



The Place Value of Numbers through Hands-On Learning

by Erin Murray, 2016 CTI Fellow
Barringer Academic Center

This curriculum unit is recommended for:
Kindergarten through 2nd grade students

Keywords: place value, base 10, kindergarten, fusing dots, addition, subtraction, zero

Teaching Standards: See [Appendix 1](#) for teaching standards addressed in this unit.

Synopsis: Students will explore place value through hands on learning. Through games and activities, students will learn the value of a number, how zero holds a place, and how place value helps solve addition and subtraction problems.

I plan to teach this unit during the coming year to 15 students in kindergarten.

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The Place Value of Numbers through Hands-On Learning

Erin Murray

Introduction

Through our seminar with Dr. Harold Reiter, my cohort and I were challenged in many different ways. This gave me new insight as a student of mathematics, long after I have taken my last math class. Dr. Reiter is engaging and inspiring with his love of math, which is something I hope can transfer through these lessons to my students.

Some of the most memorable lessons we had throughout our seminar were based on games or kinesthetic activities. If this makes them notable and engaging for a group of teachers, I can just imagine how much those ideas could transfer to elementary students! We had so much fun moving to make a rational tangle, then using math to physically solve it! I thought about this as I created this unit and the influence of games and movement is throughout.

As the title suggests, this unit is designed for students at the early elementary level to gain a deep understanding of place value numeration through various hands on learning experiences. This curriculum unit is designed for my kindergarten class, but can easily be implemented through second grade. Many students in my class have memorized math facts, yet struggle to show any understanding of the arithmetic behind it. By building a strong foundation of place value principles, it is my hope that they will be able to develop a deeper understanding and rationale for their answers to problems.

Because the students learn in different ways, the unit is designed to show place value concepts in through several different methods- visual through videos, kinesthetic through outdoor games, and hands on through manipulatives. Dealing with early elementary students can be especially interesting, due to their attention and level of activity. Therefore, the activities will be active and engaging. Many of the activities are “game-like,” which will also add to the engagement of the students.

As you navigate your way through this unit, the students will be exploring place value in several different ways. They will then apply that knowledge to problems in addition and subtraction. I hope you and your students have fun moving and playing throughout this unit!

Rationale

The students in my class hold a wide range of skills and abilities. I have students who come into my class having been exposed to very little math. They may barely be able to count to 10 and are not able to identify single digit numbers. Other students come in fluent in addition and subtraction facts. These are the students who need a bit more rigor than the kindergarten curriculum provides, as well as a deeper understanding facts they have memorized.

Regardless of the child's knowledge thus far, I have found one thing in common with most of my students. There is often little understanding of concepts beyond the memorization of math facts. Because of that, the natural place I would like to explore with the students is place value. Place value utterly confuses the students and can become cause for misunderstanding with math concepts later on when working with larger numbers and more complicated equations. While students can get by with memorizing single digit addition facts, as they progress to double and triple digits, it becomes apparent how important place value is.

In an article Ross, she analyzes how the lack of understanding of place value in early elementary students leads to a misunderstanding in later concepts¹. My students need a strong foundation of understanding in place value, because those concepts will be crucial in their understanding through the years in mathematical topics. Addition, subtraction, multiplication, division, as well as several other concepts all rely on an understanding of place value. Once students have a concrete understanding, these other mathematical topics will be understood with greater ease.

Background – Demographics

Barringer Academic Center is an elementary magnet school located in Charlotte, NC. We have students from kindergarten through fifth grade. It is in the Charlotte-Mecklenburg school district. This school district is very large and diverse with 178 schools. There are 655 students within the school with a wide range of academic abilities. Barringer Academic Center is a partial magnet school, providing a specialized public education. The focus for our school is "Talent Development," which is a program for students who are academically gifted. Our school is also host to the Horizons program, which is a select program for the highly gifted student from the entire district. These classes are typically small in size and the students work at least two grade levels above their age-appropriate grade level. The remaining students come from the neighborhood. The school is a Title 1 school, with the majority of students qualifying for free or reduced lunch. The school places an importance on quality instruction and student growth. There are 37 classrooms with 100% fully licensed teachers, 85% are highly qualified.

I teach in the “learning immersion” program in kindergarten. The students come through a lottery process, for “nurturing” towards entrance into the Talent Development program, which starts in third grade. Because the process is simply a lottery without academic qualifications to gain entry into our program in kindergarten, this allows for a very wide range of students in my class. I have students who are well above grade level in one or more subject area, as well as those who struggle. Due to the especially wide range of students’ abilities in my class, it is important that I create a unit which will challenge and engage the entire class.

Content Objectives

The main mathematical objectives of this unit are for students to be able to have a concrete understanding of the value of numbers. The students should be able to look at a number and see it in a number of different ways, which will allow them to really understand it. An example might be looking at 12. The goal would be for the student to see number 12 and think that it is 12 ones and that it is also one ten and two ones. The students will then be able to apply this knowledge through identifying the value of the numbers and demonstrating ability to solve addition problems using place value models.

In addition to the conceptual objectives, students will learn content according to the common core standards of place value. The common core standards encourage students to manipulate numbers up to 20. The lessons will accomplish that as well as expose them to some larger numbers as well. They will learn to demonstrate their understanding of place value through fusing dots, bundling straws, and base 10 blocks. They will also apply that understanding of place value to addition and subtraction. The common core standards for kindergarten is to add and subtract with and without manipulatives to 5, so this will encompass that skill as well as go beyond 5 to the double digits. Students will use a variety of ways to demonstrate the above outlined concepts. They will engage in hands on learning with manipulatives, listen to engaging picture books, and watch videos to explore concepts. By demonstrating place value concepts in a variety of ways, students will be able to find a way that makes sense to them, as well as reinforce their existing thinking on the subject.

During this unit I plan to address the following process skills and concepts:

- Base 10 as a means to demonstrate place value.
Students will use and manipulate base 10 pieces to see that ten ones’ equals one ten, ten tens equal one hundred, etc. This will allow students to visually see this.
- Explore concept of 0 as a placeholder
Students will understand that zero is an important number. It shows that there is nothing to count in that place value spot.
- Addition and subtraction using place value

Once the students have mastered the place value concepts, students will practice this through addition problems that demonstrate mastery of place value. An example would be “15+10.” A student who understands place value would understand that this is adding 0 and 5 in the ones place and 1 and 1 in the tens place. So they should be able to do this mentally, instead of counting with objects or fingers.

Teaching Strategies

-K-W-L chart

A K-W-L chart is a graphical organizer designed to help in in the learning process. The letters K-W-L are an acronym, for what students already know, want to know, and ultimately learn. A K-W-L chart is typically divided into three columns titled Know, Want and Learned.

K	W	L

-Utilizing manipulatives (base 10, straws, fusing dots)

Students will be able to manipulate pieces through hands on learning experiences. By holding the materials and physically moving them, this will keep students actively engaged. Students will also see concrete movement through fusing, bundling, and combining, which will hopefully deepen their understanding past an abstract concept and into a concrete manner.

-Think Aloud of picture book

In a think aloud, a teacher reads a story out loud to their students. As a teacher reads the story, they orally tell the students what their thoughts are. Their verbalizations include describing things they're doing as they read to monitor their comprehension. The purpose of the think-aloud strategy is to model for students how skilled readers construct meaning from a text. It also allows teachers to point out important parts of the text or illustrations, that will aide in student understanding.

-Think-Pair-Share

The think, pair, share strategy is a cooperative learning technique that encourages each student to participate in the learning process. It has three distinct steps:

1. **Think:** Students think on their own about the question that has been posed to them.
2. **Pair:** Students are grouped in pairs to discuss their thoughts. This step allows students to articulate their ideas and to consider those of others.

3. **Share:** The pairs of students then share their ideas with the entire class. Students often feel more comfortable sharing, because they not only have their own idea, but that of their partner that they can chose to share as well.

Poster: <http://www.sparklebox.co.uk/blue/4231-4240/sb4234.html#.WA6LeegrK00-Videos> This poster can be used to accompany this strategy.

Classroom Activities

The initial lesson will be an introduction to the topic of place value. The teacher will present students with a K-W-L chart. This chart will be broken into three rows- know, wonder, and learned. For this lesson, students will think-pair-share about what they already know and wonder about place value. The students may be given an opportunity to record those responses on a post it note to attach to the poster or the teacher can record this into the “k” and “w” columns.

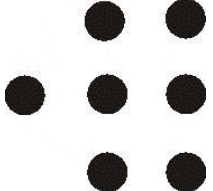
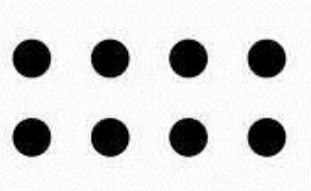
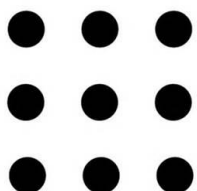
K	W	L
-Place value is about numbers.	-What exactly does place value mean?	
-Place value has tens and ones.	-How will it help me learn more math?	
	What does value mean?	
	Do numbers really have a place?	

Students will next be introduced to the concept of place value through this video: YouTube- Math Antics: Place Value <https://www.youtube.com/watch?v=T5Qf0qSSJFI> This short video will introduce the topic of place value.

In the next lesson, students will be introduced to the concept of fusing dots by James Tanton. Students can watch Tanton’s introductory video, where he shows how to use fusing dots as a basis for understanding place value. The video can be found at YouTube titled “Fusing Dots Lesson 1” by James Tanton <https://www.youtube.com/watch?v=1fXyW1vB4vI>.

In the next lesson, students will practice exploding dots with a dry erase marker and laminated game board printed and laminated from Appendix 2. The first way they can play is with dice. They can either use regular dice, or dots created from cubes that would

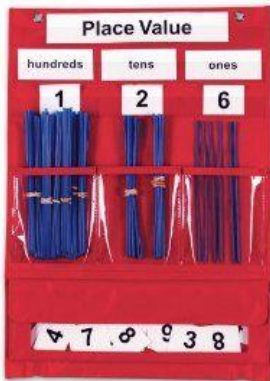
include numbers 7, 8, and 9. First they will roll a dice and put the first dots in the ones place, then they will roll a second dice to put that number in the tens spot. They will make two digit numbers in base 10, while practicing use of the game board. The second way they can play with the game board is with number cards. Students can play with number cards 0-30. Once they flip the card, they can create the number with dots on the game board.

Hundreds	Tens	Ones
		

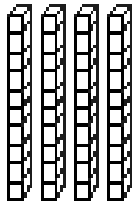
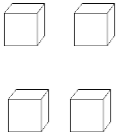
The next lesson will infuse the place value fusing dots model with a kinesthetic activity. The teacher will draw the fusing dots model onto blacktop using chalk or cement. Students will take turn rolling large dice. First they will roll for the ones place, and then count out that many students to go to the ones place. Next students will roll for the tens place. Once rolled, they will count out that many students will go stand in the tens place. Lastly, we will roll for the hundreds place and count out that many students for the hundreds place. Then they will say the number. For example, “6 hundreds, 5 tens, and 1 one... 651!”



In the third lesson, students will use straws, bundling in the tens place, to show two digit numbers. This could be modelled through a place value chart (shown below) as well as on the place value mat found in Appendix 2. To introduce the lessons, students can watch YouTube: Place Value Grouping into Numbers found at <https://www.youtube.com/watch?v=CEGhDFRHO10>. First the teacher can model this concept with dice. The teacher will roll a dice and put that many straws in the ones place. Then roll again and add more to the ones place. This process will continue until there is more than ten ones in the ones place, at which point the ten ones will be bundled into one bundle in the tens place. The students will then break into pairs to practice this activity with their own straws and laminated game board. Once students show an understanding with straws, they can move onto two-digit number cards, showing that they can “create” two digit numbers on their game board.



In the fourth lesson, students will manipulate place value (base 10) pieces on the laminated game board (Appendix 2). The teacher can begin by showing the students the ones and tens in place value pieces. The teacher can model putting ten ones together to show that it equals one ten piece. Next, the teacher can either flip over two cards to create a two-digit number or use a random two-digit number generator, such as one found at <http://numbergenerator.org/random-2-digit-number-generator>. The teacher can then model that number using the place value tens and one pieces and the laminated game board (Appendix 2). There are a couple choices that students can do next for students to model understanding. The teacher can meet with a small group, having them model their understanding by modelling this concept, while other students are in centers. Another choice is for students to work in pairs to practice.

Hundreds	Tens	Ones
		

To demonstrate understanding with base ten pieces, students can use color the worksheet in Appendix 3. Students can also use the base ten pieces in Appendix 4 to create a foldable building two digit numbers.

In the next lesson, students will explore 0 as a place holder. They will manipulate numbers and see that a 0 in the ones place, tens place, etc. is very important. The teacher can read the book Zero the Hero to the students as a think aloud to introduce the concept of zero. After, students can hold number cards up and practice making different numbers with them, demonstrating how important the zero is.

In the next lesson, students will do basic addition, up to 20, using fusing dots. This lesson can be modeled first by the teacher. A student will roll a dice and then draw that many dots on the base 10 board, in the ones spot. Then they will roll the dice again and put that many dots in the ones spot. If there are more than ten, the ten dots will “fuse,” get erased, and form one dot in the tens spot. If there is less than ten dots after two rolls, the student can continue to roll until the dots fuse. Students can break off in pairs and practice this. The teacher can observe and help students who are struggling.

Students will roll the dice and count out that many students to go to the ones place. The students will roll again and count out that many students to go to the ones place. Then we will count all the dots on the one place all together. If it is less than 10, the students will stay. If it is more than ten, then ten will “explode” and turn into one ten. Here is an example:

Students rolled a 5 and 5 students stand in ones.



Students rolled 6, added the 6 more and exploded!



The exploded dots fuse to 1 ten and leave 1 one.



Note: As an alternate or additional activity, students can use cones instead of children. The teacher can still draw the board with chalk, but instead of getting friends to make up the numbers, students can use cones instead.

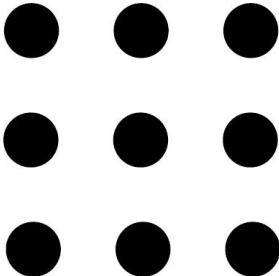
Now that students are familiar with fusing dots using dice, they will continue to practice using number cards 0-20. The teacher can model for the students and then students can work in pairs to practice. Students will flip over a card and record that many dots on the laminated game board. Then they will flip over another card and record that many dots on the laminated game board. They will count the ones place and then the tens place to make sure that there is not more than ten in either. If there is more than ten, then the dots will fuse and to make ten, or one hundred.

In the next lesson, students will work in three different centers to continue addition up to 30, using a variety of manipulatives- fusing dots, straws, and base 10 blocks. Students can rotate in centers, practicing each different way to do basic addition. This is a good opportunity for the teacher to check on student understanding. The teacher can either rotate around the centers with a checklist and place for anecdotal notes on student understanding or run a teacher center where they can evaluate the child's understanding.

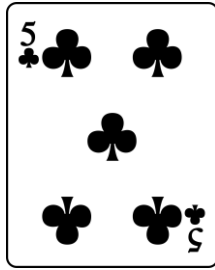
In the next lesson, students will explore subtraction of whole numbers, using fusing dots. The teacher will model the lesson using the laminated game board (Appendix 2). The teacher will start out with nine dots on their board. Then they will roll a dice or flip over a card (playing card or number cards to 9) to reveal a number. The teacher will then take off or erase that number of dots. Once they have practiced as a group several times, students will either work with a partner or in a small guided math group.

Example:

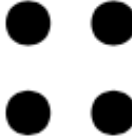
First the teacher starts out with nine dots in the ones place.

Hundreds	Tens	Ones
		

Next, a card is flipped over.




Therefore, five dots will be deleted/erased to reveal the answer of 4. $9-5=4$.

Hundreds	Tens	Ones
		

With the students having a basic understanding of subtraction, we will move onto bigger numbers to include exploding dots and the tens place. The teacher will start with 20 on her board, two in the tens place and zero in the ones place. They will then roll a dice and get a number to subtract. One of the tens will then need to explode into ten ones to be able to subtract it. Once this is done, the student will erase that number of dots. This should then result in the answer. Once this has been modelled several times and the teacher has checked for a level of understanding, students can either work in partners or in a small teacher-led group to assess their level of understanding.

Example:

First, there is twenty, shown as two tens.

Hundreds	Tens	Ones
		

Next, a five was rolled.



So, one of the tens needs to explode into ten ones.

Hundreds	Tens	Ones
	●	● ● ● ● ● ● ● ● ● ● ● ●

Now, five can be erased/removed to represent the five rolled on the dice.

Hundreds	Tens	Ones
	●	● ● ● ● ●

There are one ten and five ones left... or 15!

Next, students will do a kinesthetic activity with subtracting. The teacher will label the three squares for the students to use. Students will start out with a two-digit number, rolled from two different dice. For example, if the student rolls 3-4, then three students will stand in the tens square and three students will stand in the ones square. Next, we will roll the dice. For example, if a six is rolled, then we will need to explode the fused dots to help achieve an answer. Since we can only take 4 from the ones, the students will need to borrow from the tens. So one of the tens, and nine more, will come stand in the ones. Now that we have 14 ones, we can subtract 6, and would end up with

In the last lesson, students will discuss what they have learned about place value, in the KWL chart.

K	W	L
<p>-Place value is about numbers.</p> <p>-Place value has tens and ones.</p>	<p>-What exactly does place value mean?</p> <p>-How will it help me learn more math?</p> <p>What does value mean?</p> <p>Do numbers really have a place?</p>	<p>-Place value shows the value of each “digit” in a number based on its place or “spot” in the number.</p> <p>-Place value helps us to add and subtract because we understand where each number should go.</p> <p>-Value means how much each number is.</p> <p>-Numbers do have a place and the place they are in the number is important.</p> <p>-Zero has an important role in a number because it holds a digit spot if there is nothing there.</p> <p>-Ten ones’ equals one ten!</p>

Additional Activities/Extensions

- If students are mastering activities with two digits, they can move onto three or four digit numbers.
- Students can use multiple dice or dice with larger numbers written on it to make problems more complicated.
- For outdoor activities, cones can be substituted for students if there is not enough students, or to see it in a different way.

Materials for Classroom Use

- dry erase markers and erasers
 - Dry erase materials can be used with laminated game boards, so students can write right on them.
- straws or coffee stirrers and rubber bands
 - These are to be used with the place value pocket chart, to show bundling of tens or hundreds.
- place value pocket chart or homemade alternative

- This is to be used with the straws, but can easily be improvised. You may substitute three envelopes, with the top cut off.
- base 10 blocks
The base ten blocks will be used through various activities throughout the unit.
 - printable place value board (Appendix 2)
Several copies will be needed for various activities. These will work best when printed onto cardstock and laminated for longevity.
 - chalk
Chalk will be needed for the kinesthetic outdoor activities.
 - optional cones
As an option, instead of students, cones can be used for kinesthetic activities. These can often be easily found through the physical education teacher.
 - dice
Dice of any kind will be needed throughout the unit. You will need at least 20.
 - number cards
Several sets of number cards will be needed. Sets can be printed out onto colored card stock from <http://www.activityvillage.co.uk/number-flash-cards-1-30>.

Assessments

- observation
Students will be observed during different activities to see the areas in which they are struggling.
- KWL chart
The teacher will ask students to fill in the KWL chart to see what they have learned.
- classroom discussion
In various parts of the unit, there will be classroom discussions, where the teacher can see the connections which students are making.
- checklist of students understanding
During different activities, specifically addition centers, the teacher can walk around with a checklist to see if students are able to meet the standards.
- anecdotal notes
The teacher can take notes on teacher understanding based on their observations during activities, or through a teacher-led center.

Appendix 1 - Teaching Standards

North Carolina Common Core Standards

Number & Operations in Base Ten

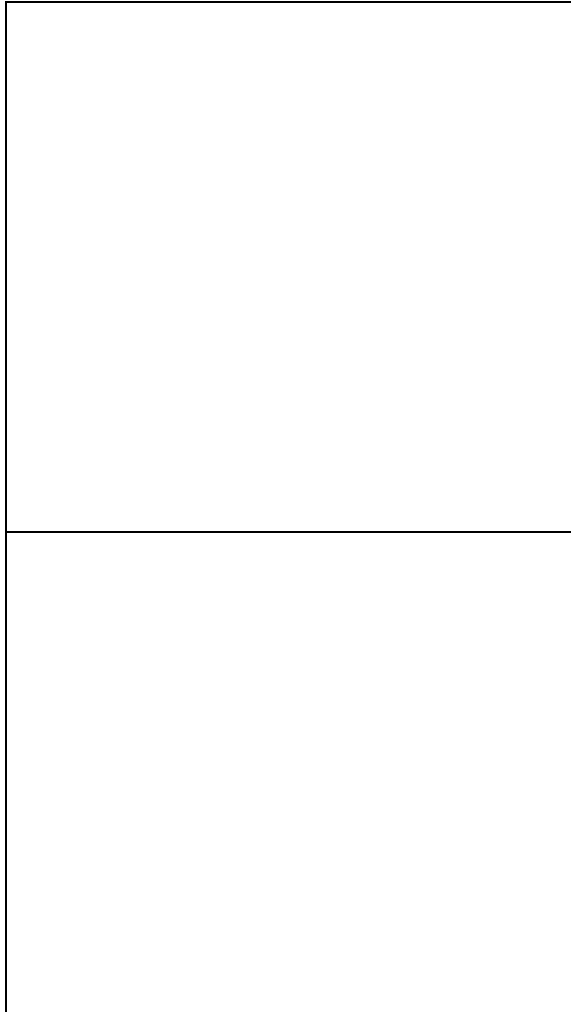
- CCSS.MATH.CONTENT.K.NBT.A.1- Work with numbers 11-19 to gain foundations for place value.
- CCSS.MATH.CONTENT.K.NBT.A.2- Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Counting and Cardinality

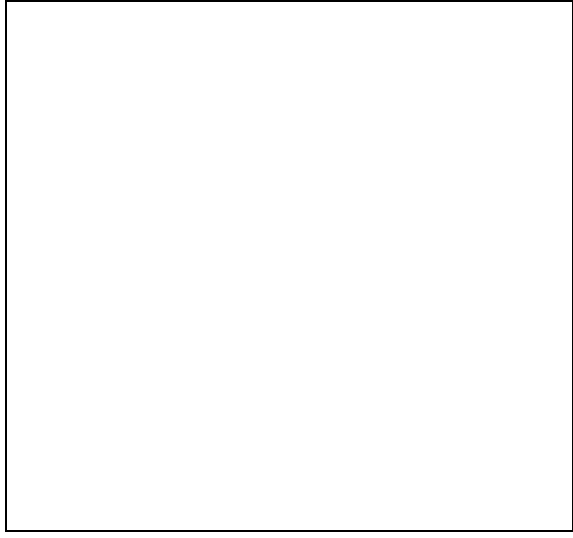
- CCSS.MATH.CONTENT.K.CC.B.4-Understand the relationship between numbers and quantities; connect counting to cardinality.
- CCSS.MATH.CONTENT.K.CC.B.4.A- When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
- CCSS.MATH.CONTENT.K.CC.B.4.B- Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
- CCSS.MATH.CONTENT.K.CC.A.3- Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Operations and Algebraic Thinking

- CCSS.Math.Content.K.OA.A.1- Represent addition and subtraction with objects, fingers, mental images, drawings¹, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
- CCSS.Math.Content.K.OA.A.2- Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
- CCSS.Math.Content.K.OA.A.3- Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).
- CCSS.Math.Content.K.OA.A.5- Fluently add and subtract within 5.



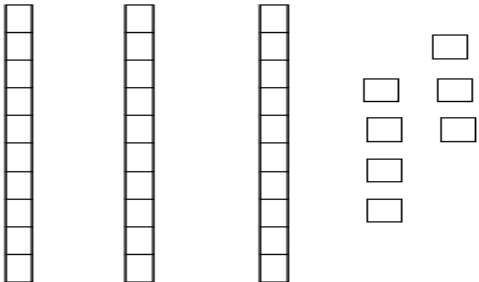
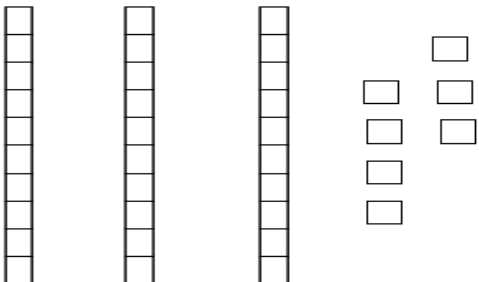
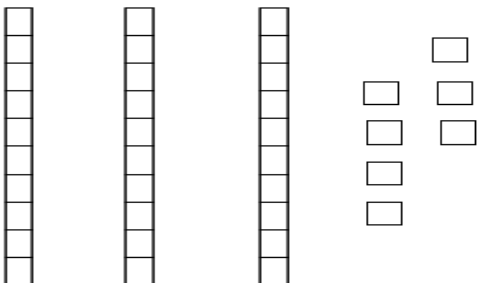
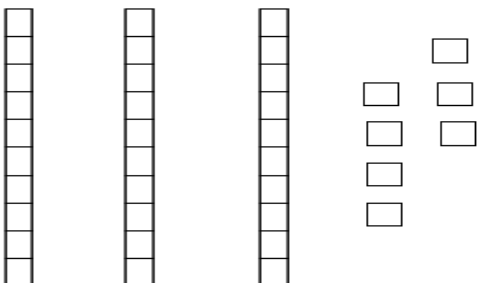
Appendix 2- Place Value Game board



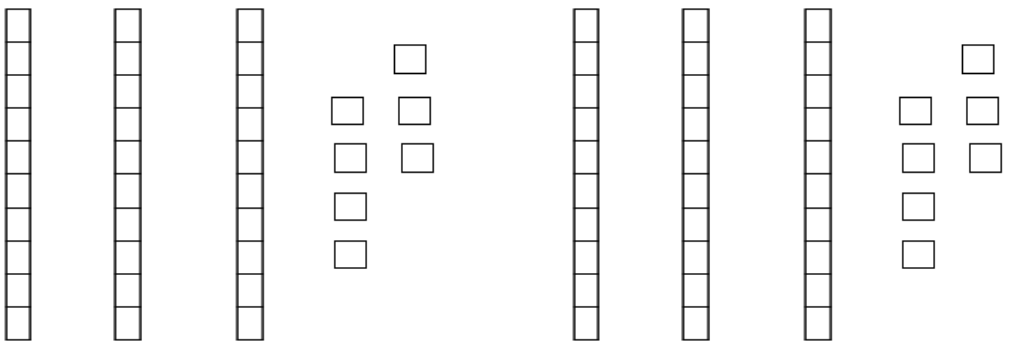
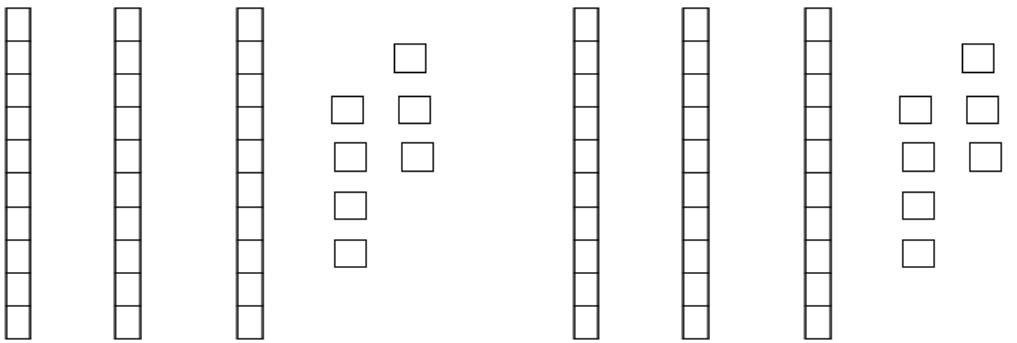
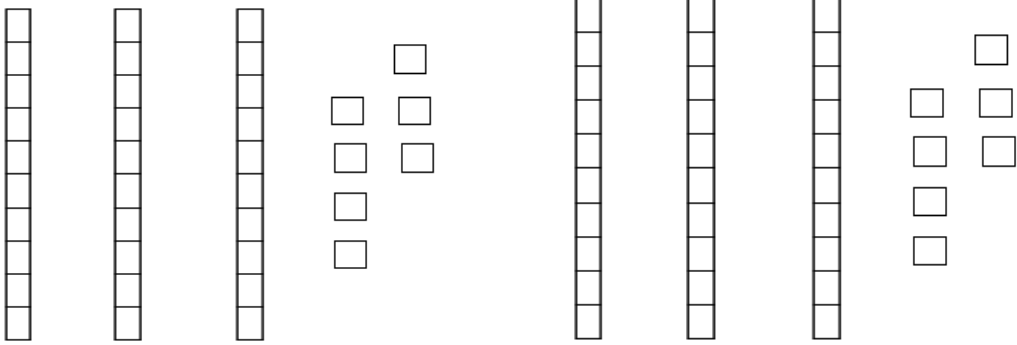
Appendix 3- Worksheet

Name: _____

Directions: Shade in the 10's and 1's to compose each number

11	
15	
23	
20	

Appendix 4- Place Value Pieces



Endnotes

1 Ross, Sharon Hill. "The Development of Children's Place-Value Numeration Concepts in Grades Two through Five." (1986).

Teacher Resources:

Franco, Betsy, and Shino Arihara. *Zero is the Leaves on the Tree*. Berkeley, CA: Tricycle Press, 2009.

This is a picture book which talks about the number zero and what it means. It goes through different objects which show none or zero. It is a great way to visualize the fact that zero means nothing.

Holub, Joan, and Tom Lichtenheld. *Zero the Hero*. New York: Henry Holt, 2012.

This picture book describes the adventures of a hero names zero. It talks about how important zero is in numbers and how different numbers would be without it.

Jeffers, Oliver. *The Hueys in None the Number*. New York: Penguin Group, 2014.

This picture book talks about zero and how there can be a number of "none." It is a silly, quick read to reinforce the concept of zero.

James Tanton wensite- <http://www.jamestanton.com/>

James Tanton is a fun, engaging mathematician who is the mind behind the concept of fusing dots. Through his website and YouTube channel, there are lots of videos describing fusing dots as a means to explain place value. These concepts can be used for Base 10 or other bases. It can be applied to several different levels of mathematical thinking- from early elementary to high school.

Fusing dots- <https://www.youtube.com/watch?v=1fXyW1vB4vI>

This video is an introduction to fusing dots concept, which are found throughout my unit.

What is place value video? <https://www.youtube.com/watch?v=T5Qf0qSSJFI>

This video is a great introduction to place value.

Student Resources

Khan academy place value- <https://www.youtube.com/watch?v=wx2gI8iwMCA>

This video from Khan academy explains place value. Khan Academy is a great resource for students to work on mathematical concepts.

IXL Math: <https://www.ixl.com/math/place-values>

IXL math is a great math website for students. This link will take students to place value practice.