Exponential Functions: Introduction to Wealth and Debt Management

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West Charlotte High School

This curriculum unit is recommended for:
High School Math 2 or Math 3 Teachers

Teaching Standards: See Appendix 1 for teaching standards addressed in this unit.

Synopsis: Exponential Functions – this unit will focus on enabling students to thoroughly examine their money management practices and skills as part of a greater social issue and use the tools of mathematics to help in the decision-making process. We will focus on solutions-based approach to tackling and managing household debt. More specifically, Students will examine how saving small amounts can create greater gains overtime while simultaneously engaging in deep mathematical thinking and discourse about how interest rates on debt such as cars, student loans and credit cards can grow exponentially, and limit challenge their ability to amass wealth and hence affect their financial well-being.

The unit will begin with an introduction to and a comparison of the effect of simple and compound interest on money invested. Students will explore the behavior of these two methods of calculating interest with the aim of helping them make connections and distinctions between how they affect future value of money. Students will use tables and graphs to compare the growth of money using simple interest (linear) function and compare it to compound interest (exponential) function.

As they conceptualize the meaning of simple and compound interest, the impact of these two formulae will contribute knowledge that underscores the importance of an education towards social justice and view this as a means to tackle problems that exist in their schools and wider communities. Each lesson will be specifically linked to the management of one form of loan or investment. These scenarios will provide opportunities for students to examine future decisions they make in a more systematic way. They will also build functions to represent the disparities they observe and present these ideas through various media to include but not limited to the production of an advertisement, video or poster to sensitize members of their community to the benefits and drawbacks of simple interest and compound interest over time.

I plan to teach 80 students this unit in the upcoming quarter and again to a new group of 90 students doing math 2 in semester 2 of tenth grade.

Introduction

Money management skills can be effectively taught in a mathematics classroom and can positively impact the quality of life that individuals enjoy. The fact that mathematics is viewed as a useful subject because of its many real-world applications is a valid reason to include experiences that will mimic very closely what happens in real life so as students integrate into the adult world, they are able to successfully navigate the demand to be financially literate. Due to this perceived usefulness it has become a mandatory part of early formal education in several countries such as the United States of America, England, Spain, Japan, and France among many others.[1]

An argument for financial education in schools

Here in America, financial education differs greatly for children born in wealthy homes compared to their less fortunate working-class counterparts. CNN's Heather Long proposes an argument for why mandatory financial education would help generations of Americans become more financially literate and make more responsible decisions about wealth management – including decisions such as planning for retirement and managing household debt.[2]

As students enter senior high school they begin to finalize plans about the next move, whether to join the workforce or to attend an institution of higher learning. Either way, how they will finance their living from this point onwards is a critical decision. They will need tools to successfully navigate these financial pathways and mathematics provides a unique opportunity for them to interact with the linear and exponential functions used to calculate simple and compound interest respectively. They can also use Microsoft Excel or online calculators to manipulate different principal amounts alongside various interest rates and terms as part of their mathematics learning. As mathematics educators seek to make these connections to real world scenarios in a meaningful way, students become better equipped to make decisions about their economic well-being. For example, the study of exponential functions, the focus of this unit, drives the thinking that underpins financial calculations such as interest on savings, mortgage payments, credit card interest, students' loans and other household debt. Having said all that, I believe early healthy financial habits students form can impact their overall ability to amass wealth in the long-run.

Stango and Zinman outline a general misunderstanding of the compounding effect of the exponential function on future value of money borrowed or saved consumers err on the side short-term investments and underestimate the interest they will repay on long-term credit card payments and other household debt. This is partly due to an intuitive linear projection made in calculation as well as biases consumers have.[3] Let us examine for example a car loan of $35,000 taken out at 5% rate of interest per annum for seven years and compare that to comparable simple interest loan over the same period:

Compound interest formula

$$Amount = PV(1 + \frac{r}{100})^t$$

$$Amount = \$35{,}000(1 + \frac{5}{100})^7$$

$$\boldsymbol{Amount = \$49,248.50}$$

Simple Interest formula

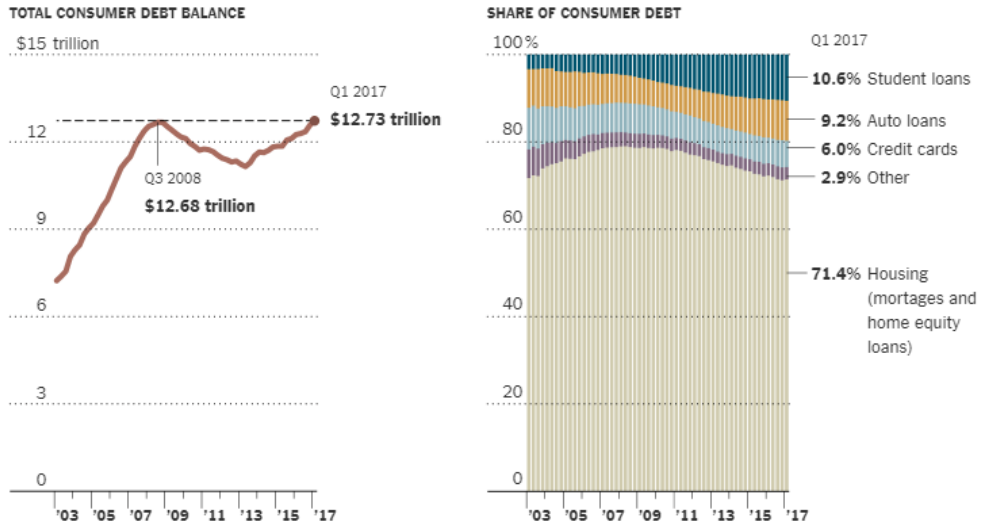
$$Amount = P(1 + rt)$$

$$Amount = \$35{,}000(1 + 0.05 * 7)$$

$$\boldsymbol{Amount = \$47250.00}$$

Here we note that there is a difference of $2000 between the two approaches to calculating interest. This reality is also seen as people make decisions about how to invest money in different interest-yielding accounts. Unfortunately, this kind of care is sometimes lost to the intuitive straight-line calculation on even bigger loans such as mortgages and we will see how the interesting behaves as either the loan, loan life or interest rate increases even minimally. A clear understanding of these differences will drive consumers to making wiser financial decisions.

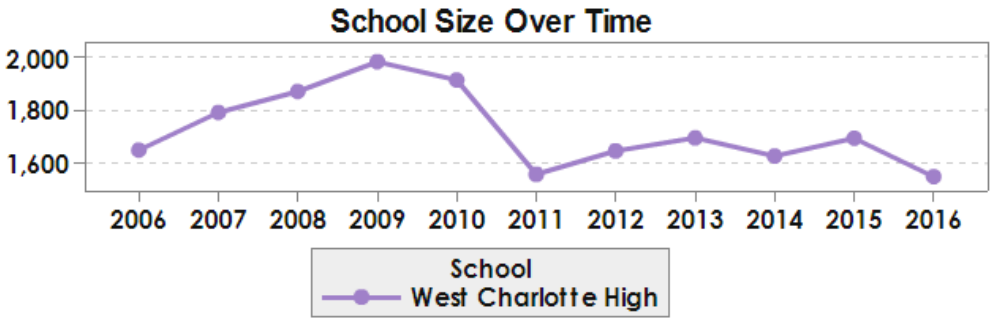
According to the New York Times consumer spending contributes about 70% of the economic activity in the United States.[4] If that 70% were used with the best financial habits guiding that spending, inevitably, the economy would see marked differences in its bid to attain economic growth and stability. Also, with such a large portion of the economy controlled by consumers, it would benefit families to become financially literate. Further examination of this 70% indicate that much of this spending is driven by debt such as car and student loans, housing mortgages and credit card debt. The graph below shows a visual comparison of the share accounted for by each of these categories of household debt in America over the last eight years.



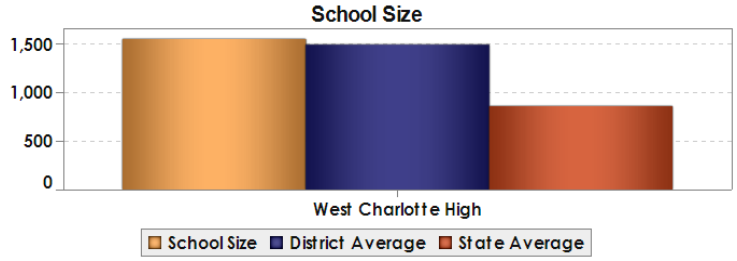
The increasing trend in debt levels indicates that consumers' ability to borrow has improved since the recession starting in December 2007 through February 2010 (National Bureau of Economic Research) coupled with greater confidence in the economy from the side of banks and other lending institutions.[5] In the final analysis, consumers must become aware of the potential risks if their debt exceeds their ability to repay these loans and the possible macro-economic risks this could have on the country's economy.

School population data

West Charlotte High School currently has a population of 1,450 students and 125 teachers. Approximately 82.3% of the student population is African American or Black, 9.4% Hispanic and the remaining 8.3 % Asian or other.[6] The population size declined dramatically in the year 2011 and has remained flat since that time. The line graph below represents the school population's growth from 2006 to 2009 and then a general decline since 2010.



Even with this decline, West Charlotte remains one of the largest schools in Charlotte Mecklenburg School District and in the State of North Carolina as shown in the graph below.[7]



West Charlotte High is a Title I school with 52.8% percent (765) of its student population male and 47.2% (685) female. Within this population, West Charlotte staff serves 83 English Language Learners, 24 (1.7%) students identified as Academically Intellectually Gifted (AIG) and 210 (14.5%) students with disabilities. Additionally, 99% of its students benefit from free or reduced lunch and 5.4% are McKinney-Vento (without permanent residence).[8] West Charlotte High is the flagship of the current Project Lift Community – a group of schools that serve predominantly unserved populations in West Charlotte. Over the past three years the school has exceeded expected growth and now at a Grade 'C' moving up from a 'D'. West Charlotte offers the International Baccalaureate (IB) Program. The school was recently selected to become an Equal Opportunities School with the aim of raising the proportion of minority students who enroll in the IB program. The following tables show the breakdown of the current population by race and grade level respectively.[9]

Ethnicity	*African-American*	*Hispanic*	*Asian*	*White*	*American Indian*	*Pacific Islander*	*Two or More*
Enrollment	1,194	137	55	21	10	7	26
Percent	82.3	9.4	3.8	1.4	1.4	0.5	1.8

Ethnicity	*Grade 9*	*Grade 10*	*Grade 11*	*Grade 12*	*Grade 13*
Enrollment	459	371	321	297	2
Percent	31.7	25.6	22.1	20.5	0.1

Background

Social justice for Education

Many school districts comprise students from diverse groups from across the world, the city of Charlotte being a prime example. Consequently, recognition of cultural identities of various groups is becoming increasingly important and hence places social justice at the forefront of the education of twenty-first century students. Equality is not so much being sought by groups that are economically defined as much as it is by groups who share and value certain cultural identities with the aim of asserting their importance in various sectors in society.[10]

By embracing a social justice perspective, the objective is to enable students to challenge, in a meaningful way, the actions of state and the economy taken against certain groups of individuals and interpretations of the law that differ greatly when applied to various groups. This forces policy makers, lawmakers and those who interpret the law to take into consideration the current cultural climate when allocating resources and how their decisions will impact groups of people. Furthermore, exposing students to this perspective on learning math the intention is towards a natural gradual departure from seeing math as simply a set of unrelated, useless ideas to a more meaningful disposition that helps them to analyze and participate meaningfully in their world. Additionally, the expectation is that they will develop a deeper understanding of the role mathematics plays in explaining social and economic issues in their local communities as well as in the national and global context.

Mathematics Content

Exponential growth and decay functions are used in various industries to determine or predict future value of money, concentrations of chemicals with the atmosphere and our bodies. Here, the focus is on monetary values over time. In its most basic form exponential growth function is represented as $f(x) = ab^x; b \geq 0$. In this function: “a” represents the starting or initial value, “b” represents the multiplier or the growth/decay factor and “x” represents the time or number of time cycles. For example, the number of times in a year that interest is paid).

Definitions:

Discrete compound interest: when interest is calculated and added at the end of specific time intervals (annually, monthly, weekly, etcetera). The formula used to calculate discrete compound interest is:

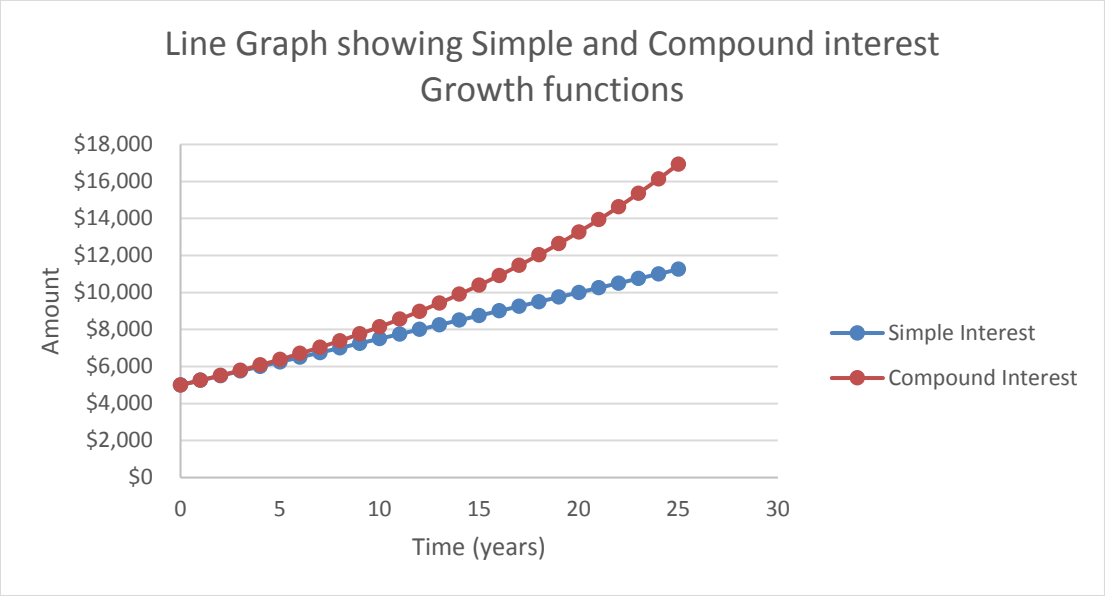
$FV = P(1 + \frac{r}{m})^{mt}$ where FV is future value; r is rate; m is number of times interest is paid each year and t represents time in years.[11]

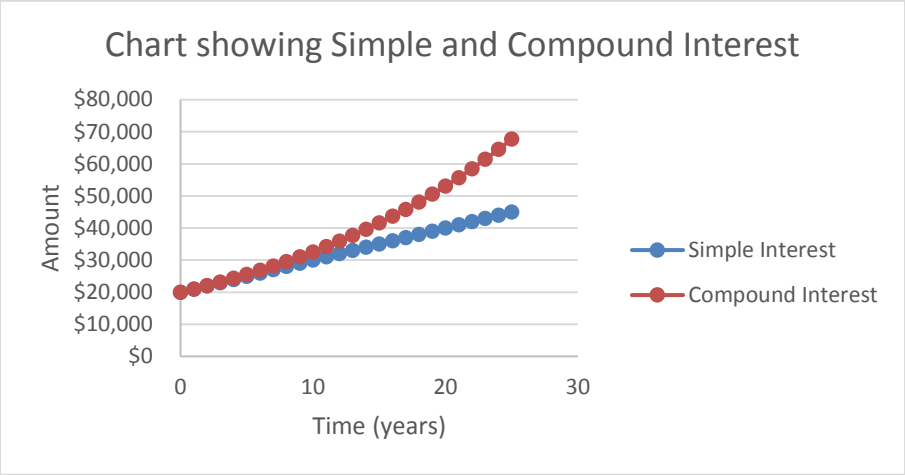
Continuous compound interest: Continuous compounding introduces the concept of the natural logarithm, represented by the letter e. This is the constant rate of growth for all naturally growing processes. The formula below is used to calculate this type of interest.[12]

$$FV = Pe^{rt}$$

It is important that students understand that while there will be an increase in the amount as the interest payment frequency increases but even more importantly, that this amount has a limit. This idea will introduce students to the idea of natural logarithms in a meaningful way.

The content examines the growth of investment using both exponential and linear functions with the aim of establish clear distinctions about how both types of functions behave with small increments of time or other changes in the parameters. The graphs below show both simple and compound interest amounts for the same initial amount invested and over the same time period. As shown by the graphs, the compound interest function increases exponentially while the simple interest function increases in a linear pattern. Each graph uses the same starting value, however in both cases there is a significant difference in the outcomes.



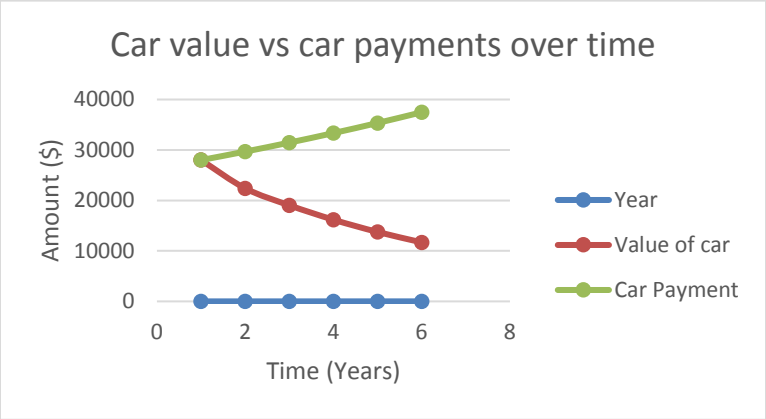


Despite the fact that it is more beneficial to invest when the interest is compounded, students should consider try to consider the impact when the growth is on a loan. Would you prefer simple interest or compound interest to be charged on a loan? These are questions teachers may want to ask students to get them to closely examine the behavior of these functions.

Depreciation – Exponential Decay

On the other hand, students have to contend with buying a car for example that depreciates while the cost of the loan is increasing with time, especially if the interest is compounded. Using the following rates of depreciation, calculate the value a car after 5 years and compare it graphically to the amount paid if the interest rate is compounded at 6% per annum.

Year	Rate of Depreciation per annum (%)
1	20
2-10	15
>10	10



Basically, the longer you keep the car, the lower its value and the longer you take to pay off the loan, the more you will accrue in interest.

The aim is that as students navigate through the various ideas of exponential growth and decay, they will develop not just an intuitive understanding of how these functions differ but they will be able to describe how they differ significantly in the short or long-term. By extension, students will be able to visualize the growth or decay of such functions in real world situations and be able to make informed decisions using the information that is available to them.

Rationale

Exponential Growth functions and Financial decision making

Understanding of the impact of exponential growth functions on household debt can help individuals make more informed decisions. Students are already learning about exponential functions in high school and would develop a greater appreciation for its use if they could understand how it affects decisions taking car loans, mortgages and credit cards.

This knowledge would help students whose education are not funded by parents or scholarships and grants find it necessary to use loans and therefore incur debt. Furthermore, even though the benefit in general is positive and contribute to individuals earning higher levels of income, there is still relatively small percentage that is negatively impacted by having this debt. This indebtedness correlates with them having a much lower net worth than their counterparts with similar household characteristics and levels of education.[13]

For a personal standpoint, writing this curriculum unit provides the opportunity to explore real life situations that I can use to help my students learn how to manage the money they earn and to also critically analyze what is means to utilize facilities such as savings and checking accounts, credit cards and other loan facilities. This is one means of creating a social outlook from the mathematics classroom, enabling students to examine how mathematical thinking can enrich their daily lives. Take for instance if Alleyna who is 15 years old were to start saving $200 monthly from her wages and Jorge also at age 15 starts saving $250 monthly. How much would they each have saved by age 40 if they invest at a 5% rate of interest compounded monthly?

The following two scenarios will require different variations of the compound interest formula.

Scenario 1: If there is a single deposit posted at the start of the 25-year period the formula below will be adequate.

Formula: $A = P(1 + \frac{r}{n})^{nt}$; where A (accrued amount – principal + interest), P(Principal), r (Annual nominal interest rate as a decimal), n(number of compounding periods per unit of time).

For example: $\$200(1 + \frac{0.05}{12})^{25\times12}$ If Alleyna only made that single initial first deposit and left it for 25 years compounded 12 times each year.

Scenario 2: When there are uniformed monthly additions, using an online compound interest calculator from Bankrate.com:[14] it was possible to make accommodations for these monthly additions.

1. The table below shows the results when a monthly deposit is made.

Input	
Initial Amount	$200
Monthly Deposit	$200
Annual interest compounded (monthly)	5%
Number of Years	25
Future Value	**$119,798.20**

Inputs	
Initial Amount	$250
Monthly Deposit	$250
Annual interest compounded (monthly)	5%
Number of Years	25
Future Value	**$149,747.75**

Observation: By increasing the monthly deposit by $50, students create a $15,000 increase in the overall input and thereby get additional interest amounting to almost $15,000.

Trivia or Challenge Question – Who wants to be a millionnaire?

1. At your current age, how much would you need to save each month to accumulate one million ($1,000,000) by the age 60, assuming the interest rate is 5% per annum?
2. What if the interest were simple interest, how much would you need to save each month compounded monthly?

Example:

Inputs	
Initial Amount	$1000
Monthly Deposit	$1000
Annual interest compounded (monthly)	5%
Number of Years	25
Future Value	**$598,991**

Students Try

Inputs	
Initial Amount	$
Monthly Deposit	$
Annual interest compounded (monthly)	5%
Number of Years	
Future Value	**$1, 000,000**

3. What if the monthly amount you found for part 1 remained the same but the interest rate was increased from 5% to 6.5%, how would that future value change?

As a matter of fact, the Equity principle proposes that all students should be provided with the support and resources needed to achieve mathematical success.[15] Therefore, instruction should be designed to meet students learning needs rather than simply providing the same instruction for all. In trying to adapt this curriculum unit, emphasis will be placed on finding means of exploring complex mathematical ideas through experiences that are culturally relevant in addition to helping students develop a positive disposition towards mathematics.

Examining the Data and Understanding Money Management

Student Loans

This category of loans is reported to have contributed a great deal to the spike in consumer debt. As a matter of fact, student loans accounted for 11% of household debt in quarter 1 of 2017 up from 5% in quarter 3 of 2008. This period of growth in students' loan debt is paralleled by a decrease in mortgages moving down from 73 to 68 per cent in the same period. Furthermore, in only nine years, student loans have more than doubled, moving from $611 billion to $1.3 trillion currently. This share of household debt has the highest tracked rate of delinquency at one in every ten borrowers.[16] While students' loans can help to bring about some measure of personal wealth and financial stability it can also stifle an individual's ability to purchase houses or curtail their spending on other consumer goods and services, which also stimulate the economy.

Interestingly, economists are uncertain of the real macroeconomic impact of student loans. Although these loans do not in any way account for the lion's share of household debt, they bear an unusual quality, they cannot be shed. As we examine how longer periods for such loans can cause a dramatic increase in the cost of the loan (total interest) we will begin to develop a clear understanding of how this might result in delaying housing purchases or starting new businesses.[17] These are compelling factors that justify connecting the worlds of finance and mathematics.

Credit Card Debt

Credit cards are becoming increasingly necessary with the rise in housing and health costs significantly outpacing growth in income. Current data indicate that over the past 13 years, medical costs have gone up by 57%, food and beverage 36%, cost of living 30% while household income a mere 28% over the same period. As a result, many households bridge the gap for living essentials using a credit card.[18] I would say that in response to this many companies offer credit card deals ranging from an interest free period, to free balance transfers, cash back, travel miles and many other incentives to attract consumers.

Though credit cards are meant to offer short term relief some consumers become dependent on them to bridge the income and expenditure gap over longer periods. If consumers learn how to manage them well, they can be very useful. For example, experts say paying more than the minimum required each month or paying off the balance can help minimize the interest.[19] That said, some credit cards do offer some benefit depending on the financial circumstances of a consumer. As students examine the credit card options on the market through this unit, they will become aware of the benefits that are available, as well as management skills that might be useful for them. Erin El Issa, in her article on credit card debt in 2016, points out alarming levels of debt for American households. The results of the study indicate that there has been an 11% increase in credit card debt with an average of $16,883 per household. This is part of a significant household debt average of $137,063 worth in debt.[20]

Pedagogical Approach

This unit will be taught over a five-day period with each session lasting 90 minutes. Each lesson will introduce a new subtopic through a real-world scenario and data relating to that financial topic. The table below outlines the pacing guide for these 5 days.

Pacing	Core ideas
Day 1 **90 minutes**	Introduction to exponential functions through: • Salaries, • Interest on savings and investment • Current interest rates • Depreciation
Day 2	Housing mortgages • Creating a mortgage calculator • Comparing mortgage cost
Day 3	Credit card debt • National data Management tips
Day 4	Auto Loans • Leasing, renting or buying
Day 5	Assessment and poster or ad presentation

As students use various formulae throughout the week to solve problems related to interest on loans and investment, the aim is to create an authentic experience for them so they experience it the way they would in real life. This prepares students to question investors and to know what to look for and when they are getting a reasonable deal. This brings to mind these following essential questions:

1. How does calculating interest several times per year affect the yield on an investment?
2. How does a small change in the percent of interest or the principal affect the amount?
3. What are the essential distinctions between simple and compound interest?
4. How do discrete and continuous compounding interest differ from each other?
5. When is it more beneficial to use long-term loans over short-term investments or loans?

As students learn the distinctions between the various types of interest charges or payments they will begin to make better use of the facilities available in the market; be it capitalizing on better ways to invest or minimizing the cost of loans. Each question is therefore designed to explore a different component of investment or loan portfolios.

Day 1 – Introduction Exponential Functions

Financial decision making and mathematical thinking

As students and teachers grapple with ideas related to compound interest both discrete and continuous compounding, it is important that there is a clear understanding of the limitations of compound interest even though its growth is significantly better than simple interest.

Problem 1

Today you will each be given $1 to invest at an interest rate of 100% per annum. Suppose Bank of America pays interest once at the end of the year, how much money will you have at the end of that year?

What if Fifth-Third Bank decided to pay you interest at the end of every 6 months that is twice annually at the same rate, which bank would you choose? And why?

Now just suppose you had the option of investing this same amount at the same interest rate paid every month, week or even daily,

1. which option would yield the highest interest over a 1-year period?
2. What is the maximum ending value you could amass in 1 year?

Compounding	Periods	Formula	Value (%)
Yearly	1	$1(2)^1$	100
Semiannually	2		125
Quarterly	4		144.14
Monthly	12		161.30
Daily	365		171.46
	n	$A = PV(1 + \frac{\frac{r}{100}}{n})^n$	
Continuously	Infinite	$A = Pe^{rt}$	171.83

A is the future value P is the initial value r is the rate % *t* is the time in years

Problem 2 – Salary Choices

Let's suppose you attended a job interview and you were given a chance to choose one of the following options for salary payments. Which one would you choose and why?

1. Receive one cent on the first day, two cents on the second day and your salary continues to double every day after that.
2. Receive a one-time payment of $1000,000

Day	**1**	**2**	**3**	**20**	**100**	**n**	**Formula**
Wages	**1 cents**	**2 cents**	**4 cents**				

Problem 3 - Future value of a computer

Adrianna purchases a computer for $1600. The value of this computer is reduced by 20% each year. If she wants to resell this computer for no less than half its original cost, how many years should she keep it for before reselling?

Worded Problems	Exponential Growth or Decay	Write a function that represents	Solution
 Raegan deposits $400 into her 5th /3rd bank account that earns 3% interest compounded annually. How much money will she have after 8 years?		$A = 400(1 + 0.03)^x$ *A is the amount in the account* *x is the number of years*	
A powerful computer sold for $3400 and loses on average 20% of its value each year. By which year would the computer be expected to be worth half its value? 			
Dinner at my favorite restaurant now costs $32 and has been increasing by 5% each year. How much did it cost 6 years ago when I first dined there? 		$f(x) = ab^x$ $a = \$32$ $b = 1.05$ $x = -6,$ *since we go back in time*	

Day 2 *Problems related to household wealth management and decision making*

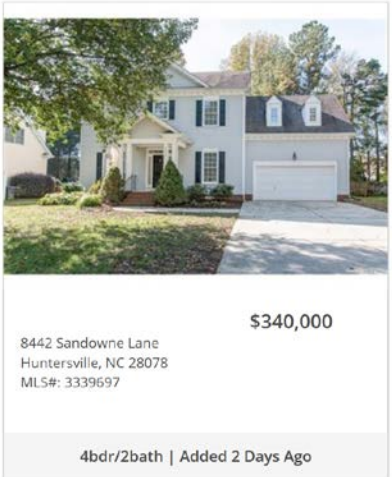
Mr. Falls of Redfin Realtors company Inc., is helping your family to find a suitable home in Charlotte to purchase. Your family has qualified for a mortgage loan of $350, 000 to buy a house and has a $20,000 deposit they have saved up. They need to decide which of the following options the Mr. Falls suggested is best for them.

NB closing costs are about 3.5% of the purchase price of a home in and around Charlotte

Can you help to:

1. *design a mortgage calculator to help simplify the calculations (see screenshots below with Option A completed)*
2. *explain which option offers the best overall deal in terms of total interest they will pay over the life of the loan.*

Option A – 3 bedroom/ 2 bathroom house in Huntersville



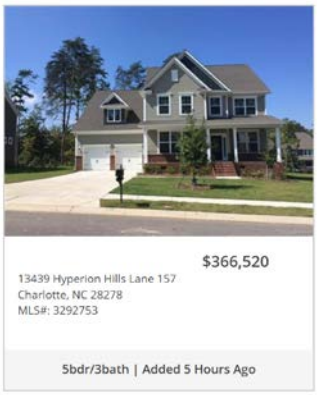
Mr. Falls calculates the costs for this home purchase as follows:

- Mortgage Loan $350,000
- Interest rate 4.5%
- Loan life 30 years
- Deposit (minimum) 5%
- Closing Costs 3.5%
- Amount financed

Option B - 4 ***bedroom/ 3-bathroom house in Mooresville, NC***

Mr. Falls calculates the costs for this home purchase as follows:

- Mortgage Loan $350,000
- Interest rate 5%
- Loan Life 25 years
- Deposit 4%
- Closing Costs 3.2%
- Amount financed



Mr. Falls calculates the costs for this home purchase as follows:

- Mortgage Loan $350,000
- Deposit 3.8%
- Loan Life 30 years
- Closing Costs 50% of 3.5%
  (seller pays other 50%)
- Amount financed

Vocabulary

- Mortgage: a loan to a purchaser of real estate that is secured on the property that is being bought. A mortgage could also be provided for any other purpose, when the borrower already owns a property and uses it as security.[21]
- Amortized loan: is one with scheduled regular installments that pay both principal and interest.[22]
- Depreciation: when assets decline in value over time. It is also an income tax deduction that enables a taxpayer to recover the cost of certain assets.[23]
- Earnest money deposit: is a sum of money paid by a home buyer to indicate commitment to buying a house.
- Closing Costs:

  This cost is most commonly associated with real estate transactions. These costs can be 3% to 6% of the cost of a house. Common closing costs include loan application fees, points, prepaid homeowners' insurance, an appraisal fee, inspection fees, transfer taxes, escrow fees, attorney fees, recording fees, prepaid interest, prepaid private mortgage insurance, title insurance, and title search costs. Other costs include the cost of obtaining a credit report , processing fees, courier fees and paperwork preparation fees.[24]

Creating Mortgage Calculator in Excel

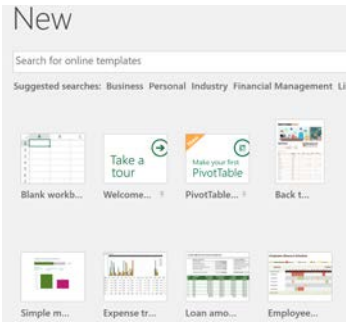
Use Microsoft Excel to create a mortgage calculator with the following summary information categories:

- Loan Amount $
- Annual interest rate
- Life Loan (in years)
- Number of payments
- Payment per period
- Sum of Payment
- Interest Cost

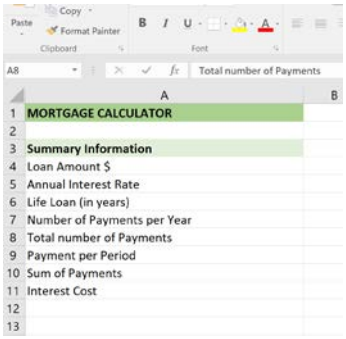
Step 1 – Input Headings and formula into spreadsheet[25]

Open Microsoft Excel

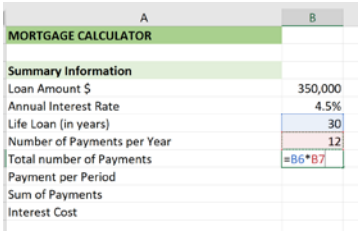
1. Select new
2. Open blank workbook
    - An alternative is to use an existing online template or find on in Excel

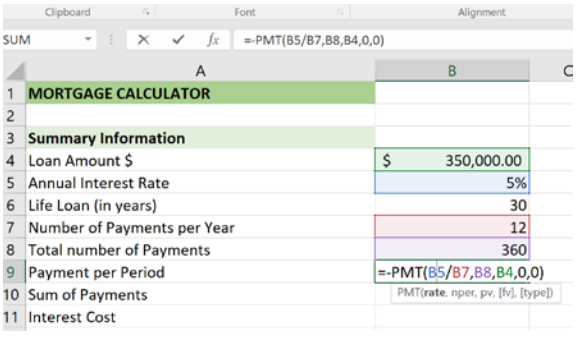


3. Create your "categories" in column A
    - Loan Amount $
    - Annual interest rate (%)
    - Life Loan (in years)
    - Number of payments per year
    - Total number of payments (over the life of the loan)
    - Payment per period
        i. Use formula: *"= -PMT (Interest rate/Payments per Year, Total number of Payments, Loan amount,0,0)" then press enter.*
        ii. *Displayed in Excel as PMT (rate, nper, pv,[fv],[type]*
        iii. *The final "0" in the column indicates payment is made at the end of the period; use "1" if payment is made at the start of the period.*
    - Sum of Payment (total payment over the life of the loan)
    - Interest Cost (total amount paid in interest over the life of the loan)

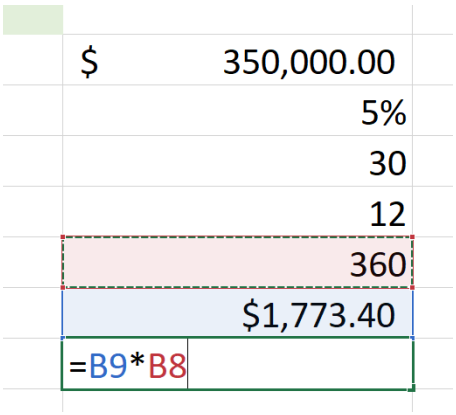


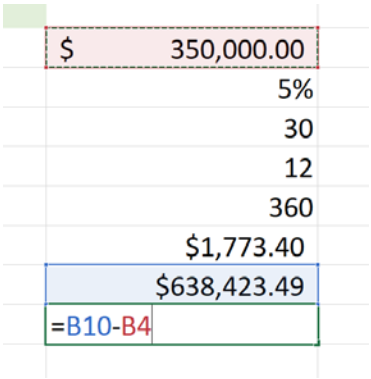
4. Enter appropriate values for each category in column B





5. Enter formula and calculate
    - Total number of payments
    - Sum of payments
    - Interest cost





Using the calculator students will explore the data provided by Mr. Falls above.

Day 3 - Credit Cards

Loans can provide a useful facility to fill the gap between income and expenditure needs. These loans may be long-term or short. Some of the earliest loans students process after finishing high school include buying a car and getting a credit card. As a result, students need to be knowledgeable about how the interest on these loans are calculated. In addition, they need to be aware that they can build their credit by:

i. Paying their bills on time and
ii. By paying their balance off within the 30-day interest free period for credit cards
iii. Or by paying more than the minimum balance and hence minimize the overall cost of these loans.

Student Activity

You just moved into your first apartment and you need $2000 to buy some new furniture. You recently received several offers in the mail for credit cards you pre-qualify for. Which of the following options do you think would suit you best or do you take them all?

Bank of America

- No annual fee
- No balance transfers fees for the first 18 months
- 23.5% APR
- Late payment $35 (after 2 later payments, a hold will be put on credit until payment is reconciled.

Ashley Furniture Company Credit Card

Terms of use:

- 0% interest if paid within 24 months (no late payments, minimum required payments. may be less than the 24 equal payments for this loan).
- This card can only be used in Ashley Furniture stores or outlets.

Discover Card

- No annual fees
- No interest for 18 months, 19-26% thereafter
- 3% or $5 transfer fees (whichever is greater)
- Earn 2%-5% cashback on select purchases

Day 4 - Auto Loans

For today's lesson students will try to understand the commitment when buying or leasing a car. They will consider factors that make it more advantageous for consumers to do one or the other. For example, consumers must consider factors like:

1. What do you do at the end of a lease term? Do you purchase, re-lease for a newer model or just turn the car back in?
2. How much is the car worth at the end of the loan period?
3. If a new car depreciates at 20% of its value in the first year and then 15% annually until after 10 years, how long should you keep it for if you wish to recover at least 40% of its original value?

<u>Vocabulary</u>

Lease:

> A Finance Lease or Car Lease is a commercial finance product. Under a Car Lease, the financier purchases a car on behalf of the customer, and then leases the vehicle to the customer in return for monthly rental payments. At the end of the term (length) of the lease, the financier gives the customer the option to purchase the vehicle in return for a final installment (residual value). Alternatively, the customer may choose to "trade in" the vehicle, or re-finance the residual and continue the lease.[26]

Residual Value: the estimated value of a car at the end of a lease term is called the residual value.[27]

Student activity

Today you will pretend you are in the market shopping for a new car and you have the option to buy or lease. Consider the options outlined below and go online to explore other options:

https://www.bmwusa.com/special-offers/finance.2017-330e-iPerformance.html?modal=special-offers-legal

Explore the various options for buying your preferred model and see which option is best economically.

1. Should you lease or finance the car of your choice?
2. How does the available information influence your decision?
3. What makes your choice the best decision for you?

Option 1 - Subaru Forester – 5 years no interest

2017 Subaru Forester 2.5I Premium HFF

- 28 MPG (combined city/hwy)
- 5-star rating (should only be compared to other vehicles of similar size and weight)

• Manufacturer's suggested retail price	$25,995
• Optional Package	$500
• Destination and delivery	$875
• Total suggested retail price	$27,370

Option 2 – Subaru Outback - 5 years no interest

2017 Subaru Outback 2.5I/HDB

- 28 MPG (combined city/hwy)
- 5-star rating (should only be compared to other vehicles of similar size and weight)

• Manufacturer's suggested retail price	$25,645
• Optional Package	$442
• Destination and delivery	$875
• Total suggested retail price	$26,962

Alternative financing

The State Employees Credit Union is willing to give you a 5-year loan of up to $35, 000 for a vehicle purchase at a 4.75% per annum rate of interest.

Refer to and modify the calculator to determine the monthly payments for each of these cars listed above. State whether you would choose one of these options or would you lease one of the following cars:

Option 3 – Lease a BMW for 3 years[28]

1. Which model

2017 330e iPerformance

Leasing at $309/month Important Info

Finance at as low as 3.34% APR Important Info

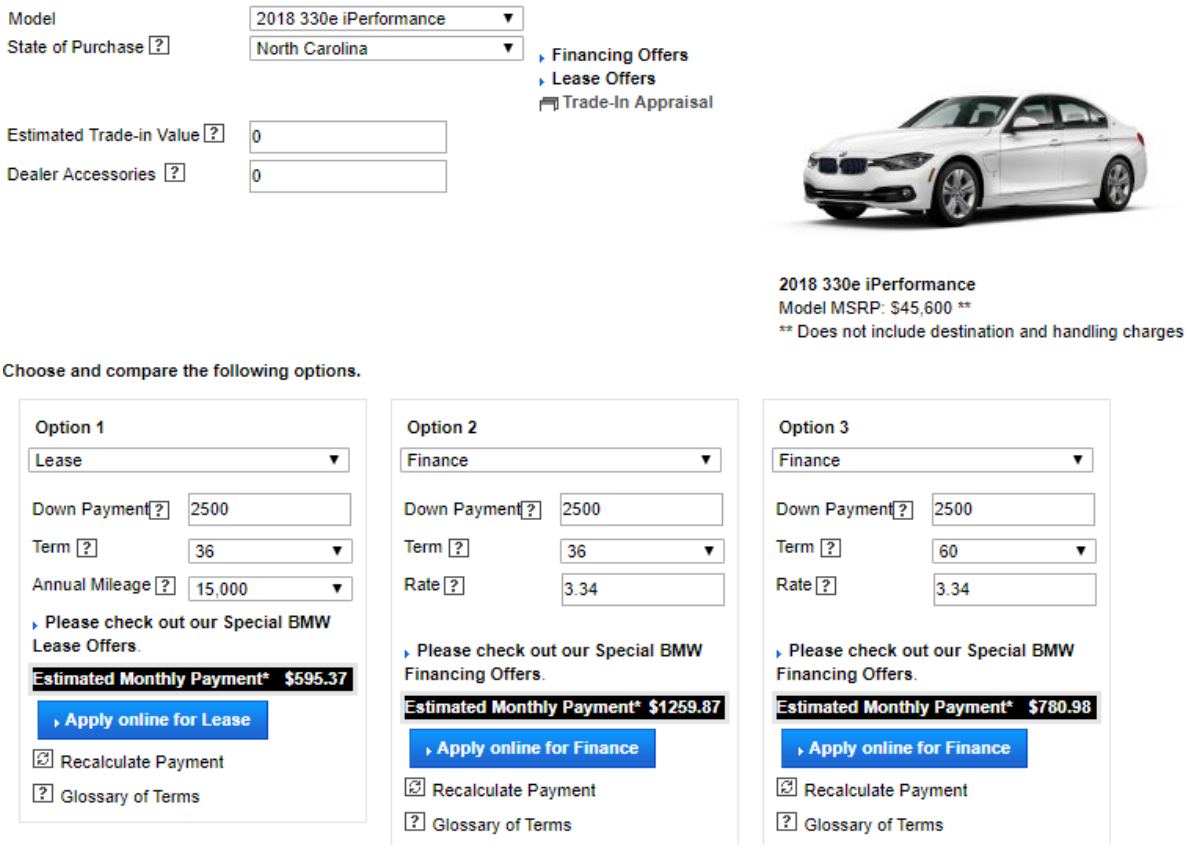
Included in payment: **$6,000 credit**

2018 330e iPerformance

Leasing at $369/month Important Info

Finance at as low as 3.34% APR Important Info

Included in payment: **$6,000 credit**



Buying a used car

The used car industry offers alternatives to buying a new car. This allows consumers to compare cars based on their condition including but not limited to mileage, physical condition (water, physical damage or other insurance claims), accident history and age of car. In addition, market value is an important factor used in determining the price of a used car.

Considering a typical 20% depreciation in the value of a new car in the first year, and 15% each year after that until it gets to 10 years-old:

- calculate how much you would save by purchasing a one-year-old car and compare to the new car you selected.
- determine the monthly payments

Day 5 - Assessment:

Question 1

Explain the difference between continuous compound and discrete compound interest. Provide a suitable worked example with calculations to assist in making this distinction clear.

_____

_____

Question 2

Having examined the purchase of a new car and the depreciation in the first year, explain whether it would be more beneficial to buy a new car or a one-year-old car. Provide suitable examples to support your choice.

_____

_____

_____

_____

Task 1 – Comic Advertising

Create a commercial in the form of a comic strip to illustrate your understanding of the differences between compound interest and simple interest. Ensure that your comic has at least:

- 5 sets of conversation happening between the characters.
- One graph showing a comparison between simple and compound interest.
- One benefit of each type of interest

Task 2 – Financial Adviser Brochure

Design a brochure to help your clients who have recently completed college and wish to undertake any two of the following: buying a car, an apartment, taking out a credit card or starting to save for retirement. As you create this brochure be mindful of other factors that these commitments will impact and how each can lend itself to multiple choices. Finally, add a list of resources to help these clients make each of their selections.

Endnotes

1. http://www.mempowered.com/children/international-curricula."International Curricula." Mempowered. Accessed November 18, 2017.

2. http://www.cnn.com/2016/09/04/opinions/sex-ed-is-required-but-not-financial-education-heather-long/index.html. Accessed June 10, 2017.

3. STANGO, VICTOR, and JONATHAN ZINMAN. 2009. "Exponential Growth Bias and Household Finance". *The Journal of Finance.* 64 (6): 2807-2849.

4. Corkery, Michael, and Stacy Cowley. "Household Debt Makes a Comeback in the U.S." The New York Times. May 17, 2017. Accessed October 25, 2017.

5. Ibid

6. https://ncdpi.sas.com/evalComposite.html?as=a&aj=a&x9=6&w4=103&ww=183281 (June 8, 2017).

7. Ibid

8. https://navigator.cms.k12.nc.us/dana-na/auth/url/_35/welcome.cgi Accessed December 1, 2017

9. Ibid

10. North, Connie E. 2007. "More than Words? Delving into the Substantive Meaning(s) of "Social Justice" in Education". *Review of Educational Research.* 76 (4): 507-535.

11. https://www.investopedia.com/ask/answers/050115/what-difference-between-continuous-compounding-and-discrete-compounding.asp#ixzz503rSDHKH. Accessed November 16, 2017.

12. Ibid

13. http://public.eblib.com/choice/publicfullrecord.aspx?p=4716729.Baum, Sandy. 2016. *Student debt: rhetoric and realities of higher education financing*.

14. https://www.bankrate.com/calculators/savings/simple-savings-calculator.aspx

15. Principles and Standards for School Mathematics (2000). The National Council of Teachers of Mathematics, Inc. Reston VA. Accessed June 1, 2017

16. Corkery, Michael, and Stacy Cowley. "Household Debt Makes a Comeback in the U.S." The New York Times. May 17, 2017. Accessed October 25, 2017.

17. Ibid

18. https://www.nerdwallet.com/blog/average-credit-card-debt-household/.Issa, Erin El. "NerdWallet's 2016 Household Debt Study." NerdWallet. December 14, 2016. Accessed November 16, 2017.

19. Ibid

20. Ibid

21. http://marketbusinessnews.com/financial-glossary/financial-glossary-m/ Accessed December 10, 2017

22. Ibid

23. https://www.realtor.com/advice/finance/understanding-the-earnest-money-deposit-2/ Accessed December 12, 2017.

24. http://www.investinganswers.com/financial-dictionary/real-estate/closing-costs-98 . Accessed December 12, 2017.

25. https://www.wikihow.com/Create-a-Mortgage-Calculator-With-Microsoft-Excel. Accessed November 5, 2017.

26. https://www.strattonfinance.com.au/car-finance/learn/faq/car-lease/what-is.aspx. Accessed on November 1, 2017.

27. http://www.realcartips.com/leasing/0052-how-residual-values-calculated.shtml. Accessed on December 12, 2017.

28. https://www.bmwusa.com/special-offers/finance.2017-330e-iPerformance.html?modal=special-offers-legal

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Baum, Sandy. 2016. *Student debt: rhetoric and realities of higher education financing*. http://public.eblib.com/choice/publicfullrecord.aspx?p=4716729.
This article reveals some of the realities students face when trying to finance higher education and also how they are impacted by this decision in their post college years.

Capeheart, Loretta, and Dragan Milovanovic. 2007. *Social Justice: Theories, Issues, and Movements*. Piscataway: Rutgers University Press. Accessed June 25, 2017 http://public.eblib.com/choice/publicfullrecord.aspx?p=320726
In this book the authors challenge popular perceptions of what is considered lawful and how justice is represented in modern society. They argue that a broader view needs to be examined as we shape our own understandings of justice.

Charlotte-Mecklenburg Schools. "School Accountability Growth, 2017." West Charlotte High School https://ncdpi.sas.com/evalComposite.html?as=a&aj=a&x9=6&w4=103&ww=183281 (June 8, 2017).
This website provides statistical information and demographic for all schools in the state of North Carolina. It is a government run organization and represents official data for nation use.

Corkery, Michael, and Stacy Cowley. "Household Debt Makes a Comeback in the U.S." The New York Times. May 17, 2017. Accessed October 25, 2017. https://www.nytimes.com/2017/05/17/business/dealbook/household-debt-united-states.html?action=click&contentCollection=Business Day&module=RelatedCoverage®ion=Marginalia&pgtype=article.
This article contains current and relevant information about household debt in the United States. It includes data about the current level of debt and compared it to what was happening nine years prior.

"Closing Costs." Investing Answers Building and Protecting Your Wealth through Education Publisher of The Next Banks That Could Fail. Accessed December 12, 2017. http://www.investinganswers.com/financial-dictionary/real-estate/closing-costs-98.
This website provides definitions and examples in a financial context that is relatable and easily understood.

"2017 330e iPerformance." 2017 BMW 3 Series - finance Offers - BMW North America. Accessed December 12, 2017. https://www.bmwusa.com/special-offers/finance.2017-330e-iPerformance.html?modal=special-offers-legal.
This is the official website for BMW USA. They allow students to manipulate buying options online and explore different makes and models without actually visiting the physical store.

Fernandez, Anthony. PhD Mathematics/ Mathematics Education
Dr. Fernandez is an Associate Professor in the Department of Mathematics and Statistics at the University of North Carolina in Charlotte. Dr. Fernandez has a wealth of knowledge spanning both the educational and financial field. During the seminars, he inspired me to begin to think of mathematics beyond the field of Education. He explored various uses of the subject but the main focus was about using math as a tool for understanding social justice and for articulating our perspectives in a variety of ways to empower students to become critical thinkers. He ensured that the focus of this unit was clear for teachers to be able to use and helped to shape the content of this unit in a marked way.

"Financial Glossary – M." Market Business News. July 07, 2017. Accessed December 10, 2017. http://marketbusinessnews.com/financial-glossary/financial-glossary-m/.
This a business glossary that provides accurate and concise definitions of financial terms.

Foltice, Bryan, and Thomas Langer. 2017. "In Equations We Trust? Formula Knowledge Effects on the Exponential Growth Bias in Household Finance Decisions". *Decision Analysis*. 14 (3): 170-186.

How Residual Values are Calculated. Accessed December 12, 2017. http://www.realcartips.com/leasing/0052-how-residual-values-calculated.shtml.
The definitions found here were concise and clear.

Issa, Erin El. "NerdWallet's 2016 Household Debt Study." NerdWallet. December 14, 2016. Accessed November 16, 2017. https://www.nerdwallet.com/blog/average-credit-card-debt-household/. Nerd Wallet does research on current financial trends and provides analysis of this data in a manner that is easily understood. It uses sources of information that are reputable and current.

"International Curricula." Mempowered. Accessed November 18, 2017. http://www.mempowered.com/children/international-curricula.
The data here was not very detailed, however it identified countries that include math as part of core curriculum in early education.

Long, Heather. “Sex Ed is Required but not Financial Education.” April 9, 2016. Accessed June 10, 2017. http://www.cnn.com/2016/09/04/opinions/sex-ed-is-required-but-not-financial-educationheather-long/index.html.
CNN has a reputation of credibility and is a well-recognized source for current information. It is one of the most prominent news networks across the world. In this article she summarily looks at how beneficial mandatory sex education is to students and delves into the possible positive effects a similar thrust could have on economic decisions if financial education were to become mandatory as well.

North, C. E. "More Than Words? Delving Into the Substantive Meaning(s) of "Social Justice" in Education." *Review of Educational Research*76, no. 4 (2006): 507-35. Accessed June 4, 2017. doi:10.3102/00346543076004507.
The idea of social justice can be explored from many different perspectives and it was important to understand the various schools of thought explored in this author's work.

North Carolina Math II Unpacking Standards.
http://www.ncpublicschools.org/curriculum/mathematics/scos/ (accessed June 6, 2016).
The North Carolina Math II unpacking of the standards helps to clarify what students should encounter in the classroom. It provides examples so teachers understand question format the will allow students to understand and breakdown the essential concepts.

Principles and Standards for School Mathematics (2000). The National Council of Teachers of Mathematics, Inc. Reston VA.
NCTM is a professional body of mathematics teachers that is well recognized. Curriculum standards provide clarification on what specific areas of content should be included in the core content of the classroom.

https://navigator.cms.k12.nc.us/dana-na/auth/url_35/welcome.cgi. Accessed December 1, 2017.
The navigator provides relevant, current and credible information relating to school enrollment for the Charlotte Mecklenburg Schools. This data contains subcategories that allows researchers to examine the demographic information for students at particular schools within the district.

https://math.la.asu.edu/~garcia/5.5reading.pdf.
This document used clear explanations to support the word problems and solutions. Some of the problems were adapted to create relevant experiences for the students in the classroom.

https://ncreportcards.ondemand.sas.com/SASVisualAnalyticsViewer/VisualAnalyticsViewer_guest.jsp?reportPath=/ReportCard/NC_SRC&reportName=NC+Report+Cards.
North Carolina Department of Public Instruction provides official data for schools in North Carolina. Retrieved October 26, 2017.

Staff, Investopedia. "Rule of 70." Investopedia. May 02, 2015. Accessed November 16, 2017. https://www.investopedia.com/terms/r/rule-of-70.asp.
This website focuses specifically on financial information. It provides definitions that are specific to the focus of the concepts explored in this unit. As I read through the definitions, I realized they addressed the essential content knowledge as well as useful information related to how the idea is used in the real world.

Stango, Victor and Jonathan Zinman. 2009. "Exponential Growth Bias and Household Finance". *The Journal of Finance.* 64 (6): 2807-2849.
Understanding factors that affect how consumers purchased a home even without recognizing these biases is critical in teaching students to become informed consumers. Also, this paper examines how intuition about perceived mathematical ideas can negatively play a role in consumers making decisions about purchasing real estate.

Stratton. "FAQ: What is a Car Lease?" Stratton Finance. Accessed December 12, 2017. https://www.strattonfinance.com.au/car-finance/learn/faq/car-lease/what-is.aspx.
This website provides useful definitions for a number of the financial terms that are used in this unit. the definitions were clear and concise.

WikiHow. "How to Create a Mortgage Calculator with Microsoft Excel." WikiHow. August 14, 2017. Accessed November 5, 2017. https://www.wikihow.com/Create-a-Mortgage-Calculator-With-Microsoft-Excel.
The steps in wikihow were clearly explained and showed screenshots which made it easy for me to following creating my own mortgage calculator. I believe students and teachers will find these steps helpful.

“Simple Savings Calculator.” Simple Savings Calculator - Savings Interest & Investment Growth Calculator, www.bankrate.com/calculators/savings/simple-savings-calculator.aspx. The format of this calculator allows the user to manipulate how often the interest is compounded and to also input monthly additions.

Appendix 1 – Standards

Topic: Functions, Statistics and Probability

Function	
Interpret functions that arise in application in terms of the context.	
NC.M3.A-SSE.1a	Identify and interpret parts of a piecewise, absolute value, polynomial, exponential and rational expressions including terms, factors, coefficients, and exponents.
NC.M3.A-CED.1	Create equations and inequalities in one variable that represent absolute value, polynomial, exponential, and rational relationships and use them to solve problems algebraically and graphically.
NC.M3.A-CED.2	Create and graph equations in two variables to represent absolute value, polynomial, exponential and rational relationships between quantities.
NC.M3.A-CED.3	Create systems of equations and/or inequalities to model situations in context.
Statistics	
Making Inference and Justifying Conclusions Understand and evaluate random processes underlying statistical experiments.	
NC.M3.S-IC1	Understand the process of making inferences about a population based on a random sample from that population.
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	
NC.M3.S-IC.3	Recognize the purposes of and differences between sample surveys, experiments, and observational studies and understand how randomization should be used in each.
NC.M3.S-IC.6	Evaluate articles and websites that report data by identifying the source of the data, the design of the study, and the way the data are graphically displayed.

Process Skills to be addressed in this unit include:

- *Making sense of problems and persevering in solving them*
- *Reasoning Abstractly and quantitatively*
- *Constructing viable arguments and critiquing the reasoning of others*
- *Modeling with mathematics*
- *Using appropriate tools strategically*
- *Attending to precision*
- *Looking for and making use of structure*
- *Looking for and expressing regularity in repeated reasoning*

Adapted from NCDPI Mathematics Standards Unpacking Math II