.com/Biorhythm.htm (accessed October 3,
> 2010). This website is an example that can be used for an enrichment activity to
> see an application of sinusoidal graphs. It must be noted, and students are
> informed that biorhythms are not "hard" science, and not part of the standard
> curriculum, but simply an interesting way to see what various sinusoidal graphs
> look like. The website is created and maintained by Indian astrologer, Pandit
> S.P.Tata, who has been practicing Vedic Astrology for more than 25 years.

"Download the Unit Circle & Blank Unit Circle - EmbeddedMath.com ."
> EmbeddedMath.com - Math T-shirts and other gifts for Mathematicians.
> http://www.embeddedmath.com/downloads/unitcircle.shtml (accessed November
> 30, 2010). This is actually a website that sells math-related clothing and
> miscellaneous gifts. But there is a section of free downloadable documents,
> including the unit circle worksheets used in this unit.

Farmer, David W. Groups and Symmetry: A Guide to Discovering Mathematics

(Mathematical World, Vol. 5). Providence: American Mathematical Society, 1995. David W. Farmer is director of programming for the American Institute of Mathematics. He is the author and co-author of many texts on a variety of mathematical subjects. This text introduces and explores the concept of symmetry and is useful for anyone who has at least a high school education. It was one of the texts used in the CTI "Math in Art" seminar.

"FooPlot: Online graphing calculator and function plotter." FooPlot: Online graphing calculator and function plotter. http://fooplot.com/ (accessed November 30, 2010). This is a free online graphing calculator application which was used to create Figure 7 to illustrate a graph discussed in the text. It allows users to manipulate viewing windows and to save and print copies of graphs.

"NMAH | Object Groups." National Museum of American History. http://americanhistory.si.edu/collections/group_detail.cfm?key=1253&gkey=192 &page=2 (accessed November 14, 2010). This website was a helpful resource for finding examples of artworks that incorporate mathematics. Of particular interest for this unit is the Crockett Johnson collection of mathematical paintings. The National Museum of American History is part of the Smithsonian Institution.

"Nice Trigonometry photos | MathFax.com." MathFax.com | Math Forum. http://mathfax.com/nice-trigonometry-photos/ (accessed October 3, 2010). I accessed this website when looking for examples of artworks that incorporate mathematics, and suggest that teachers using this unit access this site when putting together a slide show of examples of trigonometric art.

"Oolong's Playground." Fergus Ray Murray's Homepage - Photography, Animation, Science, etc. http://oolong.co.uk/play/ (accessed October 3, 2010). I accessed this website when looking for examples of artworks that incorporate mathematics, and suggest that teachers using this unit access this site when putting together a slide show of examples of trigonometric art.

"SAS® Curriculum Pathways® | Subscriber Login." SAS® Curriculum Pathways® Home Page. http://www.sascurriculumpathways.com/ProductEntrance/Navigation/navigator.j sp?node=15 (accessed October 3, 2010). SAS is the world's largest privately held software company with strong ties to the academic community. Curriculum Pathways is a subscription service with lesson plans, project ideas, and web-based student activities on a variety of topics. It is used in Lesson 3 of this unit.

"The Radian." Zona Land Education, Home. http://zonalandeducation.com/mmts/trigonometryRealms/radianDemo1/RadianD emo1.html (accessed November 6, 2010). Zona Land Education is a website which provides educational and entertaining items pertaining to physics and mathematics. Their simple animation demonstrates the derivation of one radian as a unit of measure. It is used in Lesson 2 of this unit.

"Trigonometry-sine and cosine Do "the Wave"." CatCode.com. http://catcode.com/trig/trig08.html (accessed October 3, 2010). This website is linked to one of the SAS Curriculum Pathways lessons on trigonometry. The

application demonstrates the connection between sine and cosine functions on the unit circle and graphs of those functions. It is used in Lesson 3 of this unit..

Princeton University. "WordNet Search - 3.0." WordNet®. wordnetweb.princeton.edu/perl/webwn?s=radian&o2=&o0=1&o7=&o5=&o1=1 &o6=&o4=&o3=&h= (accessed November 6, 2010). A Google search for "radian definition" linked to this website. Princeton University is a world-renowned institute of higher learning, and therefore a reliable source for the definition of a basic mathematical term. Specifically, it was the source for definition of the term radian.

"Yin and Yang - New World Encyclopedia." Info:Main Page - New World Encyclopedia. http://www.newworldencyclopedia.org/entry/Yin-Yang (accessed November 14, 2010). When searching for art with sinusoidal curves in it, a Google search resulted in this link. It is the source for the image of yin and yang.

Notes

[1] Cockerham, Teresa. Interview by author. Email interview. Cornelius, NC, November 6, 2010.

[2] Graph of sin(x) from the unit circle. Interactive demonstration of how graph relates to formula." Interactive Math Activities, Demonstrations, Lessons with definitions and examples, worksheets, Interactive Activities and other Resources. http://www.mathwarehouse.com/trigonometry/graphing-sine-function/how-to-graph-sine-from-unit-circle.php (accessed October 3, 2010).

[3] "Trigonometry-sine and cosine Do "the Wave"." CatCode.com. http://catcode.com/trig/trig08.html (accessed October 3, 2010).

[4] Princeton University. "WordNet Search - 3.0." WordNetÂ®. wordnetweb.princeton.edu/perl/webwn?s=radian&o2=&o0=1&o7=&o5=&o1=1&o6=&o 4=&o3=&h= (accessed November 6, 2010).

[5] "The Radian." Zona Land Education, Home. http://zonalandeducation.com/mmts/trigonometryRealms/radianDemo1/RadianDemo1.ht ml (accessed November 6, 2010).

[6] "Biorhythm Generator - improve peak performance by analyzing your emotional, intellectual, and physical cycles." Biorhythm Generator - improve peak performance by analyzing your emotional, intellectual, and physical cycles. http://biorhythms.sitefoundry.com/ (accessed October 3, 2010).

[7] "Biorhythms." Vedic astrology by astrologer sptata for personal predictions, rasi forecast, panchang, rahukalam, planets, navagraha, online predictions, astrology books, ayurveda, herbal medicines, health products, vedic scriptures, remedies, stotras, mantra, yantra, t. http://www.astrojyoti.com/Biorhythm.htm (accessed October 3, 2010).

[8] Yin and Yang - New World Encyclopedia." Info:Main Page - New World Encyclopedia. http://www.newworldencyclopedia.org/entry/Yin-Yang (accessed November 14, 2010).

[9]"NMAH | Object Groups." National Museum of American History. http://americanhistory.si.edu/collections/group_detail.cfm?key=1253&gkey=192&page=2 (accessed November 14, 2010).

[10]Farmer, David W.. Groups and Symmetry: A Guide to Discovering Mathematics (Mathematical World, Vol. 5). Providence: American Mathematical Society, 1995, p. 27.

[11]"FooPlot: Online graphing calculator and function plotter." FooPlot: Online graphing calculator and function plotter. http://fooplot.com/ (accessed November 30, 2010).