

Quadratics: Building a Strong Foundation for Understanding Quadratics
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East Mecklenburg High School
This curriculum unit is recommended for the following:
Advanced $7^{\text {th }}-8^{\text {th }}$ grade students
Designed for $9^{\text {th }}-12^{\text {th }}$ grade students
Algebra I / Algebra II
Math 1 / Math 2

Keywords: Quadratic(s), Function, Parabola, Vertex, Maximum, Minimum, Axis of Symmetry, Discriminant, and Quadratic Formula

Teaching Standards: See Appendix 1 for teaching standards addressed in this unit.

Synopsis: It was very easy to teach quadratics using a traditional methodology. What was not easy was teaching quadratics in such a way that will make it interesting to students and allow students the chance for a deeper understanding. This unit will explore quadratics in a way that is not commonly used. Students normally do not have a lasting understanding when quadratic functions are taught in a traditional manner. Students also do not realize the many different real world situations that quadratic functions can be used to make life much easier. In typical classroom studies students are taught how to solve quadratics, taught how to study the graphs, and perform countless computations. We as teachers speak about moving away from traditionally teaching mathematics and this unit on quadratic functions will offer teachers the chance to have a student centered classroom that will leave a lasting impression on their students. After using this unit, students will have an appreciation for the use of quadratic function and hopefully ignite a fire of learning for some students that think they do not like math or see how it can apply to their lives daily, aside from basic addition, subtraction, multiplication, and division.

I plan to teach this unit during the coming year to 60 students in Foundations of Math II that are in grades 9-12.

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# Quadratics: Building a Strong Foundation for Understanding Quadratics 

Ashley Hay

## Demographics:

The school that I currently teach at and will most likely implement this unit plan is a school that is overly populated with about 2,000 students. Lower level math classes have about 30+ students. The school I teach at is also a partial magnet school with International Baccalaureate (IB) and Advance Placement (AP) courses. East Mecklenburg High School has a 6\% Asian, 16\% Hispanic, $25 \%$ Caucasian, and $52 \%$ African American population. The school is comprised of $60 \%$ or more of students that are on free or reduced lunch. I will be using my curriculum unit in my Foundations of Math 2 courses. These students should have learned about quadratics in their Math 1 course, but if they are in Foundations of Math 2 they could use another review and a better foundation of quadratic functions.

## The Discovery:

Have you ever taught a lesson once, maybe even twice or three times, and each time you taught it your group of students still did not seem to grasp the material? You think the lesson is solid. Though this lesson is good your students are not retaining the information like you think they should. So you try it again with your next set of students. Maybe the last students just were not good students. Unfortunately, this new group of students, along with the group after those and the group after those, still are not retaining the information the way they should. Eventually you realize that the only common denominator is your lesson.

Throughout my few years of teaching there seems to be one thing consistent between each year. Each year I can ask a student what type of function is $x^{2}$ and students can consistently answer that it is a quadratic function which is great! I began to think, okay, great they remembered something from the previous year in mathematic class. I can then ask what does the graph of a quadratic function look like and what it is called. Usually one or two students will remember the shape and the name parabola with a little hinting then the entire class says "oh yeah I remember that". Then my final question(s) are phrased something like this, "Who can give me an example in real life when quadratic functions would be used? Or Give me an example of an event when it is graphed it will have the shape of a parabola." This is the question no student is hardly ever able to answer. It is not the students' fault that they cannot apply quadratics to real world situations, but the fault of the teacher. I realized that we are merely teaching students how to perform computations on quadratic functions. In turn, this is the reason we so often as educators of mathematics get the question from our students, "When will I ever use this in my life"? Students are asking us this and not retaining the information about quadratics because we are not giving them a reason to apply it to their lives.

## The Purpose/ Objectives:

As educators of mathematics we are fortunate if we are taught how to teach mathematics. Most teachers teach the way they were taught because that's all that they know. As a mathematics instructor we know that all of the mathematics connects. We know this because we have done the math. We have observed for ourselves how the math is intertwined as the complexity of the concepts develop. What we did not observe is how it is successfully taught in a manner so everyone can see how beautifully mathematics is woven together. Everything fits like a puzzle piece. When a puzzle is missing a piece then the picture is incomplete. Sometimes it becomes more difficult for you to complete the puzzle because that missing piece is vital for successful completion. It is also possible for the puzzle to be completed, but with much greater difficulties than intended.

The main objective is simple. The objective is to complete the puzzle. The objective of this unit plan is to start with all the puzzle pieces and try our best not to lose any of them along the way. This will be done by changing the way quadratics are taught. We as teachers teach the components of quadratic functions as separate entities which is incorrect. We start by introducing and comparing the functions $f(x)=x$ and $f(x)=x^{2}$. We show students how to factor quadratic functions and quickly being teaching them how to use the quadratic formula. We continue to show students all of these computations without them realizing what all of this even means.

This unit explore quadratics in a non-traditional environment. It allows the teacher to act as a facilitator in a student centered classroom. Students will explore quadratics using hands on activities that will engage them to learn kinetically and through experience. Each key component of study of quadratic functions will initially be discovered through a student driven activity. The teacher will later review and reflect with the students on their findings. Students in math 2 are presented with the standard form of a quadratic function: $f(x)=\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}$. Traditionally, students learn computation skills with quadratic functions, but $21^{\text {st }}$ century skills require the children of the future to know much more than "the answer" to the quadratic function. Twenty first century skills with quadratic functions is being able to use critical thinking skills to solve real world problems using quadratic functions.

## The Ideas / Learning Strategies:

Reasons to use quadratic functions are countless. Many adults do not realize this on their so we cannot expect the same from our students if we do not assist them properly. For example; the trajectory of a baseball when hit provides the graph of a quadratic function. If the ball is hit just right it will be a guaranteed home run. When an owner of a company need to project their future losses or profits it is through the usage of quadratic functions. The list could go on and on. Each day of this unit lesson has different learning strategies depending on the topic. The goal of the unit is to continuously intertwine the solving of quadratic functions and applying them to real world scenarios. In addition to using application
to assist in student learning the implementation of the math common core state standards are of major focus. This can be done by using methods that has worked for other teachers found through research.

## Mathematical Content Objectives

## Common Core Standards

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

These standards will be practiced more throughout the scaffold questions presented by the facilitator.

## Important Vocabulary and Formulas for the Unit:

| Quadratic Function | a function that can be written in the form <br> $\mathrm{f}(\mathrm{x})=\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}$, where $\mathrm{a}, \mathrm{b}$ \& c are real <br> numbers and a is not equal to zero |
| :--- | :--- |
| Parabola | a plane curve formed by the intersection of a <br> right circular cone and a plane parallel to an <br> element of the curve |
| Vertex | the point of intersection of lines or the point <br> opposite the base of a figure |
| Axis of Symmetry | A line that divides a figure in half so that each <br> half is the mirror image of the other |
| Minimum Value | The lowest point of a parabola (when a>0) |
| Maximum Value | The highest point of a parabola ( when a<0) |
| X-intercept | x coordinate of a point where a graph crosses <br> the x axis/ y coordinate of this point is zero |


| Roots | the solutions to a quadratic equation |
| :--- | :--- |
| Discriminant | $\mathrm{b}^{2}-4 \mathrm{ac}$ |
| Axis of symmetry and x coordinate of the <br> vertex | $\mathrm{x}=-\mathrm{b} / 2 \mathrm{a}$ |
| Quadratic Formula | $\boldsymbol{x}=\frac{-\boldsymbol{b} \pm \sqrt{\boldsymbol{b}^{2}-\boldsymbol{4} \boldsymbol{a} \boldsymbol{c}}}{\mathbf{2 a}}$ |

## Day 1: Introducing a Quadratic Function

Prior Knowledge:

- Students should be familiar with the names of the different degrees of polynomials.
- Students should have an understanding of how polynomials become of higher degrees
- For example; multiply polynomials

One of the first learning strategies to be used stems from a paper written by Harold Reiter. It is through this paper and his class that he teaches a new way to introduce quadratic functions. He used a more engaging method instead of a traditional approach of showing a video or pointing out the obvious by saying a quadratic is a quadratic because it has an exponent of two. Harold discussed during a one on one conversation with me the importance of allowing students to examine how a quadratic function actually becomes a mathematical problem. He also encouraged teachers to allow students to discover patterns on their own. The activity that is about to be presented has patterns embedded in finding the solution

Modeling with mathematics is one of the common core state standards. Many students may think about solving a problem and possibly devise a solution without realizing they modeled it by using some form of mathematics in their head. This activity allows students to also critique the reasoning of others while using math talk. It is important for students to practice mathematical vocabulary and defend their opinions with reasons of math to make them factual based. As mentioned earlier, the school this unit will be presented to has an International Baccalaureate program. Part of the program is helping students become good citizens of their community. This will be displayed in this activity by ensuring students are respecting other students' opinions regardless if they agree or disagree.

Activity:
Materials Needed:

- 64 wooden cubes
- Preferably two sets for students to better view the cubes in two areas of your classroom.
Students will then complete the following task taken from Harold CTI's Counting with Cubes Activity

Consider a large cube made from unit cubes. ${ }^{1}$ Suppose our cube is $n \times$ $n \times n$. Look at the cube from a corner so that you can see three faces. How many unit cubes are in your line of vision? Build a table that shows how many cubes are visible from one corner.

| $n$ | $n^{3}$ | number visible |
| :---: | ---: | :---: |
| 1 | 1 | 1 |
| 2 | 8 | 7 |
| 3 | 27 | 19 |
| 4 | 64 | 37 |

How does the table continue? Make some guesses and then try to prove your answer. Let's name the number we're looking for. Let $G(n)$ denote the number of cubes visible from a corner of the $n \times n \times n$ cube. Notice that the sequence of differences $G(2)-G(1)=6 ; G(3)-G(2)=19-7=$ $12 ; G(4)-G(3)=37-19=18$ has an interesting property. The differences are all multiples of 6 . When we explore such a sequence in which the sequence of successive differences is eventually constant, we can build a polynomial that produces the sequence. Since the second order differences are constant, we propose that $G(n)$ is a quadratic polynomial, $G(n)=a n^{2}+b n+c$. We can solve this without great difficulty to get $G(n)=3 n^{2}-3 n+1$. But what do these coefficients have to do with the problem? One way to see this is to extend the chart by one more column that shows the cubes that are not visible.

| $n$ | $n^{3}$ | number not visible | number visible |
| :---: | ---: | :---: | :---: |
| 1 | 1 | 0 | 1 |
| 2 | 8 | 1 | 7 |
| 3 | 27 | 8 | 19 |
| 4 | 64 | 27 | 37 |

Now you can see that we can count the number of visible cubes by counting the invisible ones first. So, in general, $G(n)=n^{3}-(n-1)^{3}=$ $n^{3}-\left(n^{3}-3 n^{2}+3 n-1\right)=3 n^{2}-3 n+1$. This doesn't completely answer

Student Version

| n | $\mathrm{n}^{3}$ | Number not <br> Visible | Number <br> Visible | Difference <br> between n |
| :---: | :---: | :---: | :---: | :---: |


|  |  |  |  | and n-1 <br> visible cubes |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 1 | 0 |
| 2 | 8 | 1 | 7 | 6 |
| 3 | 27 |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |

## Discussion/ Scaffold Questions

1. What is the pattern that is occurring to determine the number of squares not visible at the value of n increases?
2. What appears to be the pattern for the number of visible cubes as the value of $n$ increases?
3. Can you try to create a function $\mathrm{F}(\mathrm{n})$ for the number of visible cubes based on the patterns your just described in questions 1 and 2 ?

## Day 2 and Partial Day 3: Introducing the Graph of a Quadratic Function

Prior Knowledge:

- Students should be able to graph a function given coordinate points.
- Students should be able to examine a graph and determine key characteristics such as:
- Y-intercept
- X- intercept
- Determine if the graph has a maximum or a minimum
- Determine if the graph is increasing or decreasing
- Give a real world example of a situation that may be applicable to a given graph.

Warm up:
This particular warm up will focus on the common core standard of making sense of problems and persevere in solving them. Alice Keeler is an education specialist that takes time to write an article and discuss what it means to actually perform and model each standard for your students. This activity is an idea inspired by the need to make sense of problems and preserve in solving them. Keeler said that instead of giving students a problem with step by step instructions we should allow them to explore and have the ability to have different answers in the end. This particular warm up is not a multi phased question like Keller suggested, but it does allow for students to have different solutions and still be correct.

Though the main focus was originally intended to assist students in making sense of a math problem and preserve in solving it. When it is time to discuss and compare answers students will then have to support their solutions with logical reasoning as well. Though this is a
math class, Keller examines how this can be done without the use of numbers. Students can have logical reasoning and not use numbers at all. When students begin discussing their logic behind why their graph may be different than the graph of their peers they may begin discussing interpretation of the situation. Interpretation of the situation is possible to have had nothing to do with numbers at all. It should be interesting to use Keller's approach to this standard.

Upon entering the classroom, students will be given a list of scenarios. Using time versus height, students will sketch what a graph may entail given the current situation. The situations will be:

- A person walking down the street
- A grasshopper hopping across the lawn
- A basketball being shot into a hoop
- A skateboarder is trying out a trick on a 20 foot "U" shaped ramp
- Riding a bike down a hill
- Doing a cannonball off a diving board

Discuss similarities and differences observed

Please view the following Student handout / classwork and homework (Attached in Appendix 2) The discussion/scaffold questions should become embedded during the class work. Discussions questions:

1. Why is the graph of a quadratic curved and a linear equation straight?
a. Think of the real world related incidents. Why does it make sense for certain situations to have curved graphs in the shape of parabola and other situations to be a straight line?
2. In regards to the maximum and minimum in each of the quadratic functions, what do they tell us about the graphs?
3. Why does it make sense for a portion of the graph to increase and a portion to decrease?
4. In regards to the table and plotting points given a function, why does it make sense for every y-value to appear twice?
a. For example: given the expression $x^{2}$; when $x=2$ and $x=-2$ the output for both is $4 \ldots$ why does this make sense?
b. What can you conclusions can you make about quadratic functions and their graphs?

## Day 3: Identifying features of Quadratics Game day

Today is game day! There has been much research supporting the use of game based learning in the classroom. I recently created a research presentation on the pros and cons of using game based learning in the classroom. Sam Patterson mentioned that one of the benefits of game based learning is that it provides an assessment of students' abilities. In addition to assessing their ability, it allows room for students to assist in the assessment process. An easy way to incorporate writing and literacy in math is to require students to reflect. At the end of the game,
require students to write about how the game went. They can answer the following questions for a successful reflection:

1. Did you win the game? Why or why not?
a. What was your contribution?
2. Did you feel you knew enough of the material to successfully play the game?
a. If not, why do you think this was the case?
3. If you could make a change about the game what would it be and why?
4. Lastly, did you enjoy the game? Or do you feel the game should never be played ever again? Explain.
Patterson also said a leadership board is not fun for struggling students. This is one of the benefits of playing this game. This game is not leadership board based on the entire class. Therefore, the students can be paired with other students that are on the same understanding level as them. This is how the common core math standard using appropriate math tools strategically comes into this unit. The math tool that students will use here is their mathematical vocabulary. This game will allow students to practice and use appropriate mathematical vocabulary to ensure they win the game. Many students do not realize how important it is to become familiar with their mathematical terms. I think this game will be a great tool to stress the importance to students.
This activity is from: https://teacher.desmos.com/polygraph-parabolas
Materials needed:

- Computer or tablet device (phones will not be acceptable)

With this activity students will be able to do the following:

- Identify important features of parabolas
- Precisely describe these features to their peers
- Increase and practice their relevant vocabulary

How the activity works:
1.


Each student plays a practice round against the computer to learn how the game works. 2.


Next, students are paired with a classmate to play polygraph with parabolas. One person chooses a parabola; their partner asks yes/no questions in order to narrow a field of suspects down to one.
3.


Between rounds, students answer questions that focus their attention on vocabulary and strategy.

Teachers must first create a free account on desmos.com. Once the activity is located the instructor must also create a class code.

Before you put students on computers, make sure they understand the premise of the game. We do not recommend playing a sample round with the class, as the first round involves the computer asking questions of all of the students (a fact we do not reveal until they have finished the first round). You could easily, however, play a low-tech version-show the array, pick your graph, have students ask questions aloud, respond, et cetera.

Starting in the second round, we pair students with each other. One student picks the graph and answers questions, the other student asks the questions and tries to identify the chosen graph. Between rounds, students answer questions that focus their attention on vocabulary and strategy.

## Day 4: Introduction to Solving Quadratic Functions

Day four is all about reasoning! This is where the disconnection usually begins for students. Teachers usually do not help student comprehend why they are factoring expressions and setting the expressions equal to zero and solving. This time, when students are introduced to the zero product property with factoring, they will also be examining the graphs of the same quadratic function. Keller believe it is important for students to continually evaluate the reasonableness of their immediate results. Essentially, students need to know why something makes sense. Keller said it is natural instinct for students to immediately look for patterns when solving a math problem.

If this is the case, we as teachers must make sure we assist in helping students find the patterns so they can have a better reasoning. When given the opportunity to create their own reasoning and draw their own conclusions students are more likely to remember because it was an original idea. An original idea is much easier for anyone to remember versus repeating back information because a teacher said so or because that's what was written in their notes. The structure of day four assist teachers in having a student centered classroom environment. These are the type of activities and discussion questions that take away from the traditional approach while lifting up the constructivist teacher.

Prior Knowledge:

- Students should be familiar with the concept of factoring quadratic expressions.

Warm Up:
Students will factor the following quadratic expressions as their warm-up

1. $x^{2}-9 x+8$
2. $x^{2}-16 x+63$
3. $7 x^{2}-3 x-20$
4. $7 \mathrm{x}^{2}+9 \mathrm{x}$
5. $7 x^{2}-45 x-28$
6. $2 x^{2}+17 x+21$

Students will now be introduced to the zero product property. Students will asked to take the factors of the expressions they found in their warm up and set them each equal to zero. Once set equal to zero the students will be instructed to solve each equation for the noted variable.

Once this has been completed, the students' next task will be on the calculator. Students will be required to graph the first expression in their calculator.

Discussion Questions:

1. What do you notice about the graph of your equation and the solution you obtained when you solved your equation for x ?
2. Try this again with your second equation, third equations, etc. What do you notice?
3. What conclusion can be drawn about the graph of a quadratic function and your solutions from factoring after using the zero product property?

Homework Assignment/ Class Work (Please view Appendix 2)

## Day 5: Continuing to Solve Quadratic Functions

Introducing the quadratic formula will use the same strategies from day four. We will also take a moment to derive the quadratic formula. This should aid in helping the students understand that it is not a formula that we have pulled out of thin air, but actually makes mathematical sense.

Student Assignment:

Name: $\qquad$

## Derive the Quadratic Formula

Directions: To derive means to base a concept on a logical extension or modification of another concept. Therefore, you will need to derive the quadratic formula. Take the standard form of a quadratic equation and solve for x . *Hint, use completing the square* (Students are not familiar with this concept at this level and will need assistance from instructor when this step approaches)


|  | Algebraic Representations | Directions |
| :---: | :---: | :---: |
| Step 1: | $x^{2}+\frac{b}{a} x+\frac{c}{a}=0, \quad a \neq 0$ | Divide the general form of the quadratic equation by $a$. |
| Step 2: | $x^{2}+\frac{b}{a} x=-\frac{c}{a}$ | Subtracted the constant $\frac{c}{a}$ from both sides of the equation. |
| Step 3: | $x^{2}+\frac{b}{a} x+\left(\frac{b}{2 a}\right)^{2}=-\frac{c}{a}+\left(\frac{b}{2 a}\right)^{2}$ | Take half of the coefficient of the linear term, squared it, and added it to both sides of the equation. |
| Step 4: | $\left(x+\frac{b}{2 a}\right)^{2}=-\frac{c}{a}+\left(\frac{b}{2 a}\right)^{2}$ | Factor the trinomial on the left side of the equation. |
| Step 5: | $\left(x+\frac{b}{2 a}\right)^{2}=-\frac{c}{a}+\frac{b^{2}}{4 a^{2}}$ | Multiply out $\left(\frac{b}{2 a}\right)^{2}$ on the right side of the equation. |
| Step 6: | $\left(x+\frac{b}{2 a}\right)^{2}=-\frac{c}{a} \cdot \frac{4 a}{4 a}+\frac{b^{2}}{4 a^{2}}$ | Multiply $-\frac{c}{a}$ by 1 to obtain common denominators. |
| Step 7: | $\left(x+\frac{b}{2 a}\right)^{2}=\frac{b^{2}-4 a c}{4 a^{2}}$ | Combine the fractions in the right side of the equation. |
| Step 8: | $\sqrt{\left(x+\frac{b}{2 a}\right)^{2}}= \pm \sqrt{\frac{b^{2}-4 a c}{4 a^{2}}}$ | Take the square root of both sides of the equation. |
| Step 9: | $\left(x+\frac{b}{2 a}\right)= \pm \sqrt{\frac{b^{2}-4 a c}{4 a^{2}}}$ | Simplify $\sqrt{\left(x+\frac{b}{2 a}\right)^{2}}$ on the left side of the equation. |
| Step 10: | $x+\frac{b}{2 a}= \pm \frac{\sqrt{b^{2}-4 a c}}{\sqrt{4 a^{2}}}$ | Use the property $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$ on the right side of the equation. |
| Step 11: | $x+\frac{b}{2 a}= \pm \frac{\sqrt{b^{2}-4 a c}}{2 a}$ | Simplify $\sqrt{4 a^{2}}$ on the right side of the equation. |
| Step 12: | $x=-\frac{b}{2 a} \pm \frac{\sqrt{b^{2}-4 a c}}{2 a}$ | Subtract $\frac{b}{2 a}$ from both sides of the equation. |
| Step 13: | $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$ | Combine the fractions to obtain the Quadratic Formula. |

Time to test out the quadratic formula that was derived using the questions from yesterday's warm up. First we must identify the a , b , and c . We will walk through solving quadratic equations using the quadratic formula together as a class.

Discussion questions:

1. What do you notice about the solutions from the graph, the zero product property, and using the quadratic formula?
2. What conclusion can you make about quadratic functions and their x-intercepts?

Classwork/ Homework (Please view Appendix)

## Day 6: Application of Quadratic Functions

During the summer I worked with teachers from all over the Charlotte - Mecklenburg School District to create problem based tasks. The goal of this summer professional development was to aid teachers in creating tasks using mathematics and applying them to the real world. The practice of application in the classroom with the topics of study assist students with better retention. Students' retention rates begin to increase because they can relate. Students can make sense of why it is important to learn the different mathematical concepts. Additionally, these tasks may inspire students to learn math through deeper understanding because they want to pursue a future that involves that particular mathematical concept.

These tasks were created in groups then shared and presented to our peers. This particular task was created by another group in my cohort, but I am not sure of whom exactly. In this task the students are able to consider themselves as inventors. As inventors it is important that they make as much money as possible. Students become excited about the idea of them making money from something they may decide to create. For students to earn money it is important for them to also understand that they have to spend money in order to make money as an owner of a company. Quadratic equations can help students better understand the profit equation. This problem based task will be a great start for students to practice solving quadratic functions and observing how they are used daily.

## Problem-Based Task

The Charlotte Hornets fan shop offers a variety of Hornets merchandise, a co-branded line of Hornets and Jordan brand gear. The store's merchandising manager is performing an audit of last year's sales so that she may meet with her merchandising team to develop a strategy to maximize profits across all departments.
The fan shops customizable hats are her largest concern. She is considering whether to purchase the hats from a company based in Nicaragua, or to purchase from a local vendor. Locally, the merchandising manager can purchase hats for $\$ 16.99$ per hat, and receives free shipping. The company based in Nicaragua offers hats at a price of $373.44 \mathrm{C} \$$ (Nicaraguan Córdobas) per hat, however shipping will cost $45.39 \mathrm{C} \$$ per hat.

Based on last year's sales, the shop sold 2000 hats for $\$ 28.49$ each. The manager would like to increase the price of hats, and projects that for every $\$ 2.50$ increase in price, they will lose 150 customers.
Which vendor should they contract with, and for how much should the shop charge for each hat in order to make the highest profit?

## Sample Solution

Profit= (price of item)(quantity)-(cost of item)(quantity)

In order to solve you must know the cost. Find the cost per hat of each supplier USA : \$16.99
Nicaragua: $373.44+45.39=418.83$ Cordoba

- Convert Cordoba to dollars ( $\$ 1=28.75$ Cordoba $* *$ Keep in mind Conversion rates fluctuate)
- \$14.57
o It is less expensive to use the supplier in Nicaragua thus cost of item is $\$ 14.57$

Price of item is dependent on the \# of increases (x)
o Price of item $=($ Previous Price $)+($ Price increase $)(x)$

- $\quad$ Price $=(28.49+2.5 x)$

Quantity is also dependent on the \# of increases
o Quantity = (previous quantity)-(expected loss) (x)

- $\quad$ Quantity $=(2000-150 x)$

Profit $=(28.49+2.5 x)(2000-150 x)-(14.57)((2000-150 x)$

Profit $=-375 \times 2+2912 x+27840$

Students need to find the number of increases to find Price and profit

- Students can solve graphically or algebraically
- $x=-2912 /(2 *-375) x=3.88$ increases

Use x to find price and profit

- $\quad$ Price $=\$ 38.19$
- Profit=\$33,493


## Day 7: Recap and Review

Materials Needed

- Student dry erase boards
- Classroom set of dry erase markers
- Erase cloth for each student
- Homework questions from previous days/ Similar set of questions with the same amount of rigor.
For review day a game will be played using the homework and classwork questions from throughout the unit.
The game is called survivor.
Instructions:
The class will be split into groups of about four to five students per group. The students will then be rearranged in a way that they are not sitting with their group members. Each student should have a dry erase board, dry erase marker, and erase cloth. While the students are picking up all of their materials the teacher should be placing about 30 x 's on the board for each team. Example; if you have 3 teams the teacher should have 3 sets of 30 x 's for a total of 90 x 's. These will serve as each group lives.

The teacher will then explain to the students for every question they get correct they may take a "life" (which are the x's) from one of the other groups. Each group will begin with 30 lives and they may not commit suicide (take x's from themselves). You will put a question on the board. As soon as the question is on the board student are not allowed to talk to their group members. It is also best to remind students that the person next to them is not on their team so it is in their best interest to assist them in getting the correct answer.

This rule does not have to apply, but to keep the game interesting groups may earn back 2 lives if everyone in their group get the correct answer on that particular problem. Also allow students to help you keep track of who obtained the correct answer and who did not. They enjoying calling out students that try to sneak and write something on their boards once time has been called to put markers down.

After each question it is important that the instructor takes time to go over it for the students that may have missed it. For teams that may "die" early, it is best to have an alternate study guide for them to work on. This keeps them busy, learning, and minimizes classroom disruptions.

## Day 8: Test Day

Please follow the following link to view a copy of the assessment used for this unit.
www.problem-attic.com/test/zitg2mgy

## Appendix 1

## Teaching Standards North Carolina Math 1:

The following are the common core state standards that will apply to this unit plan. Some of the standards are covered more in depth than others. This unit allows for flexibility to increase or decrease the rigor depending on your level of students.
. NC.M1.A-APR. 3
o Understand the relationships among factors of a quadratic expression, the solutions of a quadratic equation, and the zeros of a quadratic function.

- NC.M1.A-CED 1
o Create equations variable that represent quadratic relationships and use them to solve problems.
- NC.M1.A-CED. 2
o Create and graph equations in two variables to represent quadratic relationships between quantities.
- NC.M1.A-REI. 1
o Justify a chosen solution method and each step of the solving process for quadratic equations using mathematical reasoning.
- NC.M1.A-REI. 4
o Solve for the real solutions of quadratic equations in one variable by factoring.
- NC.M1.F-IF. 4
o Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating intercepts; intervals where the functions increasing, decreasing, positive, or negative; and maximums and minimums.
- NC.M1.F-IF.
o Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.
- NC.M1.F-IF. 7
o Analyze quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, and decreasing, positive, or negative; maximums and minimums; and end behavior


## - NC.M1.F-IF. 8

o Use equivalent expressions to reveal and explain different properties of a function.

## Appendix 2

A.1. Quadratic Equations and their Properties (Graphing)

| Parabola: | Standard Form: |
| :--- | :--- |
| Direction of Opening: | y-intercept: |
| Vertex: | Axis of Symmetry (AOS): |
| x-intercepts: | Maximum or Minimum Value: |
| End Behavior: | My Notes: |



Student Classwork/Homework Assignment Quadratic Equations and their Properties Name:


Vertex:
AOS:
Max or Min? Value:
Y-intercept:
Direction of Opening:
X-intercepts:
End Behavior:
Number of Solutions:

Vertex:

AOS:

Max or Min? Value:
Y-intercept:
Direction of Opening:
X-intercepts:
End Behavior:

Number of Solutions:

## INTERVALS

- We can show the region of the graph that is increasing by an
$\qquad$ _.
- Intervals describe the range of $\qquad$ that meet the given requirement.
- Do NOT use $\qquad$ to describe increasing decreasing.


## INTERVAL NOTATION

- We use interval notation to abbreviate the description of increasing or decreasing.
- List the starting and ending points of your interval, separated by a comma:
$-\infty$ to -1 will look like:
- Next we decide if there should be parentheses () or brackets [ ] on the end of each \#
- Parentheses indicate that the graph does $\qquad$ the endpoint
- Always use $\qquad$ on $-\infty$ and $\infty$
- Brackets indicate that the graph $\qquad$ the endpoint
- On a $\qquad$ we can see what to use from open $\&$ closed circles
- $\qquad$ Circles indicate we are NOT including the point: ()
- $\qquad$ Circles indicate that we ARE including the point:
[]


## DOMAIN \& RANGE

- Domain:
- If the graph were to squish to the x-axis, what values would be covered by the graph?
- For quadratics this is ALWAYS $\qquad$
- Range:
- If the graph were to squish to the y-axis, what values would be covered by the graph?
- For quadratics we find the range using the $\qquad$ or $\qquad$ Y-Value

Domain:
Range:


Domain:
Range:


## INCREASING \& DECREASING

When describing where the graph is increasing or decreasing we always read the graph from $\qquad$ to $\qquad$ .

- Sometimes we have graphs that increase in more than one place.
- Rather than write out the word "and" we use the symbol " $\checkmark$ " called the

In the graph on the right-----
The interval where the parabola is increasing is from
$\qquad$ to $\qquad$
The graph is decreasing from $\qquad$ to $\qquad$ Now write in interval notation--
INCREASING:
DECREASING:

INCREASING: DECREASING:

A.2.

Kuta Software - Infinite Algebra 1
Solving Quadratic Equations by Factoring

Name $\qquad$
Date $\qquad$ Period_

1) $(k+1)(k-5)=0$
2) $(a+1)(a+2)=0$
3) $(4 k+5)(k+1)=0$
4) $(2 m+3)(4 m+3)=0$
5) $x^{2}-11 x+19=-5$
6) $n^{2}+7 n+15=5$
7) $n^{2}-10 n+22=-2$
8) $n^{2}+3 n-12=6$
9) $6 n^{2}-18 n-18=6$
10) $7 r^{2}-14 r=-7$
A. 3.

Kuta Software - Infinite Algebra 1
Name
Using the Quadratic Formula
Date $\qquad$ Per
Solve each equation with the quadratic formula.

1) $m^{2}-5 m-14=0$
2) $b^{2}-4 b+4=0$
3) $2 m^{2}+2 m-12=0$
4) $2 x^{2}-3 x-5=0$
5) $x^{2}+4 x+3=0$
6) $2 x^{2}+3 x-20=0$
7) $4 b^{2}+8 b+7=4$
8) $2 m^{2}-7 m-13=-10$

## Bibliography

Keeler, Alice. "We're All Teaching the Common Core Math Standards." Edutopia. N.p., 2015. Web. 15 Aug. 2016 https://www.edutopia.org/blog/teaching-common-core-math-standards-alicekeeler . This article is a toolbox that the covers the common core standards with details on how they can be implemented successfully in your classroom.

Patterson, Sam. "4 Best Practices in Implementing GBL." Edutopia. N.p., 20 Jan. 2015. Web. 16 Aug. 2016. https://www.edutopia.org/blog/best-practices-implementing-gbl-sam-patterson. This is an awesome article on how to implement game based learning to engage, assess, and teach students in almost any classroom.

Reiter, Harold. "CTI 2016." Counting with Cubes. Sept. 2016. Web. 16 Nov. 2016. http://math2.uncc.edu/~hbreiter/CTI2015/BigBoxProblems2016.pdf .Included in this paper is great ways to introduce and engage students in using quadratic expressions for the first time.

