Math, Paper, Pencil, Graphene, Carbon Nanotubes, and Buckyballs

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This curriculum unit is recommended for:
Honors Pre-Calculus and Honors Math 3/ Grades 10^{th}, 11^{th}, and 12^{th}

Keywords: nanometer, micron, scientific notation, orders of magnitude, logarithm, exponential decay, vector, trigonometry, hexagonal, diameter, geometric sequence, common ratio, carbon allotropes, carbon fibers, carbon nanotubes, chirality, self-assembly, Buckminster fullerenes (buckyballs), graphene

Teaching Standards: See Appendix 1 for teaching standards addressed in this unit.

Synopsis: The purpose of this unit is to introduce students to the world of nanoscience, where the significant unit of measure is the nanometer which is one billionth of a meter. This unit can be used as a review at the end of the year for rational expressions which can be used to convert standard measurements to nanometers and make comparisons using scientific notation and order of magnitude. The unit focuses on different allotropes, or forms, of carbon which have been discovered in the last 40 years which include fullerenes, specifically buckyballs, which self-assemble from vaporized carbon. One problem posed to students is to determine the number of carbon bonds in a buckyball. Geometric sequences and solving exponential functions using logarithms are reviewed when comparing human hair to carbon nanotubes which are formed from graphene. Graphene is composed of a single layer of carbon atoms which form a regular hexagonal pattern. Trigonometry, vectors, and radical functions are reviewed when examining the diameters of carbon nanotubes and the possible ways they can be formed from graphene.

I plan to teach this unit during the coming year to 60 students in Honors Pre-Calculus/ 10^{th} , 11^{th}, and 12^{th} grade.

Introduction

As a math teacher, I believe in the power of the pencil and the paper. Describing a problem mathematically on paper is a means of organizing one's thoughts and showing how to solve a problem in a systematic and complete manner. Too often students are inclined to try to do all of the work in their head and when it comes to decomposing the problem into its steps, leaps are taken that often go without being described or understood. In students' minds, it is a waste of time to "show" the work that leads to their mathematical conclusion, especially if it is a simple arithmetic problem they can do in their heads or if the steps to solve the problem are few. Teachers try to make students show their work but until students study Geometry and proofs which require explaining all the steps, students seldom see the point to taking the time to show their work.

As we get to higher level mathematics in high school, the challenge is to expand the complexity of the problem that will require taking more steps and to logically come to conclusions and make connections that are not obvious. In order to do this, however, students need to know it is okay to make mistakes, try a path that doesn't work and then go try another path, and then another path, until you figure out the problem. Learning to critique the reasoning of others, not to mention yourself, cannot take place if the work is not written down so you can identify what went wrong.

Here is where the pencil becomes one of the greatest tools of the mathematician, it allows you to put down your thoughts and perceptions, mathematical formulas, and given information and practice what you know to see if the work and solution is correct. If you make a mistake you can erase and correct. Pencils lend itself to perseverance in problem solving.

So why this deep interest in pencils? They are a common staple in most classrooms and across generations, at least right now, everyone one knows what one is and how it is used. I say "right now" because technology is inching its way into the classroom and I wonder if someday a pencil will be obsolete and students will write using their finger or a stylus on a tablet where marks can be erased with a swipe.

So let's use a pencil, or more specifically the lead or graphite in a pencil, as the subject for this unit. Although we call the material in pencils "lead", the correct term is graphite which is a form of carbon and carbon has a lot of very interesting properties as we explore it on a much smaller scale or more specifically, the nanoscale. Many discoveries resulting in Nobel Prize recognition have been made within the last forty years revolving around carbon and its different forms at the nanoscale which may have significant impacts to everyone's life in a few years based on the rate research and development having been moving and it is my goal to expose students to this exciting field of study known as nanotechnology.

Class Demographics

My focus for this unit is Honors Pre-Calculus class which is my most mixed class in terms of level of students. I have seniors who have taken only standard math classes and are taking this class for the fourth year math requirement that fits in their schedule. I have juniors who chose not to take an AP course and are seeking to strengthen their math foundation before making the leap to an AP course next year. Finally, I have sophomores who have taken a virtual Math 3 course over the summer, in some cases to graduate early, in others to be able to take more AP math courses prior to graduation. The mix of my two classes made up of 55 students is 76% female and 24% male. The Demographics can be summarized as 64% white, 22% African American, 7% Hispanic, 3.5% Asian, and 3.5% American Indian.

The majority of my students are currently in Chemistry or have taken Chemistry. Since the topic of this unit is specifically tied to science I decided those in this position of taking Chemistry and Pre-Calculus together would take more interested in this unit. Our school operates on an A/B day schedule so all classes including the core classes are year-long classes. The timing of this unit will be as a project in the last unit so that it will also be used to review some of the previously taught Pre-Calculus concepts.

Unit Goals

The alignment of this unit will review Unit 2 on Rational Functions and Unit 3 on Exponential and Logarithmic Functions using the comparison of things distinguishable by the naked eye such as hair to something on the nanoscale such as a buckyball and include a discussion about the order of magnitude and finding solutions when the variable is in the exponent.

Unit 4 on Trigonometry Functions and Unit 7 pertaining to the introduction of vectors will be used when examining the hexagonal coordinate plane to the formation of the different types of carbon nanotubes that can be formed and the diameters of the tubes formed.

Finally, the nature of how materials change as the surface area changes will be used in the discussion of Unit 8 on Recursively Defined Functions as we look at models that can be used to determine a geometric series showing the surface area as a cubic figure is broken down into small parts.

The North Carolina has defined standards for Pre-Calculus and is in the process of aligning it with common core standards. I have included both those standards and the current Common Core standards that have not been modified by the North Carolina Department of Public Instruction in Appendix 1.

Summer Experience and CTI Seminars

I am going to confess that until this seminar topic came up, I was completely ignorant of what was occurring in the science community. My field is mathematics and I subscribe to journals for math educators but none featured anything revolving around nanotechnology.

The first thing that we did for our Charlotte Teacher Institute (CTI) seminar was to watch the video, "The Strange New World of Nanoscience".[1] It was released in 2010 and discussed ways in which science was now examining our world under a microscope that took it down to the billionth of a meter, or nanometer level. Since the United States primarily uses the English versus the metric system of measurement, we mainly see the common use of meters in reference to running races such as 5K and 10K races. Although science uses the metric system, in math we see only a few problems using meters, centimeters, and millimeters but rarely anything smaller than that.

With nanotechnology and the instruments that have now been developed like the Transmission Electron Microscope (TEM) and the Atomic Force Microscope (AFM) we can now examine molecules and atoms at the nanoscale. This summer I was chosen to take part in a program that allowed me to work in a nanotechnology lab in the Chemistry Department at the University of North Carolina Charlotte under Marcus Jones, our seminar leader. We supported doctoral students working to determine if gold nanoparticles or other materials can be used with polymer layers to create a device that stored energy more efficiently and we had the opportunity to see gold particles under the TEM.

A very tangible way that nanoparticles behave differently is by their color. The color of "bulk" material such as gold is the yellow we associate with gold but we could literally see the change in color as gold nanoparticles formed suspended in a solution which changed from a clear to a reddish purple color as it was heated up. During our seminars, Dr. Jones brought in suspended quantum dots which are nanoparticles of a semiconducting material that could be seen glowing brightly as fluorescent colors under a black light. Absorbance of light changes from bulk material to nanoscale of the same material and hence the different colors.

As part of the lab experience, we were allowed to measure out chemicals to the micro, or millionth, level using precise pipettes and scales. We also used tools in the lab such as the spectrometer to measure absorbance versus wavelengths in nanometers. This was only one of a number of lab projects involving nanotechnology. Other projects presented by college students over the summer included looking at applications in the health field as well.

Content Research

So how did I miss this growing phenomena and when did it start and how does graphite and carbon play a part? If you asked students what they know about carbon, you may get responses such as, "isn't that what makes up coal?", or from art students they might relate it to charcoal, students who watch a lot of science fiction television and movies may refer to "humans as carbon-based units ", and some students may even relate it to diamonds. Wasn't it Superman who could take a lump of coal and compress it with his hands to form a diamond? These multiple responses are due to the fact that carbon has a variety of structural forms or allotropes, graphite and diamond being among them. When the bonds that hold the carbon atoms together are changed, self-assembly occurs which result in different forms of the same atoms. [2]

Graphite

Using graphite as a writing medium came about in 1564 when a large deposit of graphite was found in England. It was appreciated for its dark markings but had problems because the material was soft and easily broke. Wooden casings were finally introduced and by 1662 pencils were being "mass produced".[3]

Why does the graphite pencil leave a dark, smooth mark? The different structural forms of carbon are due to its makeup of electrons which affect how it bonds with other atoms. In the case of graphite, the bonds form an in-plane hexagonal structure that is even stronger than diamond. The interplane bonds, on the other hand, or the bonds between those strong planes or sheets, are a lot weaker which allows easy shearing of the hexagonal plane and hence the layer of carbons left on the paper when a pencil is moved across the paper. [4]

Carbon Fibers

Chronologically, it is interesting to note that Edison in 1879 was one of the first to start experimenting with carbon fibers when he was working on the light bulb. He super-heated or "pyrolyzed" bamboo fibers and created carbon fibers but found that tungsten was a better filament to use.[5]

Applications of carbon fibers did not take off until 1958 until a physicist named Bacon created single crystal carbon filaments called "whiskers" that when combined with other material created composites that were stronger and stiffer than steel.[6] A key application was in the airline industry to make planes lighter and stronger. Even within my lifetime, I remember wooden tennis racquets and then came along a composite called graphite that was lighter, stronger, and, of course, more expensive.

Fullerenes

Once carbon fibers were found to be useful the next step was to come up with ways to produce them consistently for commercial use. This led to the process known as Chemical Vapor Deposition (CVD) which involved vaporizing carbon to form carbon fibers. In the search to create the smallest carbon fiber, three scientists, Curl, Kroto, and Smalley discovered another allotrope of carbon known as fullerenes. [7] Surprise! Instead of getting a carbon chain or fiber they found a ball!

Fullerenes are truncated icosahedrons made up of 60 carbon atoms consisting of 20 hexagonal shapes and 12 pentagonal shapes resembling a soccer ball. It also resembled a structure designed by Buckminster Fuller for the 1967 Montreal World Exhibition so they named their discovery Buckminsterfullerenes, also commonly referred to as buckyballs. Fullerenes are hollow and consist of 60 carbon atoms. Although discovered in 1985, they did not receive the Nobel Prize for Chemistry until 1996 for their discovery. According to Srivastava in "Carbon Nanotubes: Science and Applications", this discovery along with the discovery of carbon nanotubes spurred the progress in the field of nanotechnology. [8]Han in the same book identifies the uses of fullerenes in "electronic, magnetic, optical, chemical, biological, and medical applications."[9]

Carbon Nanotubes

In 1991, a Japanese scientist, Iijima, discovered multi-walled carbon nanotubes (MWNT). These nanotubes were made up of layers of graphite that could be seen under the transmission electron microscope and were created using an arc discharge. In 1993, Iijima was successful in observing a single-walled carbon nanotube (SWNT).[10]

What is unique about carbon nanotubes is that they present one dimensional properties, they are straight tubes. The graphite sheets are hexagonal in shape, though, and can be joined in one of two ways based on the chiral angle or "twist" of the sheet as it connects to form the tube. For an angle of $\Theta = 0^\circ$ you get the "zigzag" pattern, and for an angle of $\Theta = 30^\circ$, you get the "armchair" pattern. A third pattern is based on an angle between $0^\circ$and 30°. Under the scanning transmission microscope you can make out the hexagonal images of the nanotube. Incredibly, the conductive properties are either metallic (armchair and one-third of nanotubes) versus semi-conductive (zigzag and other patterns which form two-thirds of carbon nanotubes) based on its chirality. Because of the strong bonds lengthwise, carbon nanotubes are incredibly strong but their length is comparably short so many of the current studies reported are on figuring out how to grow defect free carbon nanotubes and strands of nanotubes that are long enough to be useful.[11]

Graphene

Although single walled carbon nanotubes (SWNT) had been around since 1993 and was known to be made of graphite, it was not until work done by Geim and Novoselov in 2004, which earned them the 2010 Nobel Prize for Physics, that the sheet of graphite was determined to be graphene, a sheet of carbon atoms which is one atom thick and considered to be two dimensional.[12]

Of all of the important finds, this one was unique because of the method that was used to create that one atom thick sheet of graphene. They simply took adhesive tape (like Scotch tape) and graphite from a pencil and peeled off the layers by what is referred to as "mechanical exfoliation".[13] How was this possible? Remember in the discussion above on graphite that the bonds holding the layers together were weak which allow us to remove layers of graphite as we write. Instead of rubbing the graphite on paper, they used tape to remove a layer and then another piece of tape to remove a layer from what was taken until all that was left was a single layer of carbon and they called this graphene. By the time they were through, the layer was invisible but the work they did to determine the properties of graphene were considered "ground breaking" in the news release for the Nobel Prize they were awarded.[14]

In the book, "Graphene", graphene may be used eventually to increase the ability and shrink the size of semiconductors and transistors used in electronics. Radio frequency electronics, sensor applications, spin-based technologies, transparent conductors, nanoelectromechanical devices, supercapacitors to store and deliver energy, and even super strong graphene composites are all possible applications which could help us make the next leap in technology.[15] To sum up, from our everyday pencils, when you take it down to the atomic level of graphene, we have a lot to look forward to.

The possibilities almost seem endless and what an exciting challenge for students to start looking at possible careers in science and engineering. When we look at all of the electronics we now have that we didn't have ten years ago, I remind my visual and performing art students that creative minds helped develop and manufactures those things we take for granted and creativity is just as important as having the math and science skills. By the time our students graduate from college, there may be jobs and opportunities that haven't even been thought of yet because of where we will be with our technology in the future.

Instructional Implementation

One of the major roadblock math teachers face is that students enter the classroom with this notion ingrained in their minds that they cannot do word problems. They have difficulty seeing the connections made from the mechanics of using mathematical formulas to applying those formulas in a real world situation. They are also impatient because they want the answers to come quickly, if there are too many steps involved them they are more likely to give up. One of the goals involving Common Core math is to promote perseverance in tackling problems. This implies that the problems have to be written such that the answers are not readily apparent but it also means that multiple steps will be involved.

Problem Solving

Problems assigned in honors level classes can be given with little guidance but to differentiate for standard classes, coaching or questions are given to step students and guide them to the given answer. Specific “problem-based tasks” are beginning to be integrated in the math curriculum and it is a challenge depending on the level of the students in the classroom. At the Pre-Calculus level, the focus has been primarily on Math I, II, and III, so little additional work has been done to supplement the curriculum with those types of problem-based tasks.

In my readings, I came across a book called “The Science of Problem Solving” by Mike Watts (1991). As he reflects on previous works by Gagne who authored the Nine Events of Instruction, he highlights key points in problem solving that I wanted to emphasize here.[16]

- The learner discovers…
- Previously learned rules…
- Achieve a solution…
- Novel situations…
- New Learning.

He later goes on in the book to outline a constructivism process used in science class, elements of which I am going to use here starting with “anticipation”. By identifying what students already know through past experiences, they can begin to interpret present situations, and start to make predictions on what is yet to come.[17]

Anticipation Guide

When formulating problems for this unit I wanted to start with an anticipation guide to get them introduced and interested in the subject of nanotechnology. So many things are happening in this field that will be dramatically shaping how we process information, develop health related and energy related solutions, and improve consumer products. In emulating our CTI seminar leader, Marcus Jones, students will view the same video he had the CTI fellows watch at our first gathering called “Strange New World of Nanoscience” narrated by Stephen Fry that was produced in 2010.

The anticipation guide (Appendix 2) is given before students watch the video to see what prior knowledge they have on what is a nanometer and the effect of breaking a substance down to a nanoscale. It also covers the capabilities of the tunneling scanning electron microscopes, the changes in color, self-organization, and the capabilities of optical versus scanning electron microscopes.

As the word implies, it helps prepare the student to "anticipate" certain subjects and topics. In essence the guide provides goals for watching the video. Wording of the guide is important and Dr. Jones has provided input to ensure that the questions are technically correct. An answer key has been provided (Appendix 3) to facilitate discussion of the video afterwards.

Review of Previous Learned Rules:

Once the subject of nanotechnology has been established through the video and the use of the anticipation guide, the next step is to review what students have learned with regards to the use of scientific notation, unit conversion, and order of magnitude. Although students are introduced to scientific notation in middle school, they really do not use it until they get to either their 10^{th} or 11^{th} grade year when they take Chemistry. Most science End-of-Grade tests do not require more than a scientific calculator and the math is minimal.

Until last year, rational functions were in Common Core Math 2 and it has now been moved to Common Core Math 3 and explored more fully in Pre-Calculus. One of the primary uses for rational expressions I stress in Pre-Calculus is the application of unit conversion and unit rates. I also review this in depth in trigonometry when converting from degrees to radians and radians to degrees. For this unit I want to stress moving within the metric system from meters to nanometers and angstroms, and from the English system of measurement to the metric system.

Order of magnitude is another concept that arises from the discussion of exponential and logarithmic functions. In the application for this unit, I am going to use the definition where the order of magnitude represents the comparison of one number to another number using the factor of ten. In this case the comparison is made by using scientific notation to express both quantities and taking the ratio. Using the Quotient exponent rule, the magnitude is determined by taking the difference of the exponents with base 10. If the numerator is smaller than the denominator, then another tenth or order of magnitude of 1 is taken away to account for the fraction less than one.

Another method that can be considered here is using logarithms. Given a logarithm really represents the exponent in an exponential equation, the order of magnitude is the difference between the calculated logarithms.

The first planned activity, *From the Earth to Buckyballs* (Appendix 4), is to have the students practice what they know by taking the earth, a soccer ball, and a Buckyball fullerene and compare their diameters by converting lengths to the metric system first and then to scientific notation. Ratios will be taken to determine their order of magnitude and answers can be checked using the answer key provided (Appendix 5).

To introduce the lesson, provide background history starting with the carbon fibers discovered by Edison to the discovery of the buckyballs leading to the Nobel Prize in 1996. Zome tools has a model of a Buckyball that can be assembled and it is available on Amazon for about $30. As a completed model, we can take that model which is similar to the size of a soccer ball and determine what the scale factor of the model is to earth. The known distant between carbon atoms is 1.4 Angstroms and the length of the strut used in the Zome Tool model is given as 2.86 cm or 1.126 inches per their website. If time permits, students can measure the struts to determine their length using rulers. (See Teacher Resources for links to the Zome Tools info sheet and the news release of the 1996 Nobel Prize in Chemistry)

Splitting Hairs and Carbon Fibers

There is a famous math question involving a checkerboard and a grain of wheat. If, for every square on a checkerboard you doubled the number of grains of wheat you had, the question at the end is how many grains of wheat (a bushel, a barrel, a truckload, or more) would you have by the time you reached the end of the checkerboard.

With that problem in mind I put a twist on it so that instead of doubling the amount we have, we split it. In this case we are going to take the human hair. It's something that we can see with the human eye and something everyone has, some more than others. The Appendix 6 worksheet, *Splitting Hairs and Carbon Fibers*, for this activity includes a table that shows what can be seen with the naked eye, an optical microscope, and a scanning electron microscope. The table shows the metric scale as well ranging from a meter to a tenth of an Angstrom, or 1 x 10^{-10} power.[18]

With hair, you think of strands of hair so are there things in the Nanoscience world we can compare hair to? The first things that come to mind are carbon fibers which were used in the 1960's and 1970's to produce lightweight composite products and the carbon nanotubes discovered in 1991. The typical diameter of commercial carbon fibers according to Dresselhaus is approximately 7 microns.[19] Also in this book are images of the multi-walled nanotubes discovered by Iijima in 1991 which show a measure of 3 nm.[20] So, the question is how many times do you need to split human hair to get it to reach the diameter of a carbon fiber, of a carbon nanotube?

Since this problem is really one of exponential decay, logarithms can be used to solve the problem. In order to involve students personally, have them rate their own hair and use that data for the exercise. The average hair is 100 micrometers with 80 micrometers a good estimate for thin, light colored hair and 120 micrometers for thick, dark hair.[21] Using the average hair diameter, the hair would need to be split 3.84 or approximately 4 times to reach the size of a 7µm carbon fiber and would need to split 15 times to reach the size of a 3 nm carbon nanotube. See Appendix 7 for the answer key.

This problem can also be used as a review for a geometric sequence. The first term would be the diameter of the hair that is chosen. Since the hair is being split in half, the common ratio, *r,* is equal to one half. To find the size at any given point in the sequence use the formula, $a_n = a_1 r^{n-1}$, with a_1 as the initial measurement of hair and *n* represents the number of times you want to split the hair.

An extension of the geometric sequence and series would be to apply it to the problem of breaking down a cube into smaller cubes to increase its surface area. An example of this was given in the video at the beginning of the unit. In this case, cutting a cube into eight equal cubes and affects the surface area by doubling it and the common ratio, *r,* is equal to two. Use the same formula above with a_1 as the initial measurement of surface area, *r* is the common ratio which is two, and *n* represents the number of times you want to split the cube into smaller cubes.

A cubic centimeter would have the initial surface area, a_1, of $6x10^{-4}m^2$. After being split twenty times, $n = 20\ and\ r = 2$, then the surface area has increased to

$$a_{20} = 6x10^{-4}(2)^{20} = 629.1m^2$$

Novel Solutions

Just how many atomic carbon bonds are in a Buckyball if there are 60 carbon atoms and why do structures behave differently when taken at the nanoscale?

Although not truly a recursive function, I thought an interesting exercise would be to take the formation of the Buckyball model as an example of examining patterns to determine just how many bonds are formed when you have a truncated icosahedron which is comprised of 20 regular hexagons and 12 regular pentagons. There are many different approaches to take to get to the answer which is 90 bonds.

Personally, I thought this was an interesting question and, although I started with the answer since the kit provided the correct number of parts, it was interesting to try to logically think through the outcome. If you have a Zome tool model of the buckyball, you can actually have students work one row at a time and logically think through how many you need for each level. I started this way with the base being the pentagon. From each corner of the pentagon is side which forms the next level made up of five hexagons. If you don't have a model, I have included under Teacher Resources the website for the Zome Tool Buckyball information sheet on how to build the buckyball model which gives you the diagrams for each level. Have students put their work on a blank sheet of paper and have them document their thinking process. Some students may want to come up with their own net of the figure, others may need some hints. Be prepared for students who won't give up until they have figured out the problem. I used the following:

The base is a pentagon = 5 bonds, from each point is a bond = 5 bonds

The 2nd level are 5 hexagons, 3 sides already formed, 5 x 3 remaining = 15 bonds

The 3rd level alternates pentagons on top of hexagons with hexagons between,

Pentagons (5x4 remaining sides) = 20, Hexagons (5 x 2 remaining sides) = 10

Half of buckyball = 5+5+15+20+10 = 55 (6 pentagons, 10 hexagons)

55 x 2 halves = 110, but there are 20 bonds around the last row being counted twice from each half so adjust by subtracting the overlap, 110 – 20 = 90 bonds

Chirality and Carbon Nanotubes

If time permits, there is a wonderful hands on activity that allows students to take transparencies with the hexagonal pattern on them and form the main two types of carbon nanotubes, the armchair form and the zigzag form. The information for this site will be under Teacher Resources. To shorten this for the purpose of a vector and trigonometry review there is a ten minute YouTube video by Mikelson[22] on carbon nanotube chirality that explains the way coordinate points are determined on the hexagonal plane. Instead of the coordinate x-y coordinate plane, vectors forming a 30º angle are used to determine location. Included in Appendix 8 is a copy of a hexagonal plane[23] and two problems (one "zigzag" and one "armchair" to determine the diameter of a carbon nanotube. You will find the 8 hexagonal zigzag tube circumference is 1.97nm and diameter is 0.626 nm and for the (5,5) armchair tube the circumference is 2.13nm and the diameter is 0.678 nm.

Depending on your class demographics and skill level, some of these activities will work better than others and may be more needed than others. I've tried to set this unit up to provide a review of some of the key topics in Pre-Calculus. The most challenging is the trigonometry problem because of the vectors and unusual coordinate plane. The most needed for my classes were the unit conversions and the logarithm problems. The most fun and the best investment I have made was in getting and putting together the buckyball, I enjoyed the challenge and nothing beats having a hands-on model for students to study.

Appendix 1: Teaching Standards

Current NC state standards followed by corresponding Common Core Standards

NCSCOS 1.03 The learner will describe geometric figures in the coordinate plane algebraically. Operate with vectors in two dimensions to model and solve problems.
CCSS.Math.Content.HSN.VM.A.1
(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*).

NCSCOS 2.07 The learner will use relations and functions to solve problems. Use recursively-defined functions to model and solve problems.
CSS.Math.Content.HSF.LE.A.2
Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

NCSCOS 2.01 The learner will use relations and functions to solve problems. Use functions (logarithmic) to model and solve problems; justify results.
CCSS.Math.Content.HSF.LE.A.4
For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

NCSCOS 2.02 The learner will use relations and functions to solve problems. Use trigonometric and inverse trigonometric functions to model and solve problems; justify results. a) Solve using graphs and algebraic properties.
CCSS.Math.Content.HSG.SRT.C.8
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

CCSS.Math.Content.HSG.MG.A.3
Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

APPENDIX 2:

Anticipation Guide

For the Video: Strange New World of Nanoscience

BEFORE			AFTER	
True	False	STATEMENT	True	False
		A nanometer is one millionth of a meter		
		When a substances is broken down into smaller parts, the total surface area of the substance gets smaller.		
		We can manipulate the structure of molecules using a scanning tunneling electron microscope.		
		Nanoscale materials are always the same color as bulk materials		
		Self-organization or self-assembly is when molecules group to form an ordered structure without any outside influences.		
		You can see the hexagonal structure of single-layered carbon (graphene) using an optical microscope.		

Not on the video: Because of national security, there are many government controls and restrictions with producing nanoparticles for use in consumer products. True /False

APPENDIX 3:

Anticipation Guide

For the Video: Strange New World of Nanoscience

BEFORE			AFTER	
True	False	STATEMENT	True	False
		A nanometer is one millionth of a meter		False, it is 1/1,000,000,000 Or one billionth
		When a substances is broken down into smaller parts, the total surface area of the substance gets smaller.		False, as the substance is broken down the amount of surface area gets larger
		We can manipulate the structure of molecules using a scanning tunneling electron microscope.	True	
		Nanoscale materials are always the same color as bulk materials		False, at the nanoscale level, light absorption may change so the color may change
		Self-organization or self-assembly is when molecules group to form an ordered structure without any outside influences.	True	
		You can see the hexagonal structure of single-layered carbon (graphene) using an optical microscope.		False, a scanning electron microscope is needed

Not on the video: Because of national security, there are many government controls and restrictions with producing nanoparticles for use in consumer products. True /False

Appendix 4

From the Earth to the Buckyball

Objective: Review English length to metric length unit conversion using equivalent ratios, metric length conversion, scientific notation, and orders of magnitude

Example: Convert the length of a pencil (7.5 inches long) and the Empire State Building (1454 feet to the tip) to meters, then express in scientific notation, and determine the order of magnitude of the building to the pencil.

Pencil: $7.5\ in\ (\frac{2.54\ cm}{1\ inch})\left(\frac{1\ m}{100\ cm}\right) = .1905m = 1.905\ x\ 10^{-1}m$

Building: $1454\ ft\left(\frac{.3048\ m}{1\ foot}\right) = 443.1792\ m = 4.432\ x\ 10^{2}$

Order of Magnitude of Building to Pencil is $= \frac{4.432\ x\ 10^{2}}{1.905\ x\ 10^{-1}} \rightarrow 2-(-1) = 3$

Conclusion: The Empire State Building is 3 orders of magnitude greater than a pencil

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7917.5 miles

8.65 inches

0.7 nanometers

Convert lengths to meters, then express in scientific notation, and determine the order of magnitude between the Earth, a soccer ball, and a buckyball.

	Given	Metric Equivalent	Meters	Scientific Notation	Order of Magnitude
Earth	7917.5 mi.	km			
Soccer ball	8.65 in.	cm			
Buckyball	0.7 nm	µm			

Measure the distance between atoms on the given model or use 1.126 inches. If the distance between carbon atoms in a buckyball is 1.4 Angstroms $(10^{-10}\ m)$, find the scale factor of the model. The scale factor is __________________ (Hint: $SF = (\frac{image}{original})$)

Appendix 5

From the Earth to the Buckyball (Answer Key)

Objective: Review English length to metric length unit conversion using equivalent ratios, metric length conversion, scientific notation, and orders of magnitude

Example: Convert the length of a pencil (7.5 inches long) and the Empire State Building (1454 feet to the tip) to meters, then express in scientific notation, and determine the order of magnitude of the building to the pencil.

Pencil: $7.5\ in \left(\frac{2.54\ cm}{1\ inch}\right)\left(\frac{1\ m}{100\ cm}\right) = .1905m = 1.905\ x\ 10^{-1}m$

Building: $1454\ ft \left(\frac{.3048\ m}{1\ foot}\right) = 443.1792\ m = 4.432\ x\ 10^{2}$

Order of Magnitude of Building to Pencil is $= \frac{4.432\ x\ 10^{2}}{1.905\ x\ 10^{-1}} \rightarrow 2-(-1) = 3$

Conclusion: The Empire State Building is 3 orders of magnitude greater than a pencil

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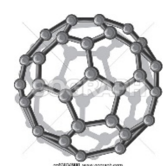
7917.5 miles

8.65 inches



0.7 nanometers

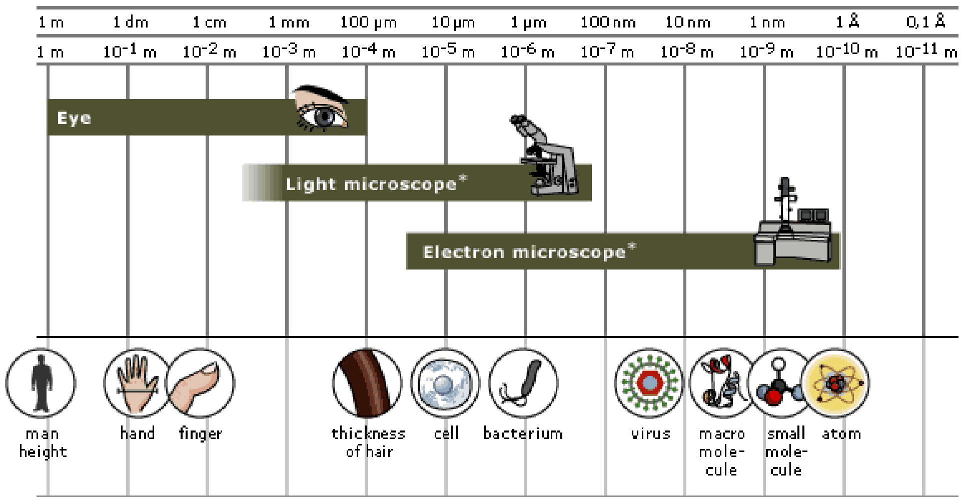
Convert lengths to meters, then express in scientific notation, and determine the order of magnitude between the Earth, a soccer ball, and a buckyball.

	Given	Metric Equivalent	Meters	Scientific Notation	Order of Magnitude
Earth	7917.5 mi.	12741.949 km	12741949 m	$1.27\ x\ 10^{7}$	
Soccer ball	8.65 in.	21.971 cm	.21971 m	$2.197x\ 10^{-1}$	7
Buckyball	0.7 nm	0.0007 µm	$7.0x\ 10^{-10}m$	$7.0\ x\ 10^{-10}$	16

Measure the distance between atoms on the given model or use 1.126 inches. If the distance between carbon atoms in a buckyball is 1.4 Angstroms ($10^{-10}\ m$), find the scale factor of the model. The scale factor is $\frac{2.8\ x\ 10^{-2}}{1.4\ x\ 10^{-10}} = 2\ x\ 10^{8}$ (Hint: $SF = \left(\frac{image}{original}\right)$)

Appendix 6

Splitting Hairs



http://www.nobelprize.org/educational/physics/microscopes/powerline/images/pl.gif

Objective: Determine how many times a human hair needs to be split to become the size of 1) 7 µm carbon fiber and 2) 3 nm carbon nanotube.

A) Choose the approximate diameter of your hair
   a. Thin hair 80 µm   b. Medium hair 100 µm   c. Thick hair 120 µm

B) Find the exponential function that describes the process using carbon fibers.

C) Solve the problem using logarithms.

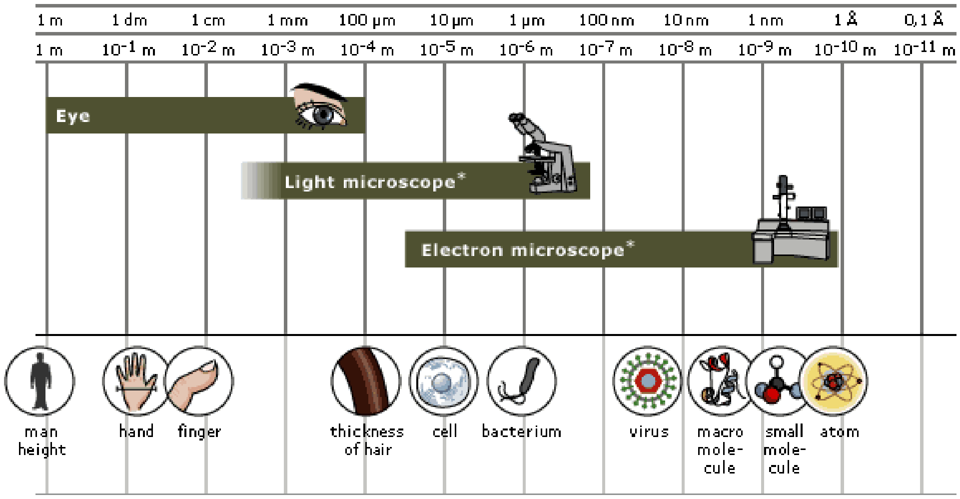
Note: Hair and carbon nanotubes are measured using different units. Switch to meters before setting up your equation.

D) Find the exponential function that describes the process using carbon nanotubes.

E) Solve the problem using logarithms.

Appendix 7

Splitting Hairs



http://www.nobelprize.org/educational/physics/microscopes/powerline/images/pl.gif

Objective: Determine how many times a human hair needs to be split to become the size of 1) 7 µm carbon fiber and 2) 3 nm carbon nanotube.

A) Choose the approximate diameter of your hair
   a. Thin hair 80 µm   b. Medium hair 100 µm   c. Thick hair 120 µm

B) Find the exponential function that describes the process using carbon fibers.
   $y = a_1 b^x$
   a. $7\mu m = 80\mu m(\frac{1}{2})^x$ b. $7\mu m = 100\mu m(\frac{1}{2})^x$ a. $7\mu m = 120\mu m(\frac{1}{2})^x$

C) Solve the problem using logarithms.

   a. $x = log_{\frac{1}{2}}\left(\frac{7}{80}\right) = 3.51$ b. $x = log_{\frac{1}{2}}\left(\frac{7}{100}\right) = 3.84$ c. $x = log_{\frac{1}{2}}\left(\frac{7}{120}\right) = 4.10$

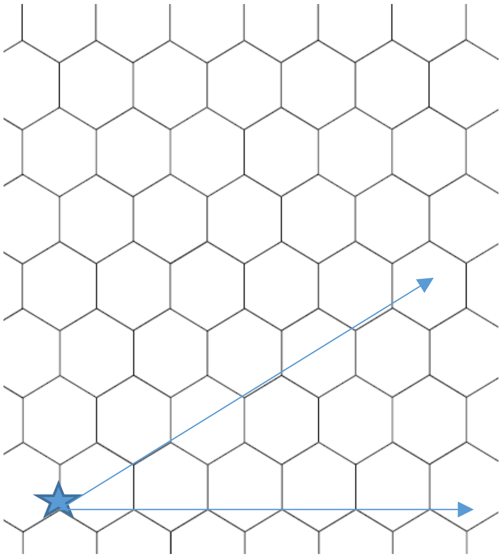
Note: Hair and carbon nanotubes are measured using different units. Switch to meters before setting up your equation.

D) Find the exponential function that describes the process using carbon nanotubes.
   a. $3x10^{-9} = 8x10^{-5}(\frac{1}{2})^x$ b. $3x10^{-9} = 1x10^{-4}(\frac{1}{2})^x$ c. $3x10^{-9} = 1.2x10^{-4}(\frac{1}{2})^x$

E) Solve the problem using logarithms.
   a. $x = log_{\frac{1}{2}}\left(\frac{3x10^{-9}}{8x10^{-5}}\right) = 14.7$ b. $x = log_{\frac{1}{2}}\left(\frac{3x10^{-9}}{1x10^{-4}}\right) = 15.0$ c. $x = log_{\frac{1}{2}}\left(\frac{3x10^{-9}}{1.2x10^{-4}}\right)$

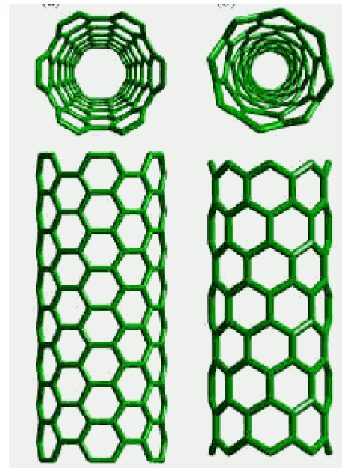
Appendix 8

Trigonometry and the Diameter of the Carbon Nanotube



$\theta = 30^\circ$

$\theta = 0^\circ$



Education.mrsec.edu

Objective: Find the diameter of the carbon nanotube given the length of the bond is 0.142nm

1) Zigzag Pattern of 8 Hexagons
2) Armchair pattern of (5,5)

Armchair $\theta = 30^\circ$

Zigzag $\theta = 0^\circ$

1) Hint: The side-to-side measurement forms two 30-60-90 triangles. Since half of that side to side measurement is $\frac{hypotenuse}{2}\sqrt{3}$ or $0.142 sin 60°$ then the total is equal to the $0.142\ x\ \sqrt{3} = .246nm\ or\ 2(0.142 sin 60°)$. If you have a zigzag pattern of eight hexagons then the circumference of the nanotube is $8\ x\ 0.246nm =$ _________ nm and the $diameter =$ _________nm

2) Hint: Note the pattern along the 30° vector is the length from point to point plus one side of the hexagon. The total length of that pattern is $0.142nm\ x\ 3 = .426nm$. For a (5,5) armchair nanotube the circumference is *.426nm x 5* = __________nm and the $diameter =$ _________nm

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End Notes

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