



### ***Fractions, Fractions Everywhere***

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Quail Hollow Middle School

This curriculum unit is recommended for:  
Middle grade levels (6th – 8th)

**Keywords:** fractions, ratios, proportional reasoning, decimals, percents, division of fractions, markdown, markup, unit rate, geocaching

**Teaching Standards:** See [Appendix 1](#) for teaching standards addressed in this unit.

**Synopsis:** Realizing that larger number of my students struggled with the various ways to interpret fractions or unit rates of items, I decided to create a unit focused on fractions with specific attention to ratios, proportional reasoning and unit rates. Students will interpret fractions as decimals and percents relating the basic operations to these values in their daily lives. I want to challenge my students to explore real world word problems in their everyday lives and express them in fractional formats. Creating a unit to include interests of students will involve their input in my unit. If they can relate to fractions, then I'm hoping they will be less intimidated by them.

*I plan to teach this unit during the coming year in to 107 students in 7th grade math and Math I courses.*

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## **Fractions, Fractions Everywhere**

*Connie George*

### **School and Classroom Background**

Quail Hollow Middle School (QHMS) comprised of sixth through eighth grade is located in Charlotte, North Carolina. Quail Hollow Middle School's mission is to build a student body that will persevere, achieve, and be thoughtful and healthy in order to accomplish diverse goals by developing a growth mindset. In order to develop this growth mindset, we focus on creating a desire to learn within the classroom environment. QHMS contains a diverse population of students with wide range of socio-economic and academic backgrounds. According to 2016-2017 school improvement plan for QHMS, our population identified 26% as White, 33% African American, 6% included in the American Indian, Asian and more than one race ethnic groups including 33% identified as Hispanic race. The student population includes 8% more female seventh graders than the male population which closely aligns within my classroom of 107 students. This year QHMS will receive Title I funding allowing additional resources which will reinforce school efforts to advance student learning.

The school improvement plan will continue to require more rigor, engagement and differentiation. Although we made improvement in our proficiency level on end of grade tests, closing the gaps among our ethnic, racial and student with disability categories still need improvement. Last year, I designed a number theory unit advancing students in their mathematical processing of conceptual and abstract learning. For all math classes of seventh grade, an interactive notebook will be utilized and include this rational number unit and number theory unit from last year. Having an interactive notebook keeps students organized and provides a personal resource in their future. By supporting students with organizational skills, I feel the joy of learning math will grow. I want this unit to inspire students to feel more confident and learn to love math.

I begin my fifth year of teaching mathematics at QHMS, three years in the sixth grade and second year in seventh grade. As a student learner of math or mathematician, I continually reflect to improve my teaching strategies. By striving to improve my teaching approaches, I expect to improve student learning in the process. Student learning ranges from basic fraction understanding to higher applied math levels, a collection of background knowledge will need to be utilized. By developing their fundamental problem solving skills, my intention is to motivate or trigger a love for learning math with more emphasis on rational numbers.

My seventh grade math curriculum is based on the Common Core State Standards and by our Professional Learning Community (PLC) to determine the pacing of our

objectives. Engaging lessons require constant reflection to allow for more interactive participation where students should retain more as it applies to their backgrounds. Objectives from the seventh grade math Common Core State Standards will be addressed within the unit with emphasis on rational numbers within the ratio and proportions, number sense and expressions and equations curriculum objectives using real world mathematical problems.

## **Rationale**

Watching students struggle this year on their math end of grade exam, I was inspired to explore another approach in teaching rational number applications. The middle school years provide an opportunity for students to experience a deeper understanding of basic mathematical concepts. Students should acquire higher level thinking by developing their problem-solving skills. These critical thinking skills are the building blocks essential for their mathematical growth as they transition to high school and beyond. Piloting my unit within the 7th grade curriculum and Math I courses will enable students to grasp personal applications of fractions within their world. Increasing self-confidence in their mathematical skills should lead to well-rounded conceptual and applied novice mathematicians. According to the Oxford dictionary, a mathematician is “an expert in or student of mathematics” which my students will identify themselves as a mathematician. Students will grasp that a mathematician is a student learner of math, which applies to all who seek more understanding in the mathematical field including their math teacher.

While students envision themselves as mathematicians, they will apply concepts of fractions from their elementary years towards the introduction of unit rates in middle school. Unit rates as an objective title may be unfamiliar to students, but the concept itself is regularly used by them. Moving into middle school, students will use fractions, decimals and percents as a part of their everyday language within a math classroom. Students will stretch that concept development with richer applications which apply to their lives. Using inquiry based activities; we will explore the interconnectedness of fractions to unit rates and related concepts of fractions, decimals and percents.

By allowing students to struggle with some concepts, they will discover some key ideas or misconceptions on their own. One of the ideas behind the Fundamentals in Mathematics seminar is to “make sense of problems and persevere in solving them.”<sup>1</sup> In order for students to gain the confidence in persevering through problems, trust of setting them up for success will be a key in my teaching strategies. Students’ willingness to trust me as their teacher will be my initial goal and by tapping into their background knowledge of multiplication and division facts confidence will be established. Having students confidence with prior knowledge concepts will lend itself into exploring unit rates and rational numbers.

Diversity among student learning has required scaffolding of lessons. According to the authors of “Common Core for the Not-So-Common Learner”, “students abilities to manage their own learning through organization techniques”<sup>2</sup> such as using an interactive notebook is a best practice technique. My dedication of implementing a more effective interactive notebook in this research strategy will improve the opportunity of reaching my diverse population. Students will use their interactive notebook containing mathematic tools, formulas, strategy approaches, scaffold guided notes and practice exercises to develop their math study skills. Creating lessons with scaffolding approaches using the mathematical practice standards enhance instructional approaches and student processing of learning.

Assisting students with effective mathematical standards of practice strategies is essential to completing the inter-connectedness of prior concepts as they relate to new concepts. I love when students ask me, “When am I going to ever use this stuff?” By sparking their curiosity with new information and relating it to previous concepts, I intend to let students know where this information may be useful. Eventually, all students will purchase groceries, clothing, and electronics, therefore, connect simple examples to their everyday lives begin the conversation of connections. Seventh graders are seeking to apply their knowledge immediately versus years from now; however offering future profession ideas of intertwining sports related activities, science-driven professions, finance occupations, and my prior experiences lead students to engage in learning a new concept. A previous student wants to be a veterinarian, relating measurement conversions of medicine, gathering statistical data on specific animals, designing an office space for seeing her patients were some of the connections made to her math learning. I understand that I don’t have to answer all of their questions, but sparking their interest could lead to their own seeking of answers.

Inspiring students to be open to learning old and new math concepts is always a goal for me. By continuing to pursue my professional development growth and sharing these experience with my students represents my willingness to continue learning and remain open to new concepts or approaches as well. My unit is intended to provide a safe, developmental and intriguing approach to exploring the uses of fractions.

### **Content Objectives and Strategies**

Fractions, Fractions Everywhere curriculum unit will integrate the mathematical standards of practice and tools gained from the seminar, Fundamental Ideas in Math Grades PreK-12. Modeling mathematical concepts are essential for most concepts specifically ratios, proportional reasoning, fraction, decimal, and percent. According to Cathy Checkley, author of *The Essentials of Mathematics Grades 7-12*, “support beyond instruction”, “high expectations” and promote “positive learning and social interaction”<sup>3</sup> are essentials in effective curriculum and instruction. When a positive learning environment exists students are open to thinking and testing with “integrated options”

(p.29) which allows sharing of ideas within groups. Framing concepts with a “positive mood induction experiment” (p.84) can promote openness to learning. She suggests presenting this experiment with a “think about the happiest day of your life” to alleviate any anxiety caused from prior experiences. I believe that creating that positive feeling will encourage a growth mindset.

Creating an environment of positive student thinking “through mathematics instead of about mathematics”<sup>4</sup> will empower students’ lifetime skills according to Ed Burger, president of Southwestern University. He says the inquiry-based learning “will instill habits of living, such as perseverance, curiosity, and intellectual passion.”(p.25) I believe that the growth mindset fits well within this framework of inquiry-based learning. Designing interesting discussions and activities will encourage and promote this approach to learning.

Reflecting daily upon my teaching approaches, I began to notice the struggles of a large number of students connecting fractions to ratios, proportions, unit rates, decimals and percents. Fraction foundations are introduced in the elementary years and will need to be continually developed. Fraction concepts expand to include equivalent fractions which are tied into proportions and unit rates during the middle school years. Students limited connection of multiplication and division of fractions will require developing. Exploring the multiplication chart and connecting that concept with division and equivalent fractions will lend itself to the discovery of proportions.

By revisiting multiplication and division concepts, students will be encouraged to explore new uses of their multiplication chart as it pertains to other seventh grade math concepts. Connecting multiplication chart uses will allow struggling students the time to process prior concepts in addition to learning the new ones. By understanding fractions, students will connect all the “concepts that fractions can represent” denotes the authors of *Elementary and Middle School Mathematics: Teaching Developmentally*. Connecting multiplication and division facts to equivalent fractions using the multiplication chart, students will apply these facts to rates and unit rate concepts. As this concept is still confusing and needing more developing time in the middle school years, we will revisit those concepts within problem solving of word problems involving sports, music and food. As these concepts can be related to their lives, I’m hoping to create that long-term connection.

Coupled with all operation applications of fractions, the concept of decimals and percents are reintroduced as alternative measurement options. Adding the visual placement of fractions on a number line will connect the intertwining of fractions, decimals and percents as rational numbers<sup>5</sup>. A visual display of this number line will be utilized yearlong with the multiple representations of these rational numbers. Scaffolding and continual building on these concepts will be required to maintain that sense of perseverance and confidence building. Displaying visual items for students such as a

multiplication chart will provide an easy transition to proportional reasoning. Proportional equations, graphs, charts will provide the link to the multiplication chart containing equivalent fractions.

Linking proportional reasoning, students will distinguish between pictures or situations of comparisons to apply their middle grade concepts of ratios. Comparing glasses that contain a juice as to its flavoring strength will allow students to compare ratios implication in the flavoring process. Discussing the impact of changing proportions, ratios or fractions of ingredients in recipes will provide an at home situation which students can immediately apply. Explaining to students how fractions are applicable to them now as well as examples of their future uses makes learning a necessity instead of a theory.

Developing understanding of fractions and applying them to percents within markup (tax and gratuity) and markdown (discounts, percent off, coupon) problems will be our next step in linking fractions to ratios and unit rates. Markup and markdown problems are an essential consumer math concept. The concept of marking a product's cost up as a retailer, paying taxes on products and providing a tip for services rendered are some examples of markup situations which will be used in sample problems. In addition to markup problems, students will learn about markdown situations. These markdown concepts include receiving percentage discounts through product sales. Utilizing interesting product choices for students will allow for frequent possibilities of student friendly applicable real-world situations to draw students into the learning frame of mind. By choosing products to hypothetically purchase or compare for better buys will entice students to learn and develop this concept throughout the year.

As I proceed through this unit during the school year, I intend to expand on its development of joining fractions to their middle school year applications. My culminating activity will include an interesting idea inspired by geo-caching and another website, Mathbits.com.<sup>6</sup> Students will complete a website scavenger hunt of seventh grade ratio and proportion objectives. Students will be given four to six fraction related problems scaffolding concepts as they proceed through solving this scavenger hunt. Students will complete a series of questions and turn their answers into encrypted website locations. These locations will lead to the next set of problems to solve therefore leading them to their completion of this unit of concepts. By using this strategy of acquiring the answers, students will immediately know if they have solved their problems correctly with instant feedback leading to their next set of tasks.

## Activities for Incorporating Fractions

My unit will use warm-up style lessons or sprinkling concepts into lessons using a PEAK strategy. By providing short lessons, it allows students to absorb this concept without being overwhelming. My overall objective of this fraction unit is supplying students a connection between elementary and middle school fractional concepts. I believe students respond more positively if they can take a smaller concept and build upon it. I want students to think and apply their knowledge of fractions. By creating that sense of inquiry and checking for accuracy with fellow students, they will gain confidence in their basic skills.

Warm-up 1 (WU1): Begin the warm-up units with a partial multiplication chart being displayed or possibly displayed on a smartboard for an initial explanation (Figure 1a). Connecting addition and multiplication with the multiplication chart allows students who are struggling with their multiplication facts to be more successful in locating their answer by producing it another way. Using the partial chart, ask students: What numbers can you add together to arrive at the sum of 4? (Figure 1b) You can get a few answers such as  $2+2$  or  $1+3$  and others.

Figure 1a.

Multiplication Chart	1	2	3	4	5	6
1	1	2	3		5	
2	2	4	6		10	
3	3	6	9		15	
4	4	8	12		20	
5	5	10	15		25	
6	6	12	18		30	
7	7	14	21		35	
8	8	16	24		40	
9	9	18	27		45	
10	10	20	30		50	
11	11	22	33		55	
12	12	24	36		60	

Figure 1b.

Multiplication Chart	1	2	3	4	5	6
1	1	2	3	1+3	5	
2	2	4	6	2+6	10	
3	3	6	9	3+9	15	
4	4	8	12	4+12	20	
5	5	10	15	5+15	25	
6	6	12	18	6+18	30	
7	7	14	21	7+21	35	
8	8	16	24	8+24	40	
9	9	18	27	9+27	45	
10	10	20	30	10+30	50	
11	11	22	33	11+33	55	
12	12	24	36	12+36	60	

Ask students to use this strategy of thinking to complete the fours column by highlighting any combination of their sums. Calculate the sum  $1 + 3 = 4$  and use the two columns of data to complete the fours column. Students should be able to complete the six column of data using the same reasoning with slight prompting of combinations where the sum is equal to six can be applied.

WU2: The following day should be used to complete a seven through twelve multiplication chart. After having students check each other's chart, follow it up with a check for further understanding by applying those skills to an extended column.

Use an example of another chart where students will supply the data for each column using their connected understanding and previous completed charts (Figure 2a & 2b).

Figure 2a

Multiplication Chart	17	26	43
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

Figure 2b

Multiplication Chart	17	26	43	
1	5+12	17+9	17+26	
2	10+24	34+18	34+52	
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				

My reason for beginning with the multiplication chart is to make connections of students' elementary understanding of operations and applying them to equivalent fraction concepts. Another connection for students would be to revisit the concept of place value. Using the same example, relate the area model to place value connection the multiplication and addition concepts. The area model gives a visual representation of place value using the product of two sums<sup>7</sup> such as  $a(b+c+d) = ab + ac + ad$ . The area model below is applied to the previous example of developing the 17 multiplication values:

Area Model	5	10	2	5+10+2= 17
2	10	20	4	10+20+4=34
3	15	30	6	15+30+6=51
4	20	40	8	20+40+8=68
5	25	50	10	25+50+10=85

Figure 2c

WU3: Give students a quick warm-up to complete a column of multiplication information using their previously completed charts. For example: Complete the multiplication facts for the column of 23. This time ask students to apply their chart skills by considering completing this column using subtraction. There are multiple ways to create a subtraction or addition and subtraction problem to arrive at the number 23 such as  $26 - 3 = 23$  or  $12+12 - 1 = 23$ . Ask students to find a quicker approach by only using subtraction to arrive at the data for 23. The area model can also be applied to this approach of subtraction. Twenty-three can be written as  $26 - 3$  or  $20 + 6 - 3$  therefore allowing the area model to be applied and incorporated with their understanding of place value using multiplication facts.



Area Model	20	6	-3	$20+6-3=23$
2	40	12	-6	$40+12-6=46$
3	60	18	-9	$60+18-9=69$
4	80	24	-12	$80+24-12=92$
5	100	30	-15	$100+30-15=115$

Figure 3a

Then, ask students to create a subtraction only column of information for the number 14. For this example, students could select to use their new information for 23 and create  $23 - 9 = 14$  or  $26 - 12 = 14$ . Encourage students to find other options where they can apply addition and subtraction to the multiplication chart or area model.

WU4: Introduce fractions into the conversation by means of a multiplication chart. Ask students to consider looking for fractions within the chart (Figure 4a). If they need a hint, consider covering up some of the chart as in the example below. Now try expanding it to separated rows and cover up other sections to show that  $12/21$  is equivalent to  $4/7$  (Figure 4b). Can they provide other equivalent fractions to the  $12/21$  example?

Multiplication Chart	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36
7	7	14	21	28	35	42
8	8	16	24	32	40	48
9	9	18	27	36	45	54
10	10	20	30	40	50	60
11	11	22	33	44	55	66
12	12	24	36	48	60	72

Figure 4a

Multiplication Chart	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36
7	7	14	21	28	35	42
8	8	16	24	32	40	48
9	9	18	27	36	45	54
10	10	20	30	40	50	60
11	11	22	33	44	55	66
12	12	24	36	48	60	72

Figure 4b

WU5: Review a few equivalent fraction problems such as  $18/20$  or  $9/36$ . Now try a few fractions where the denominator is smaller than the numerator to see if they can connect the equivalency in reverse order such as  $21/12$  or  $60/25$ . You may want to provide a piece of construction paper, cut to the size of a row or even cut to a few rows until they can locate the fractions on their own. Now wrap up the warm up with a partner talk of having students explain how the multiplication chart could be used to add  $3/4$  and  $5/12$ ? Students should be able to connect the denominator of  $3/4$  to the equivalent fraction of  $9/12$  and add the fraction numerators, therefore arriving at  $9/12 + 5/12 = 14/12$ . Show

students that their answer of  $14/12$  reduces to  $7/6$  (Figure 5a) by locating them on the chart in the same column and moving to the left.

Multiplication Chart	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Figure 5a

WU6: Always begin with a review of their prior knowledge and build upon it. Ask students to provide three equivalent fractions for  $40/16$  where one fraction has a numerator greater than 40 (Figure 6a). Students could provide multiple answers which include  $50/20$ ,  $30/12$ ,  $10/4$  and hopefully a student will realize that  $10/4$  is equivalent to  $5/2$ .

A	B	C	D	E	F	G
Multiplication Chart	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36
7	7	14	21	28	35	42
8	8	16	24	32	40	48
9	9	18	27	36	45	54
10	10	20	30	40	50	60
11	11	22	33	44	55	66
12	12	24	36	48	60	72

Figure 6a

Now return to an addition of fractions problem such as  $5/2 + 3/5$ . Encourage students to locate equivalent fractions where their denominators are the same and add the numerators.  $5/2 + 3/5 = 25/10 + 6/10 = 31/10$  Demonstrating how to use the multiplication chart in numerous ways should build mathematical confidence and trust in their teacher.

WU7: Proportions are “the equality between two ratios in which the first of the four terms divided by the second equals the third divided by the fourth<sup>8</sup>” or equivalent fractions. Students in the 7th grade learn to analyze proportional relationships as fractions and relate them to the slope of a line. Students are introduced to unit rate early in the school year, so this concept can be easily applied within this unit. I like to use examples which interest my students to capture their attention and food is always a good way. Pose the following examples: Cici’s Pizza offers a lunch buffet for adults (including young people over the age of 10) for \$5.69 per person. You and three friends went to eat there

and your total bill was \$22.76. Later, you decide to have your birthday party there for a total of 12 people, how much will your meal cost? Understanding that the relationship is proportional makes the meal calculation easier. Although students can't locate \$5.69 in the multiplication chart, there are other relationships that are equivalent by using proportions. Encourage students to apply unit rates to this example and create three equivalent proportions of  $\frac{\$5.69}{1} = \frac{\$22.76}{4} = \frac{\$68.28}{12}$ . A proportional equation in the format of  $y = kx$  or  $y = mx$  where  $k$  or  $m$  is identified as the slope of the relationship. Ask students how they calculated the value of the meals for 12 people. Could they explain how to arrive at the cost for 9 people. Have students try to develop the equation using their knowledge of a unit rate to an equation of  $y = kx$ , where  $k$  is the constant of proportionality (the consistent amount of change) or slope. Ask students what would the cost of a meal be for no one? How does this information relate to their equation? One goal of this question is to get students to recognize that the proportional relationship must include the point (0,0) for their equation of  $y = 5.69x$ .

x= Number of People	y = Cost of meal
4	\$22.76
12	?



Figure 7a

WU8: Now reflect back to the area model with a brief warm-up of calculating the following: You have just won the lottery and want to share part of your money with your family. You realize that giving each family member \$13,999 as a gift will involve some calculating. You have 2 brothers, 1 sister, both parents, 5 cousins and 1 aunt and 2 uncles, which equals 13 relatives to give money.

Area Model	10,000	3,000	900	90	9	Sum
10	100,000	30,000	9,000	900	90	139,990
3	30,000	9,000	2,700	270	27	41,997
Sum	130,000	39,000	11,700	1,170	117	181,987



Figure 8a

Students will arrive at the value of \$181,987 as long as they add all of the values together. Ask students what they would do with this extra money and save those ideas for warmup examples for another day.

WU9: Present a number line from zero to one. I placed a number line on the top ledge of a white board for year round display. I labeled a few items on the number line and left it for a few days without any questions being asked. Then warmup day came and students were asked to present other fractions, decimals or percents for placement on the number line. Students are able to quickly provide equivalent fractions for the numbers already placed on the number line. Now give them a few familiar fractions such as  $\frac{4}{5}$ ,

$\frac{3}{8}$ ,  $\frac{1}{8}$ ,  $\frac{2}{3}$ . Then give them a few they may not immediately know such as  $\frac{4}{11}$ ,  $\frac{19}{20}$ ,  $\frac{3}{7}$ ,  $\frac{5}{12}$ .

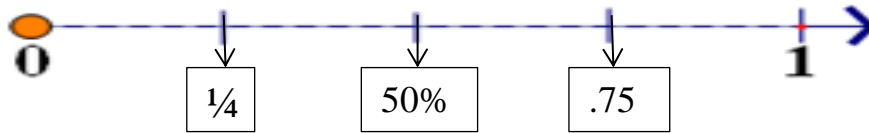


Figure 9a

Could students provide a strategy as to how they arrived at the location of  $\frac{4}{11}$ ? A possible strategy for students struggling with  $\frac{4}{11}$  and  $\frac{9}{20}$  could include:

- Convert  $\frac{4}{11}$  to a decimal .36 repeating
- Compare  $\frac{4}{11}$  to  $\frac{4}{10} = \frac{2}{5}$  and  $\frac{4}{12} = \frac{1}{3}$ , therefore  $\frac{4}{12} < \frac{4}{11} < \frac{4}{10}$  could be examined
  - Ask students what they notice about the denominators of these fractions and how does that impact the overall number. Understanding that as denominators increase with a constant numerator the fractions decrease is an important location tool.
  - Try another example with  $\frac{14}{15}$ . Students should quickly find  $\frac{14}{16} = \frac{7}{8}$  and  $\frac{14}{14} = 1$  as possible solutions, therefore locating  $\frac{14}{15}$  between these fractions.
- Convert  $\frac{9}{20}$  to a decimal .45
- Compare  $\frac{9}{20}$  to  $\frac{8}{20} = \frac{2}{5}$  or .4 and  $\frac{10}{20} = \frac{1}{2}$  or .5
- Using fractions only this time, ask students to compare  $\frac{9}{20}$  to other fractions with the same denominator
  - Examples could include  $\frac{8}{20} = \frac{4}{5}$  and  $\frac{10}{20} = \frac{1}{2}$
  - Where would  $\frac{9}{20}$  be located compared to the other fractions?

Have students place these fractions on the number line.

WU10: Try something different such as a magic trick to intrigue your students with the fraction theme. After recently re-reading Dr. Harold Reiter's *Fractions* publication, he referenced Art Benjamin, a professor from Harvey Mudd, who combines mathematical ideas with magic. I found a fraction problem and decided to try some "math magic" on my students.

- Students are asked to select a number 1 through 70. Divide that number by 7.
- If there answer is a whole number, they should divide by 7 again.
- To add to the mystery, ask students if there is a 1 in their answer?
- Say "I believe your answer has the number 4 following the 1 digit."
- Allow students to acknowledge this part.
- Tell students to add the first 6 digits following the decimal point of their answer.
- Announce the answer...27 is the sum!

Try to discover the secret for yourself, if unsuccessful locate books by Dr. Arthur Benjamin.

WU11: In 7th grade students encounter division of fractions again. To grasp students interest in exploring the division of fractions, try using dominoes for this activity. Students should work in partners. Student A will select 4 dominoes from a bag of dominoes. Student B will place the dominoes on the worksheet. WS1 in resource section of appendix 2. Practicing simple division of fractions can work for one day activity. An additional activity using division of fractions can include a day working with 1 dice and 1 domino. WS2 in the resource section. Modifications to this worksheet could allow for mixed numbers and repositioning of dice and dominoes for further exploration by students.

WU12: Covering this next topic around the Black Friday holiday time allows full student buy into your covering of this topic. Tell the students that they have \$50 to spend on gifts for 4 family members. They can buy a family gift or individual or multiple gifts, but gifts cards can't be purchased. Provide students with an example to get them thinking. I want to purchase the following items, but need some discounts (markdown) so my \$50 can cover the cost of the item including tax (markup).



\$14.99 basketball for my brother

WM Discount 25%: \$11.24



\$12.00 slippers for my mom

Belks Discount 33%: \$ 8.04



\$10 dice game for my sister

Target Sales Price: \$5.99



\$20 wallet for my father

Kohl's Price: \$20

Basketball Total price: \$11.24 with 8.75% tax = \$12.22

Slippers Total price: \$8.04 with 8.5% tax = \$8.72

Dice Game Total price: \$5.99 with 8.75% tax = \$ 6.51

Wallet Total price: \$20 with 8.75% tax = \$ 21.75

Total spent:  $12.22 + 8.72 + 6.51 + 21.75 = \$49.20$  with \$ .80 left over!

Name: _____ Use this worksheet to show how you spent your \$50 budget for 4 family members. NO Gift Cards allowed. • Attach copies of sales flyers or draw pictures of items purchased • Contain minimum of 2 markdowns (discount, sale price) Highlight markdowns in yellow or green • Contain minimum of 1 markup (tax) Highlight markups in pink or blue		
Item purchased (store name included)	Original Price, discounts, coupons, sales price, tax calculations	Total Cost of each item
1. _____		
2. _____		
3. _____		
4. _____		
		Total Amount Spent: _____

Provide students with a worksheet to help show their calculation, example in teacher resource section. Review with students how to calculate markdowns and markups, possibly writing the information on the back of their worksheet.

Markdowns: (Original Price)  $\{ (100\% - \text{discount } \%) / 100 \} = \text{Sales Price}$

Mark ups: (Sales Price)  $\{ (100\% + \text{sales tax } \%) / 100 \} = \text{Total Cost for Item}$

Upon completion of this task, have students swap with a classmate to check their math.

WU13: Culminating Activity on <http://fractionsfractionseverywhere.weebly.com/>  
 Students will complete a series of questions relating to the warm up items and concepts taught. Upon answering the questions in each webpage, students will use their answers to calculate or interpret the location of the next webpage.



## FRACTIONS FRACTIONS EVERYWHERE

Using the multiplication chart below.

1. Find the missing value of  $21 \times 9 = 7 \times ?$ .
2. Convert  $18/81$  to simplest form.
3. Convert  $55/30$  to simplest form.



Multiplication Chart	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

4. USING THE AREA MODEL BELOW, WHAT ARE THE MISSING VALUES?

Area Model	70	4
30	2100	120
?	560	?

MULTIPLY YOUR ANSWERS FROM QUESTIONS #1, #2 AND #3. THEN ADD THAT RESULT TO THE ANSWERS IN QUESTION #4. THEREFORE  $(\#1)(\#2)(\#3) + (\#4? + \#4?) = \text{-----}$ , TAKE THIS ANSWER AN INPUT IT IN THE ?QUESTION MARKS? OF THE URL BELOW:

[HTTP://FRACTIONSFACTIONSEVERYWHERE.WEEBLY.COM/PAGE??](http://fractionsfractionseverywhere.weebly.com/page??)



Unit Rates are used to find the value or cost of one item.

5. Takis are sold in 1.2 ounce bags of 25 for \$10.98.

What is the unit rate of one 1.2 ounce bag? (Round to the nearest cent)

6. World record holder for the 50 meter freestyle swimming is Cesar Cielo. He swam 50 meters in 20.91 seconds. How many meters can he swim in one second?

(Round to the nearest hundredths)

7. Gas prices are falling, which allows Mrs. George to fill her my gas tank. Currently gas is priced at \$2.18 per gallon and she paid \$56.68 for gas only. How many gallons of gas did she buy (to drive to school everyday, lol)?

Calculate your next website location with the following instructions:

Multiply the answers from all questions above and use the cipher code to locate your next set of questions.

Cipher code using the following 1=a, 2 = B, 3 = c, 4 = D, 5 = e, 6 = F, 7 = g, 8 = H, 9 = I,

0 = j and the any decimal point is K.

[http://fractionsfractionseverywhere.weebly.com/\(cipher code here\)](http://fractionsfractionseverywhere.weebly.com/(cipher code here))

Unit Rates 2: Fractions

8. Juan can drive 512 miles in 8 hours to visit his cousins in Miami.

At this rate, how many miles can he drive in 15 minutes?

(Watch your units)

9. When pressed for time, Cynthia was able to type a 1890 word essay in 45 minutes.

What is her word rate of typing per minute?

10. At this rate, how many words can Cynthia type in 90 seconds?

The sum of the (answers to questions #9 and #10) multiplied by the (square of the answer to question #8).

Insert your answer in place of the \$ symbols: [http://fractionsfractionseverywhere.weebly.com/\\$\\$\\$\\$](http://fractionsfractionseverywhere.weebly.com/$$$$)

Complex Fractions (Division of Fractions)

11. Keep answer as an improper fraction  $4/15 \div 6/25 =$

12. You have four yards of ribbon to create hair bows. If each hair bow is to be two-thirds of a foot long, how many bows can you make from the ribbon you have?

13. If Emani uses  $3 \frac{1}{2}$  tablespoons of decaffeinated coffee to make 8 cups of coffee, how much would she need to make twelve cups of coffee?

14. Ana's Macadamia Nut Cookie recipe will make about 4 dozen cookies. Her recipe calls for  $3 \frac{3}{4}$  cups of flour and  $3/8$  cups of sugar. If Ana was to adjust the recipe to include 1 cup of sugar, how many cups of flour would she need to use?

The square of the answer to question #14 multiplied by the

*products of the answers to questions #11, #12 and #13.*

Insert your answer in place of the \*\*\*\* then copy and paste in the URL.

[http://fractionsfractionseverywhere.weebly.com/\\*\\*\\*\\*](http://fractionsfractionseverywhere.weebly.com/****)



Percents/Markup/Markdowns

**15.** Red grapes are on sale for \$1.99 per pound. In addition to the sale price, today only, shoppers receive an extra 25% off their purchase of grapes. Z'Andre bought 3 pounds of grapes, how much did he pay for the grapes today?

**16.** Black Friday sale of a 50 inch LED TV regularly priced at \$499 will drop its price. Buyers will receive 60% off the regular price. If buyers paid a 6.5% sales tax, what is the Black Friday sales price of the TV?

**17.** Beets Best Headphones are on sale for \$239.20 (discount of 20% off regular price) for 3 hours only today. After the 3 hours, the headphones will return to their regular price of how much?

Input the *sum of the answers to all of the questions using the cipher code:*

Cipher code using the following 1=a, 2 = B, 3 = c, 4 = D, 5 = e, 6 = F, 7 = g, 8 = H, 9 = I,

0 = j and the any decimal point is K.

Insert your answer in place of the ++++++ then copy and paste in the URL.

<http://fractionsfractionseverywhere.weebly.com/++++++>



CONGRATULATIONS!  
YOU COMPLETED THE CHALLENGE.

## **Appendix 1: Implementing Teaching Standards**

### **CCSS.MATH.CONTENT.7.RP.A.1**

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units

### **CCSS.MATH.CONTENT.7.RP.A.2**

Recognize and represent proportional relationships between quantities.

#### **CCSS.MATH.CONTENT.7.RP.A.2.A**

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

#### **CCSS.MATH.CONTENT.7.RP.A.2.B**

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

#### **CCSS.MATH.CONTENT.7.RP.A.2.C**

Represent proportional relationships by equations.

#### **CCSS.MATH.CONTENT.7.RP.A.2.D**

Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate.

### **CCSS.MATH.CONTENT.7.RP.A.3**

Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

### **CCSS.MATH.CONTENT.7.EE.B.3**

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

The above listed standards are directly quoted from the Common Core State Standards site.

## Appendix 2

**WS1**

Place two dominoes in the first two domino places to set up the problem.

Example:  $1/3 \div 2/5 =$  **Keep** first domino ( $1/3$ ) **Change** division to multiplication **Flip** 2nd domino then multiply numerators together and then multiply denominators.

$$\begin{array}{|c|c|} \hline \cdot \\ \hline \cdot \quad \cdot \\ \hline \end{array} \div \begin{array}{|c|c|} \hline \cdot \quad \cdot \\ \hline \cdot \quad \cdot \quad \cdot \quad \cdot \\ \hline \end{array} = \begin{array}{|c|c|} \hline \cdot \\ \hline \cdot \quad \cdot \\ \hline \end{array} \times \begin{array}{|c|c|} \hline \cdot \quad \cdot \quad \cdot \quad \cdot \\ \hline \cdot \quad \cdot \\ \hline \end{array} = \frac{5}{6}$$

$$\begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} \div \begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} = \begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} \times \begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} =$$

$$\begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} \div \begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} = \begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} \times \begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} =$$

$$\begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} \div \begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} = \begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} \times \begin{array}{|c|c|} \hline \\ \hline \\ \hline \end{array} =$$

## WS2

Roll 1 dice and place it in the blank. Place a domino in the specified position.

Example:  $2 \div 2/5 =$  **Keep** the dice 2, **Change** division to multiplication **Flip** the domino (5/2) then multiply.

$$\begin{array}{|c|} \hline \bullet \\ \hline \bullet \bullet \end{array} \div \begin{array}{|c|} \hline \bullet \bullet \\ \hline \bullet \bullet \bullet \bullet \end{array} = \begin{array}{|c|} \hline \bullet \bullet \\ \hline \bullet \end{array} \times \begin{array}{|c|} \hline \bullet \bullet \bullet \bullet \\ \hline \bullet \bullet \end{array} = 5$$

$$\begin{array}{|c|} \hline \text{Place} \\ \text{dice} \\ \text{here} \end{array} \div \begin{array}{|c|} \hline \text{Domino} \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array} \times \begin{array}{|c|} \hline \\ \hline \end{array} =$$

$$\begin{array}{|c|} \hline \text{Place} \\ \text{dice} \\ \text{here} \end{array} \div \begin{array}{|c|} \hline \text{Domino} \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array} \times \begin{array}{|c|} \hline \\ \hline \end{array} =$$

$$\begin{array}{|c|} \hline \text{Place} \\ \text{dice} \\ \text{here} \end{array} \div \begin{array}{|c|} \hline \text{Domino} \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array} \times \begin{array}{|c|} \hline \\ \hline \end{array} =$$

Below is a suggested worksheet for WU 12 for students to use. In the layout provided students can organize their project demonstrating their understanding of a real-world application problem involving markdown and markup calculations.

<div><div>Name: _____</div><div>Use this worksheet to show how you spent your \$50 budget for 4 family members. NO Gift Cards allowed.</div><div><ul style="list-style-type: none"><li>• Attach copies of sales flyers or draw pictures of items purchased (separate sheet of paper)</li><li>• Contain minimum of 2 markdowns (discount, sale price) Highlight markdowns in yellow or green</li><li>• Contain minimum of 1 markup (tax) Highlight markups in pink or blue</li></ul></div></div>			
<div>Item purchased (store name included)</div>	<div>Original Price, discounts, coupons, sales price, tax calculations</div>	<div>Total Cost of each Item</div>	
1. _____			
2. _____			
3. _____			
4. _____			
			<div>Total Amount Spent</div>

## **Student Resources**

Game oriented ratio games can keep your student interested in learning math. Below is a list of a few suggestions:

[http://www.mathplayground.com/tb\\_multiplication/thinking\\_blocks\\_multiplication\\_division.html](http://www.mathplayground.com/tb_multiplication/thinking_blocks_multiplication_division.html)

[http://www.mathplayground.com/ASB\\_RatioStadium.html](http://www.mathplayground.com/ASB_RatioStadium.html)

Math Term Dictionary

<http://www.mathematicsdictionary.com/math-vocabulary.htm>

## **Teacher Resources**

This website allows teachers to create a particular objective to focus on while playing a fantasy inspired game. Students beg me to assign an activity for them each day, all while practicing their math skills.

<https://www.prodigygame.com/Play/>

Whenever students are struggling with a topic, I assign some videos or lessons for them to review. Sometimes hearing about a topic from another point of view is beneficial. Therefore, I love using [www.khanacademy.org](http://www.khanacademy.org).

Looking to challenge your students with simple to more complicated math puzzles look at <http://www.kenken.com/>.

The most helpful website in pushing your students to a higher level way of thinking is using my seminar leader's website. <http://math2.uncc.edu/~hbreiter/> His website is a resource among resources with collections of years' worth of sites and articles to explore.

Culminating activity for this unit is located at

<http://fractionsfractionseverywhere.weebly.com/>

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## Notes

<sup>1</sup> Reiter, Harold. FUNdamentals in Mathematics 2016,  
<http://math2.uncc.edu/~hbreiter/CTI2015/>

<sup>2</sup> Honigsfeld, Andrea, and Maria G. Dove. Common Core for the Not-so-common Learner: English Language Arts Strategies, Grades 6-12. CA: Corwin, 2013.

<sup>3</sup> Checkley, Kathy. 2006. The essentials of mathematics, grades 7-12: effective curriculum, instruction, and assessment. Alexandria, VA: Association for Supervision and Curriculum Development.

<sup>4</sup> Jensen-Vallin, Jacqueline, "Empowering with Inquiry-Based Learning" MAA Focus, October/November 2015.

<sup>5</sup> Johanning, Debra I. "Estimation's Role in Calculations with Fractions." Mathematics Teaching in the Middle School 17, no. 2 (September 2011): 96-102.  
A visual representation of fractions on a number line is used to develop a comparison of fractions and equivalent fractions, decimal and percent concepts.

<sup>6</sup> Roberts, Donna Roberts and Frederick. "Roberts & Roberts." Accessed July 11, 2016.  
<http://mathbits.com/MathBits/AboutUs/aboutus.htm>.

<sup>7</sup> Reiter, Harold. "Exploring with the Area Model" CTI, April 2016.  
<http://math2.uncc.edu/~hbreiter/CTI2015/AreaModel1.pdf>

<sup>8</sup> Merriam-Webster's Collegiate Dictionary. 11th ed. Springfield, MA: Merriam-Webster, 2003. <http://www.merriam-webster.com/>.

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- Checkley, Kathy. 2006. *The essentials of mathematics, grades 7-12: effective curriculum, instruction, and assessment*. Alexandria, VA: Association for Supervision and Curriculum Development. I loved this resource as it reminded me that all students are mathematicians and should be approached in the manner in which they can learn.
- Honigsfeld, Andrea, and Maria G. Dove. *Common Core for the Not-so-common Learner: English Language Arts Strategies, Grades 6-12*. CA: Corwin, 2013. This book reminds the reader to reexamine teaching approaches as not all students learn the same way.
- Jensen-Vallin, Jacqueline, “Empowering with Inquiry-Based Learning” *MAA Focus*, October/November 2015.
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- Lamon, Susan J. 1999. *Teaching fractions and ratios for understanding: essential content knowledge and instructional strategies for teachers*. Mahwah, N.J.: Erlbaum. This book highlights that students understanding of fractions and ratios begins much sooner than formal teaching of those concepts. Allowing students to explore these concepts using reasoning this resources suggests students will develop richer fraction reasoning.
- Ma, Liping. 1999. *Knowing and teaching elementary mathematics: teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, N.J.: Lawrence Erlbaum Associates. This source provides interesting theories behind the teaching of concepts related to other countries implementation of math concepts.
- Merriam-Webster’s Collegiate Dictionary. 11th ed. Springfield, MA: Merriam-Webster, 2003. <http://www.merriam-webster.com/>. I used this resource to define proportion for the reader.



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Moyer, Patricia S. 2001. "Are We Having Fun Yet? How Teachers Use Manipulatives to Teach Mathematics." *Educational Studies in Mathematics* 47 (2): 175–97.

<http://dx.doi.org.librarylink.uncc.edu/10.1023/A:1014596316942>

Denise S. Peppers, Anna Wan, and Hope E. Phillips. "A Closer Look at Manipulatives in Remediation." *Mathematics Teaching in the Middle School* 20, no. 3 (2014): 166-73.

Posamentier, Alfred S., Terri L. Germain-Williams, and Daniel Jaye. 2013. What successful math teachers do, grades 6-12: 80 research-based strategies for the common core-aligned classroom. This book provides research on the strategies of how to teach and encourage students to question the reasoning behind concepts being taught.

Reiter, Harold. "Exploring with the Area Model" CTI, April 2016.

<http://math2.uncc.edu/~hbreiter/CTI2015/AreaModel1.pdf>

Reiter, Harold. *FUNDamentals in Mathematics* 2016,

<http://math2.uncc.edu/~hbreiter/CTI2015/> Dr. Reiter's website provides a collection of math resources for teachers to use in all level of classrooms. He continues to update his site with new articles, lectures, events and interesting concepts to explore.

Roberts, Donna Roberts and Frederick. "Roberts & Roberts." Accessed July 11, 2016.

<http://mathbits.com/MathBits/AboutUs/aboutus.html> This site offers interesting resources for all levels of math courses using geocaching and easy access within a technology driven classroom.

Sgroi, Laura Shannon. 2001. *Teaching elementary and middle school mathematics: raising the standards*. Belmont, CA: Wadsworth/Thomson Learning. This book is a wonderful resource in providing insight into how to best teach mathematics to students from pre-school to middle grades.

"Standards for Mathematical Practice." Common Core State Standards Initiative.

Accessed June 9, 2016. <http://www.corestandards.org/Math/Content/7/RP/>

This site is regularly accessed for the Common Core Process Standards and Objective Standards for Mathematics.

Van de Walle, John A., Karen Karp, and Jennifer M. Bay-Williams. 2013. *Elementary and middle school mathematics: teaching developmentally*. Boston: Pearson. I was first introduced to this resource when I began studying math education. It has remained an excellent source of presenting math in elementary ways.