

Fantastic Fractions: Comparing Fractions for Elementary Students

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This curriculum unit is recommended for: 2nd -3rd grade Elementary School classroom

Keywords: fraction, equivalent, whole number, unit fraction, numerator, denominator, greater than, less than, and compare

Teaching Standards: See Appendix 1 for teaching standards addressed in this unit.

Synopsis: Mathematics curriculum and instruction is a core subject area within the elementary school classroom. This curriculum unit will serve as an additional mathematics resource to the elementary school classroom. Within the unit, students will explore fractions to create models, poetry, and a PowerPoint to demonstrate their comprehension of comparing fractions. In this unit students will think of how fractions can be applied to their own lives, draw and create fraction models to compare fractions, utilize number lines and fraction bar to determine the fraction order of smallest to greatest, and use a multiplication chart to find equivalent fractions. This unit explores fraction comparison through mentor texts. Through this unit, critical thinking, trial and error, and confusion will be used to help with the concept of comparing fractions. Learning through failure is a skill that isn't used much in classrooms. Traditionally students think getting the answer right shows that learning is taken place. The unit includes lessons in comparing fractions where students will apply and develop fraction concepts into a PowerPoint. The lessons can be adapted to fit any classroom or learning style. Throughout the lessons there will be suggestions on how to differentiate instruction based on students. There are opportunities for independent work, small group instruction, guided instruction, and independent work levels based on students in the classroom.

I plan to teach this unit during the 2017 -2018 school year with third grade students.

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Deidra McGuire

Introduction

After several years as an educator I have noticed that students and teachers have a fear of math. In math we have been conditioned to think that in order to show how smart we are in math, we must get the correct answer. Common Core expects teachers and students to step away from "old math". Students are expected to not just know a routine or strategy. It is important for students to know the how and why behind their answers. Classroom discussion is an important tool in math. Having academic conversations is a great way for students to learn from one another. An academic conversation is more than just students telling their answers. Academic conversations allow students to elaborate and clarify their thinking. "Academic conversations are rare in many classrooms. Talk is often dominated by the teacher and a few students, or it does not advance beyond short responses to the teacher's questions." (Zwiers 2009). Throughout the unit, mathematicians will engage in academic conversations to get away from surface teaching. In this unit, students will dig deep when it comes to comparing fractions. Students will be able to incorporate reading and writing into this unit. This unit provides scholars the opportunity to write and create. Allowing students to express themselves through writing can open many doors, allow teachers to make many connections, and deepen their understanding of how their students think. This unit was created to allow students the flexibility to learn math through creating, writing, and conversations within the curriculum. The unit's purpose is to appeal to the entire child, and not just one piece of a subject or child.

Comparing fractions is when students are able to explain equivalence of fractions in special cases as it relates to a fraction's size. When students compare fractions it allows students to model fractions through pictures, number-lines, and size to determine if a fraction is smaller or bigger than another. Through comparing fractions, students will be able to create equivalent fractions, fraction models, poems, and Power points to show what they have learned about comparing fractions. Teaching them to compare fractions will enable them to apply their critical thinking skills and express their knowledge of fractions in a creative way. The collaboration and conversations that will be held in the

classroom will prepare the students to showcase their knowledge of comparing fractions beyond a paper and pencil test.

I have written this unit for third grade students during the first five months of school. The unit should come after the place value and geometry units. The unit is expected to take approximately two to three weeks, (approximately fifteen lessons). Some students may move through the unit more quickly depending on how fast they are able to grasp comparing fractions. This unit allows students to get creative in showcasing how a fraction is larger or smaller than another, create their own fractions to compare, and have meaningful conversations about a mathematical topic. This unit was created in order to build mathematical critical thinking, develop academic conversations, allow students to think outside the box, integrate literacy and math through mentor texts, and allow students to make mistakes to learn. The unit includes detailed outline of lesson plans, time-line of classroom activities, teaching strategies, as well as math workshop models to use when teaching. The unit also includes mentor text for teachers to use when teaching fraction comparison for second and third grade students.

Background

I am a third grade teacher at J.H. Gunn Elementary school in Charlotte, North Carolina. Currently there are twenty-two students in my class. Of the twenty-two students there are six students classified as English Language Learners. There are two students classified as exceptional children, and one student with a 504 educational plan. Throughout the duration of the math block we have one assistant in the classroom when she is not pulled for classroom coverage.

- J.H. Gunn is a transient elementary school where students from kindergarten through fifth grade learn; a place where all teachers believe children come first. Currently the third grade team is made up of six teachers. Each of the six teachers teach all subject areas.
- J.H. Gunn educates approximately 779 students from grades Kindergarten to Fifth grade. We are a Title One school serving students of low income families and aim to close the gap between our highest and lowest demographics. Our overall demographics indicate for the 2015-2016 school year include approximately 43% African American, 41% Hispanic/Latino, 11% white, 2% two or more races, and 2% Asian, and about 1% American Indian. Our largest gap we are trying to close is the performance of our Exceptional Children (EC) population and the performance of their White counterparts.

My school and professional learning community is data-driven. This means we focus instruction on the needs of our students as determined by assessment data. In math we have informal and formal assessments. After we take our formal assessments in math, my

PLC sits down with our math facilitator and principal to dissect the data. At the beginning of the school year (the beginning of September) students will take the MAP test which will determine the Lexile level and projected growth of the student. This test is taken 3 times per year; I will use the scores to aid in determining the activities and learning strategies for this unit. Other assessments such as anecdotal notes, exit tickets, quizzes, math small groups, and other formative assessments taken during in class discussions will be used as well in order to provide feedback and guide instruction for students will meet the child where they are and cater directly to their growth in math.

As a third grade teacher at J.H. Gunn, I face the challenge of protecting the math time during the day. Literacy is a big focus in my learning environment. If we are not careful, literacy could take over our entire schedule. This unit will give students more meaningful instruction than simple test taking strategies and redundant math practice. This hands-on fraction unit offers more than just simplistic understanding of fraction comparison. Within this unit the exploration of mentor texts, fraction bars, number lines, poetry, and technology helps the comprehension of fraction comparison. Time for this unit is open and can last longer if needed as we have a core math time built into our schedule.

Rationale

Providing students with choice and autonomy is a big part of my teaching strategy in my classroom. When students have choice and input into the learning that takes place in the classroom, they are more open to the activity because they feel as if the task belongs to them. "Autonomy support can be manifested in the classroom in at least 3 distinct ways: organizational autonomy support (e.g., allowing students some decision-making role in terms of classroom management issues), procedural autonomy support (e.g., offering students choices about the use of different media to present ideas), and cognitive autonomy support (e.g., affording opportunities for students to evaluate work from a self-referent standard)" (Stefanou, et al. 2010). Throughout this unit, the students will have plenty of autonomy when it comes to decision-making, choice in how they present their ideas, and self-evaluation. Allowing students to choose how they present their fraction comparison will provide them with autonomy. In order to get to the end product of choice, students will have to master the skills needed to compare fractions.

Students master skills easily when they are able to find meaning in learning a concept. Comparing fractions is a skill that can be directed towards food. Everyone loves food! Throughout this project they will be allowed choice. Allowing my students to have choice will open up a meaningful learning experience for them. Academic conversations in classrooms enable students to work collaboratively. 21st century learning is an important way to make sure students are college and career ready. 21st century learning in this unit allows students to work in learning teams and express themselves in a variety of ways. as according to 21st century learning. Autonomy and academic conversations allow

students ways to communicate with others openly and effectively through the use of language frames and teach modeling.

My underlying reason for writing this unit is to show students that we can think creatively in math by learning from our mistakes. Through fraction comparisons, we are able to expose students to a formal assessment that is not paper and pencil. This unit will be more than just learning tricks and routine. Students will think critically, work in learning teams, talk about math, and be ok with making mistakes. The meaningful exposure to comparing fractions will enable the students to dive deep into the study of fractions. Throughout the mentor math texts, students will make a connection to literacy and math.

Through collaborative learning and teacher modeling students will learn how to compare fractions in a variety of ways. By providing a safe environment for students, they will be ok with exploring fractions through frustration and mistakes without feeling defeated.

Unit Goals

The unit encompasses lessons will be designed using multiple teaching strategies and supports all learners through the process of learning about fractions. The difficulty children have with fractions should not be surprising considering the complexity of the concepts involved. When comparing fractions without the same denominator, children need to coordinate the inverse relationship between the size of the denominator and the size of the fraction. Ultimately the goal is for children realize that if an object is divided into four equal parts, each piece will be smaller than when an object of the same size is divided into two equal parts. An overarching goal with this unit is for children to gain the understanding that the more fractional parts, the smaller the size of each piece.

Part of this unit is for children to gain understanding of how to fractional parts make a whole. Also I want scholars to understand how to take different fractional parts in order to compare them to determine order and ultimately a deeper understanding of parts of a whole. At the conclusion of this unit, students will understand fractions referring to the same-size whole can be compared through reasoning about the size and number of pieces. Part of the goal is for them to become more comfortable using different tools such as number lines, diagrams, area models or even hands on food to reason about the size of fractions. In prior lessons, students have come to understand unit fractions and comparing whole numbers. This unit will extend those ideas to compare fractions with like numerators or like denominators.

Clearly, the way current way of teaching fractions should be improved. Due to the intense complexity required of the North Carolina adopted Common Core Standards, more time is not a simple enough solution to meeting the needs of the children. However, the emphasis of instruction should also shift from the development of algorithms for

performing operations on fractions to the development of a quantitative understanding of fractions.

Teaching Strategies

Understanding Basic Fractions

Students learning fractions can be divided into two basic components. One of the parts is considered as the knowing exactly what fractions are. The other part being the manipulation and application of fractions and their components. Knowing fractions different variations of what fractions actually mean can help boost student understanding of fractions in the overall. Knowing and having a clear understanding of what a fraction in the number world is will help the foundation of using fractions to complete more complex tasks with them. Fractions are numbers that explain the relationship with two quantities. Understanding where in the real world and making them relevant will be beneficial as well with in the beginning of the unit. Students have more buy in when they know why something is important or they understand how this can help them later in life.

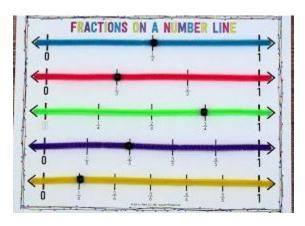
When beginning the unit, you should expect to first build students basic understanding of fractions. This is where you are able to implement read aloud to build understanding and basic identifying landmark fractions such as ½, ¼, and 1 whole. Knowing these landmarks will come in handy when beginning the process of comparing and ordering fractions. Furthermore, a key concept they should grasp is knowing that the larger the denominator, the smaller the actual piece and the smaller the denominator, the larger the piece. Having the concrete models and manipulatives mentioned below will provide a strong a support when engaging students in this concept. Strong foundation is key. Using the strong foundation will help provide a greater support when continuing into complex fraction structures.

Concrete Models

Learning models and manipulatives are important instructional strategies to implement when learning something as complex as fractions. Often times to support all learners scaffolding instruction and beginning at the basic level of understanding to build a foundation will be more beneficial in the long run. Manipulatives can be key in providing effective, active, engaging lessons in the teaching of mathematics. Concrete model examples using manipulatives to compare fractions will not only support English Language Learners and exceptional children, they will also provide me with an opportunity to present challenge questions to the students who catch on quickly to the concept. It allows an opportunity for students to prove their thinking and explore in their learning. Having a facilitated learning process throughout can also make the learning more meaningful and relevant to students. The use of manipulatives in the classroom help

boost the safety of exploration within the classroom. Often times students may be afraid to speak out, when using manipulatives and a concrete model to help prove their thinking will encourage students to have safe practice time and become more confident when we begin to move away from the concrete into a more complex understanding of comparing and ordering fractions. Below you will find a few examples of concrete modes that can be used in order to help students with the exploration of fractions.





Comparing Fractions

The focus of the unit will be comparing fractions. Being able to compare fractions comes with many misconceptions. Identifying those misconceptions and dispelling them is highly important when teaching children, the concept. Visualizing and actually drawing the fractional parts will play a part in understanding and comparing the size of the fractions. When beginning to build understanding of comparing fractional parts, beginning with comparing to a whole may be beneficial. Understanding what is more than a whole, less than a whole. Building off of this concept into what is more or less than a half can help students. This refers back to land mark fractions. The foundation of knowing what their landmark fractions are on a number line can help support them when learning to compare and order fractions.

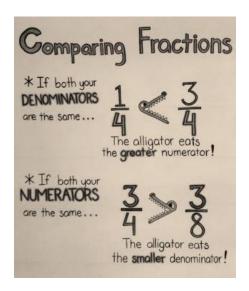
Anchor Charts

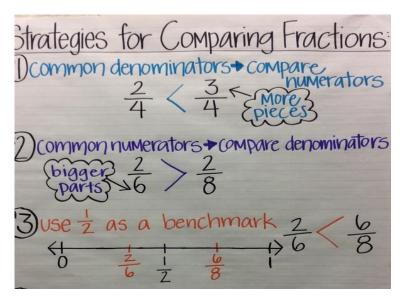
Anchor charts when used correctly are an extremely important tool for students to use within the classroom. This is not only an instructional tool, but it is also a tool for students to use a reference guide when they are working on practice assignments, and even assessments on given within the classroom. Not only are anchor charts useful as a resources, when done with specific intentions, they can serve as a strong support for exceptional children as well as English Language Learners within the classroom. In my current school, the achievement gap is focuses on these two of our lowest performing subgroups. Having anchor charts in order to support them gives them some extra support

they need. Meaningful anchor charts are key. "Anchor charts should be co-created with students and should contain only the most relevant content, strategies, cues, processes, or guidelines to focus students' learning" (Wisconsin Department of Public Instruction 2015). Requirements of effective anchor charts should be as follows:

- Anchor charts contain only the most relevant or important information.
- Charts should be neat and organized
- Charts should have picture examples
- Charts should use vocabulary in them that you are using in class.

Anchor charts within the classroom as an instructional tool will overall serve as a support for students who need a reference guide. Much like a dictionary is used when we do not know the meaning of a word. When students need to figure out the process for comparing fractions, referencing the anchor charts below will come in handy for them.





Interactive Notebook

Interactive Notebooks are very complex way for students to take notes and demonstrate understanding of concepts. However, they are usually used in upper grades when students become more facilitators of their own learning. Even though this concept of interactive notebooks is not fully implemented, parts of the notebook will be and can serve as exit tickets and a way for students to show their own creativity in their learning and understanding of concepts. These notebooks provide teachers with an organizational tool for their lessons, encouraging the use of different types of lessons, and can be helpful when planning for the learning styles of students. The interactive notebook can be an important tool to help students remember and review information needed for assessments. The right side of the notebook can be used for notes provided by the teacher; this is the side that students use to study for assessment. Teachers that provide their students with notes were shown to have better achievement than those whose students personally

recorded them. Interactive notebooks allow students to record information and process it to improve their level of understanding. As students learn new ideas, they can use several types of writing and graphic organizer to record them. Then students will do something with those ideas. In doing so students will use critical-thinking skills to organize and process information. As a result, students can become more creative, more independent thinkers, and they will develop a deeper understanding of the information being taught.

Rubric

Using rubrics as a means of assessment is extremely important. "Assessment—and especially student self-assessment—is most powerful when viewed as a learning activity" (Educational Research Services 2004). Rubrics allow for a focused guide for teachers and students to be able to complete assignments with accuracy and with high rigor. Knowing your expectations before you complete something is a natural way for you to be more successful with demonstrating your understanding. Using rubrics within class provides the opportunity to help support a focus on the educational process in its entirety. As students are working to shift their learning products to become more rigorous as they read and move up the scale on their rubric, the more in-depth their products and their learning should in turn become. "they are learning how to improve their own learning skills simultaneously with achieving specific standards" (Educational Research Services 2004). Within Appendix 4 you will be able to find an example for the lessons provided of specific rubric that you are able to use in order to have students grade themselves as well as for the teacher to use in order to provide instructional feedback and grade the students.

Instructional Implementation

Week 1: Understanding Fractions

Objective: As a mathematician, students will write and model fractions using a numerator and denominator.

Materials Needed: Chart paper, Math Notebook, Fraction anchor charts, dice, picture cards, mentor text that show or model fractions. Possible examples include: *Fraction Fun by* David A. Alder, The *Hershey's Milk Chocolate Fractions Book* by Jerry Pallotta, *Polar Bear Math* by Ann Whitehead Nagda, *A Fraction's Goal—Parts of a Whole* by Brian Cleary

Tasks: Choose a mentor text that can be utilized to introduce Fractions. Explain to the students that you are going to read the story and that they should "listen as a mathematician." Mathematicians listen to stories differently than others. They listen for vocabulary, look for concepts that may seem a little confusion, and ways to problem solve. After reading, point out that this particular piece is will help us to deepen our meaning of fractions and how to create them. You can have students take notes during discussion, or have students put an anchor chart within their notebook for modeling or writing fractions using a numerator and denominator and the meaning.

Guiding discussion Questions: What is the story about? What is a fraction? What does a fraction have to have? How can you draw a fraction?

After reading, discuss the questions mentioned above. Create a list of fractions and have students help you to model or represent the fraction in picture and standard fraction form. In learning teams, have students create and model fractions using picture cards or dice.

Week 2: Introduce comparing fractions

Objective: As a mathematician, students will compare fractions using fraction bars, number lines, greater than, less than, and equal to symbols.

Materials: Math Notebook, Smartboard, mentor texts from previous lessons

Tasks: Through previous lessons, students have learned how to write and model a fraction using a numerator and denominator. In this lesson, the students choose fractions from previous mentor texts and compare them using a variety of manipulatives, comparison symbols, number lines, and picture models. Teacher will model using the fraction one half and two fourths using pictures, fraction bars, a number line, and comparison symbols. Have students complete a few examples of comparing fractions. Once students have solved given fraction problems allow the students to create comparison problems of their own. Use time to also pull small groups of students that may need help and enrich those students who have grasped the concept. To foster safe practice with partner discussion, model the acts of the partner who is listening and learning and the person who is talking. Also be sure to model product expectations. Have students share their fractions that they created.

Week 3 & 4: Assessment

Objectives: As a mathematician, I can use my knowledge of comparing fractions to complete an acrostic poem or PowerPoint to demonstrate my knowledge of comparing fractions and present it to the class.

Materials: Chart Paper, Chromebooks, Smartboard, PowerPoint, Google Classroom, Rubric, Self-Assessment

Tasks: From the knowledge of comparing fractions that students have they will get to choose in presenting their information for a formal assessment. Students can choose to write an acrostic poem or create a PowerPoint on fraction comparison to present to the class. It is important to provide an example of both assessments and allow students to grade the projects with the rubric. Giving the students the rubric before beginning the project will ensure that they include all necessary components. The self-assessment will allow them to rate themselves and the work that they did. Once students have finished their project, they will present the information to the class.

Appendix 1

Implementing Teaching Standards

Math Common Core Standards

This unit will mainly focus on building an understanding of fractions. Students will view fraction in general and use visuals and manipulative to represent fractions as part of a whole. They will be able to understand the size of a fractional part in relation to its size. Students will be able to solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators. The culminating activity will be a presentation of an acrostic poem or PowerPoint where students can show what they've learned.

CCSS.Math.Content.3.NF.A.1

Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

CCSS.Math.Content.3.NF.A.2

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

CCSS.Math.Content.3.NF.A.2.a

Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.

CCSS.Math.Content.3.NF.A.2.b

Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

CCSS.Math.Content.3.NF.A.3

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

CCSS.Math.Content.3.NF.A.3.a

Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

CCSS.Math.Content.3.NF.A.3.b

Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

CCSS.Math.Content.3.NF.A.3.d

Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Name______ Date_____

Determine which letter best represents each fraction.

1) Which choice best shows $\frac{1}{3}$?



B. (



D.

2) Which choice best shows $\frac{1}{4}$?









3) Which choice best shows $\frac{4}{6}$?









4) Which choice best shows $\frac{1}{8}$?









5) Which choice best shows $\frac{2}{3}$?





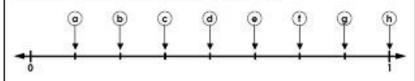




Appendix 3

Fractions Number Line

Write the correct letter on the blank line next to each fraction.



Compare the fractions using <, >, and =.

$$\frac{1}{2}$$
 $\bigcirc \frac{3}{4}$

$$\frac{7}{8}$$

$$\frac{1}{4}$$
 $\bigcirc \frac{7}{8}$

Mrs. Browning asked her class to help with safety patrol. $\frac{4}{8}$ of the class went with her to help younger students onto the bases. Mr. Tobias took $\frac{1}{2}$ of the class to help students at the crosswalk. Compare the fractions of the class that went with each teacher using 4, 2, or 4.

Mrs. Browning $\frac{4}{8}$ \bigcirc $\frac{1}{2}$ Mr. Tobias

Super Teacher Worksheets - www.superfeacherworksheets.com

Appendix 4

Fraction Formal Assessment Project

Fraction Formai Assessment Project				
Criteria	4-90%-100%	3-89%-80%	2-79%-70%	1- 69% and below
Fraction Project	Project focuses on	Project is not	Project focuses on	Mathematician
110000011100000	fractions and	focused on	fractions, but not	does not seem to
	ways to compare	multiple ways to	in PowerPoint or	be aware of project
	fractions using	compare fractions	acrostic poem	expectations.
	_	*	-	expectations.
	acrostic poem or	using acrostic	format.	
	PowerPoint.	poem or		
		PowerPoint.		
Published work	Project is neatly	Project is neatly	Project may not	Project is not
	done and	done and	be appropriate	published for
	published in an	published in an	and neat for	others to see or
	appropriate and	appropriate	display or sharing	read.
	attractive format.	format.	with others.	
	Could be used as	10111111	William outlots.	
	a model for			
	others.			
	oulcis.			
Conventions	Project is free	Project contains	Numerous small	Many errors in
	from errors.	minimal mistakes	errors and	spelling,
		that do not	interfere with	capitalization, and
		interfere with	being able to	punctuation often
		meaning or	understand	interfere with
		understanding of	fraction	project and make it
		comparing	comparison.	difficult to read.
		fractions.	Makes it difficult	difficult to found.
		machons.	to read.	
			to read.	
	l	I	1	1

List of Materials for Classroom Use

The mentioned materials list below is also within the unit instructional lessons. You may choose to use these instructional materials in order to prepare and complete lessons in their entirety. All materials are subject to change based upon lessons taught and availability.

- Mentor text to pull from (listed within above document)
- Math Journal
- Smartboard
- Chart paper
- Markers
- Student technology (iPad or Chromebook)
- Access to PowerPoint or Google Slides
- Fraction Activity Pages
- Rubric for self- assessments
- Anchor Charts
- Prepared project examples

Resources for Students

How to Create A Powerpoint.2012.

This video resource is a tutorial that explains step by step how to create a Powerpoint.

Acrostic Poem. Directed by ReadWriteThink. 2016.

This interactive piece provides students the opportunity to learn about acrostic poems through an interactive model. This tool will be good for students to review and create a practice acrostic poem before doing their own on Fractions.

Resources for teachers

Stefanou, Candice. "Supporting Autonomy in the Classroom." *Educational Psychologist* 39, no. 2 (June 2010)

Through this article the author provides vivid insight into how important providing students the autonomy in their work to add to and make learning a meaningful part of their education. The author speaks about how important student buy-in is. If students believe a project or task is important they will complete it.

Zucker, Josh "Notes on Fractions" CTI May, 2016 Fractions a workshop for teachers at American Institute of Mathematics, (2009)

These notes provide the following problem: A bicycle team of 7 people brings 6 water bottles, while another team of 13 people brings 11 water bottles. What happens when they share? This would be a good challenge activity for students that finish early. It is called the water sharing method.

Zwiers, Jeff. "Academic Conversations." Classroom Talk that Fosters Critical Thinking and Content Understandings (2012): 1-30.

From a teacher's perspective the author gives an account of how important academic conversations are in the classroom, but by 4th grade they are non-existent. The author describes how important conversation are in class, and how academic conversation help students learn from one another.

Bibliography

Acrostic Poem. Directed by ReadWriteThink. 2016.

How to Create a Powerpoint.. 2012.

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