Are Math and Literacy in Proportional Relationship?

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This curriculum unit is recommended for 6-8 Mathematics

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Teaching Standards: See Appendix 1 for teaching standards addressed in this unit.

Synopsis:

This unit can be taught to any middle grade math students with convenient adjustments according to the standards being taught. Numeracy skills should be communicated in every math lesson because it is the only way to promote conceptual understanding of the content. Specifically, in this unit I will be illustrating numeracy and literacy skills while teaching the proportional relationship for my 8th grade classes. In Grade 6, students develop an understanding of relationships and Cartesian planes. In Grade 7, students should be able to transition to analyzing linear graphs and solve basic linear equations. Grade 8 students should be able to build on their prior knowledge to investigate the deeper concept behind proportional relationships, building on their understanding of ratios, proportions, and unit rate from 7th grade to apply it to functions and linear relationships. This unit incorporates teaching Numeracy skills and literacy for best practices. Teaching numeracy is not only teaching arithmetic skills; it also develops proficiency with numbers; it promotes the ability to analyze data; it develops critical thinking in solving problems; and it empowers students to apply mathematical concepts in any settings. In other words, "numeracy is about making sense of numbers and understanding the effect numbers have in the world around us."

I plan to teach this unit during the coming year in 2015-2016 to 85 students in 8th grade math class,

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Introduction:

As the title suggests, this unit is based on literacy skills while teaching the standard on proportions, graphing proportional relationships, interpreting the unit rate as the slope of the graph, and comparing two different proportional relationships represented in different ways. Common core mathematical practices follow the same theme of math practices to apply literacy. They both embrace problem solving and perseverance, reasoning abstractly and quantitatively, and modeling with math. Numeracy is not only the skill of calculating arithmetic. Numeracy is an individual's capacity to identify and understand the role mathematics plays in the world, to make well-founded judgments, and to engage in mathematics to meet the needs of that individual's current and future life as a constructive, concerned, and reflective citizen (PISA, 1999). Numeracy includes the capacity to identify similarities and differences through categorizing and organizing information (Marzano 2007). Therefore, teaching with arithmetic skills, like proportions, literacy plays an important role in defining it, interpreting world life scenarios which apply it, and reflecting on the problem solving strategies.

In general, I would use math literacy as my educational philosophy with every unit. I could not have asked for better opportunity to appreciate the value of numeracy skills in learning and teaching math than these seminars lead by Dr. Reiter by Charlotte Teachers Institute. As some fellows describe it, sometimes our brains get fried by the problems we analyze and solve. It is amazing how his enthusiasm and love to solve these complicated problems gets transmitted to us while enjoying the fun of this experience. In addition, the hands-on activities we are participating in are quite suitable to the content; one can never stop widening the horizons of knowledge under this great opportunity with CTI. To broaden my horizons of knowledge is one goal, but the meeting at the Discovery Place where I get the privilege of all the hands-on activities and applications has been the icing on the cake; Not to mention our professor's great strategies of facilitation and support of all this information. And I have to share with my students the fun of confusion I went through. Yes, the confusion we were exposed to each time a challenging problem has been presented. If I had the fun discussing them, trying to solve them, and sharing answers with my colleagues as much as I would have when presenting them to my students, my objective behind these seminars would have been justified gracefully. Therefore, when learning will and should never end at any time or place, I have learned a lot with these seminars and plan to teach it to my students. I have planned to teach a unit on proportional relationships, analyzing linear functions, and comparing them.

Math discourse will be used to incorporate literacy while teaching this unit, despite the fact being a challenging for the students I teach; however, integrating these common core mathematical practices should be facilitated and students should be able to get used to the strategy. Students would be motivated and curious to find out the connection between a table, graph, and daily life illustration to our lesson. My students will endure the same

amazing experience with their skills in math and even their state mind towards mathematics. My students will grow to become mathematicians while communicating their knowledge, ideas, and methods. They will undergo learning concepts of math to the deepest perceptions of the content and they will be motivated, enthusiastic, and lovers of math.

Although this unit is designed for 8th grade math standards, any teacher of 6th or 7th grade can personalize it and model it to fit the grade level standards. Students build on their work with unit rates from 6th grade and proportional relationships in 7th grade to compare graphs, tables and equations of proportional relationships. Students identify the unit rate (or slope) in graphs, tables and equations to compare two proportional relationships represented in different ways. The 8th grade math and common core standards and skills related to this unit are listed in Appendix 1.

I chose this standard because since North Carolina adopted the common core standard, line graphs and linear equations understanding has raised up to another level of reasoning. Before 2010, it was appropriate to teach linear equations as just y = mx +b; students were taught to find the slope m with the formula; to write this type of equation in standard and point slope form. Also we taught them how to graph it and how to write the equation from the graph. However, after common core, students should learn all of this in the context of life scenarios to apply reasoning so that they should be able to justify, analyze, and create more similar situations. In addition, common core math practices should be applied in order to achieve the highest level of learning. Therefore, after 3 years of teaching common core and administering their type of questioning, I realized that this standard has been the most challenging for other math teachers and me. I have developed this unit, especially the opening lesson to it hoping that students develop an understanding of the relationships between two sets of values and will be able to justify x and y in the context of the scenario. My goal is to get the students to use abstract reasoning and develop a strategy to manipulate the variables in relationships. Only then, they will be able to fully understand functions, especially linear at this grade level.

Demographic Background

My classes are a varied group of students of different abilities and math backgrounds. My students' heterogeneous upbringings reflect their math skills; they even include students who have not been in regular schools not to mention any math class appropriate to their age.

The school serves over 1,425 students in grades 6-8. Geographically, the school is located in a disadvantaged setting of Charlotte, NC in the Charlotte-Mecklenburg School District. We are a racially diverse school of approximately: African American (39.9%), Hispanic (26.9%), Asian (4%), and White (27.43%). 56% of students are receiving a free or discounted lunch. Population's gender is about 49% Male / 51%Female, Our school's 8th graders performed at 44% last year, which I find

disappointing. However, the main reason is the huge capacity of students and the student: teacher ratio of 18:1 is higher than the NC average of 14:1. Lots of other variables attribute to our school's low math and English End of Grade tests scores.

Unit Concept

My students suffer from huge lack of number sense, gaps as far back as 2nd grade:

- Relationships of whole numbers
- Adding and subtracting whole numbers
- Multiplying and dividing whole numbers
- Fact knowledge
- Understanding of doubling/halving, adding/multiplying by 5 or 10.
- Seeing patterns
- Adding/Subtracting of Integers
- Multiplication/Division of Integers

The 8th grade common core math standards I am covering in this unit will be the general framework for this unit while we review the above listed math skills through warm-ups and classroom activities. The generalizations of this unit standards state that my students will understand that

- Properties of any function, in multiple representations, create opportunities for comparisons across representations.
- Graphs of linear functions qualitatively represent the relationship between two quantities.
- Connecting proportional reasoning to functions from daily life.
- Prove the constant rate of change (linearity) in the relationship.

The length of the units should not exceed 3 weeks bearing in mind the MAP testing window for the winter might coincide during this unit; however, time will be adjusted for convenience.

Unit Content

The common core standard of the unit as listed above is 8EE5; I teach it after solving equations with one variable, 8EE7. Moreover, I teach it to introduce linear equations, equations with two variables. Teaching Proportional relationships and reasoning is handy and useful, especially if the teacher connects it to "how to apply proportional thinking to situations that affect one's life." Knowledge of ratios and proportions can help in lots of

real life situations. I should be able to cover all these real life situations with the word problems and performance tasks during the unit. In addition, the numeracy skills adopted during this unit is the number line vertically and horizontally, the hundred chart, which will aid my students visualize the multiples of 2, 3, 5, prime numbers, and perfect square numbers. Students become familiar with the idea of equivalent ratios such as those represented by the fractions 3/4, 6/8, and 75/100. Area multiplication will also be taught and modeled. During the CTI seminars with Dr. Reiter, we looked at the proportional relationships of the numbers produced when we perform the area model multiplication method. Number sense application activity will be included here to show the properties of the numbers multiplied in the area model allow us to recognize ratios inside the table. The literacy part of this unit will be covered by applying the lesson to real life applications and solving word problems. Integration of literacy is the embedded purpose of project-based learning strategy.

Teaching Strategies

No one can argue against the effective role of students' engagement and active learning provided by project-based learning. I have gone through lots of learning and research to implement it and to create the best project, instruction, and rubrics possible for this unit. Along with project-based learning, mathematical discourse is automatically provided because it enhances the instruction for diverse learners especially the non-English speakers and low socio-economic population. The most important part of mathematical discourse is getting to know the students' prior knowledge of the content by communicating it in writing, debate, or group activities; it will open up the opportunity for them to improve their mastery of the skill also by explaining and justifying their answers; then it will make the new knowledge add up to their previous learning much easier. The end result will be tremendous when students learn by connecting, reasoning, and arguing professionally and respectfully. Building relationships in the classroom among students and between the students and me is the hidden goal behind Mathematical discourse. I will be implementing verbal and written communication through these practices for mathematical discourse:

- Answering questions
- Asking questions
- Expressing own methods and strategies
- Debates and discussions
- Problem solving
- Group work activities
- Reflecting by writing
- Computational discourse

These activities are a natural fit for the Project-based learning. This unit about proportional relationships will begin with the justification of relationships in math; then a

general description of the concept of proportions and ratios. The 8th grade standard they need to learn is about proportional relationships would not be fully understood without making sure that students know deeply the meaning of the concept behind "proportion". 8th grade students should have learned this in 7th grade; however, not only I would review the understanding of proportions, I will also go over the skills of using proportions, solving proportions, and understanding examples of proportions in everyday life. The lessons will continue to cover these proportional relationships representations as ordered pairs, tables, equations, line graphs, and real life scenarios or word problems. At the end, I should teach how to compare proportional relationships represented in different ways. At this point the individual project will be directed into group project when they are asked to compare their sets of values; moreover, the group will provide another artifact, a power point presentation, video, or movie showing each relationship and how they approached the comparison of their relationship representations.

Our school administration encourages data-driven instruction as well as project-based learning. The math discourse in my classes will accommodate the diversification in the lesson plans and will definitely provide the appropriate strategies to plan and adjust lesson plans according to the data based on the students' projects.

Although my students are not as motivated or well-nourished due to economic circumstances, they are, on the other side, good artists especially when given the support, supplies, and encouragement. They enjoy creating an artifact out of a lesson concept; and from my experience, they are always willing to create posters and produce art projects. I have noticed how foldable, graphic organizers, and thinking maps for lesson notes have swept their minds and changed their attitude towards new math lessons. Therefore, my students cannot ask for a better differentiation opportunity of learning other project-based learning.

In addition, these seminars have encouraged me into presenting more challenging problems to my students and not giving them the answers until they have tried, discussed, and solved it on their own. Honestly, so far this has been the most challenging since these students are reluctant to work and persevere until they find out the result. As Dr. Reiter advises, "it is alright to confuse your students" I will.

Classroom activities

Since this unit comes after three weeks of solving multi-step equations with one variable, I will introduce it by talking about real life scenarios when we have to compare two values and use two variables; for example, the amount of money we get paid related to the amount of hours we work; the amount of words typed per minute; the price of a commodity per unit of weight (pounds or ounces). Explaining this relationship between two values introduces the concept of ratios. Situations come up in everyday life where

ratios describes the numerical relationship between two variable quantities. Measuring the distance a car travels related to the amount of gas spent is one example. Another one is comparing prices of a commodity at different stores to decide which one is the better buy. Ratios can be written in words: 2 to 5. Ratios can be written horizontally 2:5. Also, ratios can be written vertically 2/5. Then I start discussing equivalent ratios since they express the same relative relationship between their terms. Following this introduction, students should follow up with a discussion on more examples and their justifications. When the dialogue diverts into comparing different units like price and quantity, we call it a rate. Discussing equivalent ratios or proportions becomes justifiable since many relationships of two quantities in real word are proportional.

The first classroom activity I will be using to assess their prior knowledge is "class brainstorm web". I write the word "proportion" on a poster paper; then I have the students write as many words connected to it that they can think of around it. Then I would keep the poster visible during the unit so that we could go back and refer to some of the words they need to use while learning new skills. I would even add to the poster new words and concepts as we go along with the unit.

A proportion is a name we give to a statement that two ratios are equal. A proportion involves four numbers or four terms. It can be written in two ways:

- two equal or equivalent fractions, $\frac{a}{b} = \frac{c}{d}$

Or,

- using a colon, **a:b = c:d**

Students should be able to learn how to say two ratios are equal. They should say it by either "the ratios are proportional" or "the ratios are in proportion". Saying the appropriate expressions to express the equality is very important. For example, One out of three is the same as two out of 6. Since a proportion involves four numbers, to explain why they are proportional we start with the first number and state its ratio to the second; then we state that the third term has that same ratio to the fourth.

Also I will review with them how to build up equivalent fractions and how to prove that two fractions are equivalent. To find out if two fractions are equal, cross multiplication method is used. The product of the means should equal to the product of the extremes. The next activity is "the matching equivalent ratios". I created index cards with a ratio on each one. Students work in partners on a stack of these cards and together they should be able to match the equivalent ratios. Another extension to this activity is

getting the students to build equivalent ratios bigger by multiplying denominator and numerator by the same factor or by smaller equivalent ratio by reducing.

In lesson 2, students conceptualize that some quantities are proportional to another one when there exists a constant such that each measure in the first quantity multiplied by this constant give the corresponding measure in the second quantity. When students identify the measures in the first quantity with x and the measures in the second quantity with y, they will recognize that the second quantity is proportional to the first quantity if $y = kx$ for some positive number k. They apply this same relationship when using variable choices other than x and y.

A good example to discuss with the students is the amount of money John earned related to the quantity of hours of work he put each week. The scenario states that John gets paid $336.00 after he worked 33 hours. I will pose the following questions:

- How much do you think John had earned by the end of 2 weeks? *Double than the pay per one week. However, students should realize that they need to find the unit rate (John's pay per one week) first.*
- How will a table help us predict John's pay check for the coming weeks?
- How can we write some ordered pairs? What if he worked 0 hours, what would be his pay? *If he did not work any hours, he would not get paid. So (0,0) is the first ordered pair students need to find.*
- Let us make a table to find out his earnings after 3 weeks, 4weeks?
- Are John's total earning proportional to the number of weeks he worked? How do you know? *His total earnings are proportional to the number of weeks he worked because there exists a constant value that can be multiplied by the number of weeks to determine the corresponding earnings for that week. The table shows an example of a proportional relationship.*
- Then we should be able to write the equation of the relationships between the hours of work and the pay check.

Another investigation students should be able to complete for deeper understanding of a proportional relationship:

Mr. Alex decided to make juice to serve along with the pizza at the student council party. The directions said to mix 2 scoops of powdered drink mix with a half a gallon of water to make each pitcher of juice. One of Alex's students said she will mix 8 scoops with 2 gallons of water to get 4 pitchers. How can you use the concept of proportion to decide whether the student is correct?

Students should be able to make this table to represent this proportional relationship:

Amount of powdered drink mix	1 scoop	2 scoops	4 scoops	8 scoops
Amount of water (gallons)	¼	½	1	2

Students should be able to identify the constant k (1/4) which is multiplied by the first quantity (powder mix: x) to determine the second quantity (gallons of water: y). eventually, to determine any of the measures of water, you will always multiply the number of scoops by ¼.

The next lesson is identifying proportional and non-proportional relationships in tables where students examine situations to decide whether two quantities are proportional to each other by checking for a constant multiple between measures of x and measures of y when given in a table. Students study examples of relationships that are not proportional in addition to those that are.

Another classroom activity: Students work in pairs. Each pair gets index cards. On one index card, the students work together to create a table of two quantities that are proportional to one another. On the other index card, the students create a "story problem" that would generate the table. Once complete, I will collect all the table cards and all the story cards. I will display the table cards around the room and randomly passes out story cards. Students are to match their story to the correct table representation. This activity is a shortened version of the final project. This classroom activity will help the English Language Learners students to fully understand what the project is asking them to model. It is also a base where low-level students can build on to start their planning process for the project.

The next lesson will be focused on identifying proportional and non-proportional relationships in graphs. Students decide whether two quantities are promotional to each other by graphing on a coordinate plane and observing whether the graph is a straight line through the origin. The opening exercise will be a table containing data about candy bars sold (x) and money received (y). Students should identify if the two quantities are proportional to each other and to give reasoning for their answer. In this lesson I will present a coordinate plane and ask students to recall standards in grade 6 and 7. These concepts will be reviewed by discussing these questions:

- What is the origin and where it is located?
  - *The intersection of the x and y axis, at the ordered pair (0,0)*
- Why are we going to focus on quadrant 1?
  - *We are using numbers that are positive. Since we are measuring or counting quantities in daily life.*
- What should we label the x-axis and y-axis?

    - *The x-axis should be the chocolate bars, and the y-axis should be the amount of money received.*
- Could it be the other way around?
    - *No, the amount of money received should depend on the candy bars being sold, so the amount of money should be y, the dependent variable.*
- How should we note that on the table?
    - *The first values of the table should be the x-coordinates (the independent variable) and the second values of the pairs should be the y-coordinate (the dependent variable).*
- How do we plot the first ratio pair?
    - *If the relationship is 3:2, where 3 represents 3 chocolate bars, and 2 is 2 dollars, then from the first point, we go 3 to the right on the x-axis and go up to 2 on the y-axis.*
- When we are plotting a point, where do we count from?
    - *The origin (0,0)*

Have students plot the rest of the points and use a ruler to join the points and ask them;

- What observation can you make about the arrangement of points?
    - *The points al fall on a straight line.*
- Do we extend the line in both directions? Explain why or not why not.
    - *Technically, the line for this situation should start at (*0,0) *to represent 0* do*llars for 0 chocolate bars and extend infinitely in the positive direction because the more chocolate bars we sell, the more we make.*
- Would all proportional relationships pass through the origin? Think back to those discussed in previous lessons. Take a few minutes for students to share some of the context of previous examples and whether (0,0) would always be included on the line that passes through the pairs of points in a proportional relationship.
    - *Yes, it should always be included for proportional relationships. For example, if a worker works zero hours, then he or she would get paid zero dollars, or if a person drives zero minutes, the distance covered is zero miles.*
- What can you infer about graphs of two quantities that are proportional to each other?
    - *The graph will be a straight line and go through the origin.*
- Why is it a straight line?
    - *Each chocolate bar is being sold for $1.50 each, which is the unit rate and also the constant of the proportion. This means that for every increase of 1 on the x-axis, there will be an increase of the same amount (the constant) on the y-axis this creates a straight line. Each*

point may not be part of the set of ratios; however, the line would pass through all of the points that do exist in the set of ratios.

Therefore, after going more examples, students should realize that quantities presented in a graph are proportional if they represented in a graph where the points lie on a straight line that passes through the origin.

I would spend another day on this lesson when students are working in groups to make posters of proportional relationships representations. This activity would also be another approach to provide the students ideas for planning their projects. Each group will have a different scenario (which I plan and provide). Within the groups, give students 15 minutes to discuss the problem and record their responses onto the poster paper. For the last 5 minutes, have groups adhere their posters on the wall and circulate around the room looking for the group that has the same ratios. Have groups with the same ratios identify and discuss the differences of their posters. As a closure for this lesson, I will ask the students to reflect on the posters and write their questions to other groups on sticky notes. One of the questions I would ask the groups, "what does (0,0) mean in the context of the situation?"

The next lesson will be focused on the unit rate as the constant of proportionality. Students should be to identify the constant of proportionality as the unit price because it is the value per unit of the second value. Examples will be provided by students after giving them ample time to jot them down. The use of unit rate was discussed in 7^{th} grade when students learn how to find and determine "the best buy" of commodities. At this level, the unit rate is the slope of the line "m".

The next lesson will be representing proportional relationships with equations. I will open the lesson with the discussion: "how could we use what we know about the constant of proportionality to write an equation?" I encourage students to begin to think about how we can model a proportional relationship by using an equation by framing with the following probing questions:

If we know that the constant proportionality, k, to be equal to y/x for a given set of ordered pairs, x and y, then we can write k= x/y. how else could we write this equation? What if we know the x-values, and the constant of proportionality, but do know the y-values. Could we rewrite this equation to solve for y? I will provide with examples and get their feedback and application. The main lesson's note is to determine which value is the independent variable (x) and which value is the dependent variable (y). The closing will be summarizing the lesson about the unit rate and constant of proportionality. Next, I will teach them that the unit rate is the slope of the line "m" and students will get tables, graphs, and word problems examples to find slope and write the equation. Here students learn that slopes can be determined using any two distinct points on a line by relying on

their understanding of properties of similar triangles. Students verify this fact by checking the slope using several pairs of points and comparing their answers.

Proving the constant of proportionality from the line graph is the most challenging part of this unit. Similar triangles is used to prove that slope is the same between any two points on the line. Fortunately, students should have a prior knowledge of similar triangles from 7th grade when they learned scale factors and dilations. An activity is planned where coordinate planes with similar and non-similar triangles are drawn and students need to analyze and draw conclusions. For non-similar triangles, I would ask students to write down the transformation which they suggest as needed to perform on one triangle to make it similar to the other one. The questions applicable for this activity:

- If the figures are similar, how do you prove it?
  - *By dilating one of them to show it will be the same size as the other one.*
- How would you dilate one triangle?
  - *By applying the appropriate scale factor on the measures of the sides.*

Please note that the examples provided should allow for dilating by less than 0 so that students realize that shrinking or reducing is also another form of dilation of figures. Then the activity will expand into showing examples of straight lines and students should be able to draw similar triangles along the line and prove similarity by setting up the proportion of the corresponding sides. Vocabulary check and understanding should take place here. Vocabulary words is best defined and illustrated using a sketch on a poster where the whole class recognizes and defines while highlighting it on the diagram. A diagram of two similar triangles formed by a tree and its shadow and a person standing in the path of the tree while showing his shadow as the smaller triangle similar to it. Similarity, corresponding sides, corresponding angles should be illustrated and students should be able to set up the ratios of corresponding sides showing similarity and corresponding angles showing congruency. After completion of this activity, students are reminded to show this concept in their projects. Consequently, the rubrics show that this part is a substantial chunk of the grade. Also, students will represent this similarity in ratio tables and with equations as well.

After completing the similarity activity, students will verify their findings by checking the slopes using several pairs of points and comparing their answers.

Finally, students should recognize that the graph of a proportional relationship must be a straight line through the original.

An example of classroom activity including some of the exercises we will go over:

Wesley walks at a constant speed from his house to school 1.5 miles away. It took him 25 minutes to get to school.

a) What fraction represents his constant speed, C?
b) You want to know how many miles he has walked after 15 minutes. Let y represent the distance he traveled after 15 minutes of walking at the given constant speed. Write a fraction that represents the constant speed C, in terms of y.
c) Write the fractions from parts (a) and (b) as a proportion and solve to find how many miles Wesley walked after 15 minutes.
d) Let y be the distance in miles that Wesley traveled after x minutes. Write a linear equation in two variables that represents how many miles Wesley walked after x minutes.

More examples:

1. A train travels at a constant rate of 45 miles per hour.
   a. What is the distance, d, in miles, that the train travels in t hours?
   b. How many miles will it travel in 2.5 hours?
2. Water is leaking from a faucet at a constant rate of $\frac{1}{3}$ gallons per minute.
   a. What is the amount of water, w, in gallons per minute, that is leaked from the faucet after t minutes?
   b. How much water is leaked after an hour?
3. A car can be assembled on an assembly line in 6 hours. Assume that the cars are assembled at a constant rate.
   a. How many cars, y, can be assembled in t hours?
   b. How many cars can be assembled in a week?
4. A copy machine makes copies at a constant rate. The machine can make 80 copies in $2\frac{1}{2}$ minutes.
   a. Write an equation to represent the number of copies, n, that can be made over any time interval, t.
   b. Complete the table below.

t (time in minutes)	Linear equation:	n (number of copies)
0		
0.25		
0.5		
0.75		

a. Graph the data on a coordinate plane.
b. The copy machine runs for 20 seconds, then jams. About how many copies were made before the jam occurred? Explain.

The learning outcomes of this unit should be assessed within the unit and with a final test. These outcomes should show mastery when:

- ✓ Students correctly computes the slope from 2 points on a line graph. Then computing the slope from another 2 points on the same line (or an equivalent ratio: proportion). Also, students find that the slopes in both parts are equal.
- ✓ Students write and provide a strong mathematical explanation as to which function has a greater rate by referencing a graph (slopes of each line and one of them is steeper) or a numerical calculation of their rates.
- ✓ Students correctly write the equation of the line from a word problem, for example, students should be able to explain "twice as fast" in terms of distance travelled as Y = 2X when they are given the distance of the first situation, students should find the right slope, compare the data, write the equation and graph both lines.
- ✓ When given different prices of a commodity with lines representing different stores, stude3nts uses the information provided to determine the unit rate of each store (slope of each line); students write analyze the situation and writes an estimate that makes sense, for example, store A sells as half as much; also students should use evidence from th3e graph in the justification (comparison of slopes and sizes of angles).

The project

A project will be assigned as a formative assessment for this unit. The rubrics will be provided in Appendix 2. Students will receive these instructions at the beginning of the unit and justification for this strategy will be explained to them. The project is an assessment tool for their understanding of the content by answering the questions provided and a research-based learning opportunity to create and predict further concepts. Students are expected to show mastery according to the rubrics. They should be able to come up with a life situation where a value is dependent on another value proportionally. They should be able to explain how they found the constant of proportionality, unit rate, or the slope from all four representations. The equation of their function should like Y = mX where m is the rate of change or slope. They need to show that they can identify m from the description of the scenario and show how they got it from two points on the line and table. Finally students should be able to predict to answer the last question. They should conclude that some functions will have additional value added (when the

equations has a constant added or subtracted “b” and it is called the Y-intercept of the line graph). This last question in the project should be the opening discussion for the next unit.

The Project Instructions

Make a brochure, poster, or a booklet in which you are teaching proportional relationships. Create a life scenario in which you are analyzing two sets of values, satisfying the proportional relationship restrictions. Your artifact should include explanation of the concept by answering the below questions with clear explanations; also it should include illustrations of the scenario with a table and a line graph. Make sure you are describing how to find the rate of change (slope) from all representations.

Your brochure should not be limited only to answering the questions below. Explain in depth and give examples:

1) Define ratio, rate, and proportion.
2) Define relationship between two sets of data.
3) Define proportional relationship.
4) Illustrate and define a relationship which is not proportional.
5) Demonstrate the four representations of the relationship
    a) Description of the scenario in words.
    b) Table of ordered pairs
    c) Equation of the function.
    d) The line graph
6) Show how one should find the rate of change (slope) in each representation.
7) Compare your relationship to another scenario when the two sets of values are not proportional. Include “what if” questions to challenge your audience.
8) Predict and inquire about what would happen to the relationship of your sets of values if their line graph should pass by another point on the Y-axis (not the origin).

Appendix 1: Implementing Teaching Standards

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

By implementing this project-based learning, I would have enforced the common core standards for mathematical practice with the focus on:

Make sense of problems and persevere in solving them. Students analyze given constraints to make conjectures about the form and meaning of a solution to two-variable linear equations. Students are systematically guided to understand the meaning of a linear equation in two variables with respect to proportional relationships.

Reason abstractly and quantitatively. Students decontextualize and contextualize throughout the module as they represent situations symbolically and make sense of solutions within a context. Students use facts learned about rational numbers in previous grade levels to solve linear equations and systems of linear equations.

Construct viable arguments and critique the reasoning of others. Students use assumptions, definitions, and previously established facts throughout the module as they solve linear equations. Students make conjectures about the graph of a linear equation being a line, then proceed to prove this claim. While solving linear equations, they learn that they must first assume that a solution exists, then proceed to solve the equation using properties of equality based on the assumption. Once a solution is found, students justify that it is in fact a solution to the given equation, thereby verifying their initial assumption. This process is repeated for systems of linear equations.

Model with mathematics. Throughout the module, students represent real-world situations symbolically. Students identify important quantities from a context and represent the relationship in the form of an equation, a table, and a graph. Students analyze the various representations and draw conclusions and/or make predictions. Once a solution or prediction has been made, students reflect on whether the solution makes sense in the context presented. One example of this is when students determine how many buses are needed for a field trip. Students must interpret their fractional solution and make sense of it as it applies to the real world.

Look for and make use of structure. Students use the structure of an equation to make sense of the information in the equation. For example, students write equations that represent the constant rate of motion for a person walking. In doing so, they interpret an equation such as $y = \frac{3}{5}x$ as the total distance a person walks, y, in x amount of time, at a rate of $\frac{3}{5}$. Students look for patterns or structure in tables and show that a rate is constant.

Appendix 2
Proportional relationship rubrics: Brochure/booklet/poster

Name: _____

	Below Standard	At Standard	Above Standard
Organization of Ideas	• The content lacks logical sequence of information. • Includes little information; only one or two facts. • Information is incomplete or incorrect.	• The content is written with logical sequence. • Includes some information. • Information is accurate.	• The content is written clearly with a logical sequence. • Includes information with well-developed details. • Information is accurate and organized with tools and features.
	0…….17	18……...35	36……..50
Mechanics	• No source documentation. • Sentences are incomplete. • Errors in spelling, capitalization, punctuation, and/or grammar. Requires editing.	• Some source documentation. • Some sentences are complete. • Text is written with little need for editing in spelling, capitalization, punctuation, and/or grammar.	• All sources are documented. • Sentences are complete and varied. • Text has no errors in spelling, capitalization, punctuation, and/or grammar.
	0……17	18……..……35	36……50
Formatting and Graphics	• Graphics do not enhance content. • Layout is cluttered and confusing. • Fonts are inappropriately sized and difficult to read. • Poor color selection making it difficult to read.	• Graphics support content. • Layout is organized. • Fonts are easy to read, but not always uniform. • Colors are appropriate.	• Graphics support content and unify the theme. • Layout is organized with appropriate headings. • Fonts are easy to read with some uniform variation. • Colors enhance the readability.
	0…….17	18…….……35	36…….50

Resources for classroom use

Suggested tools and representations should not be only limited to these listed items below.

- Scientific calculator
- Graph paper
- Straight edge ruler
- Coordinate plane poster
- Coordinate plane cut-outs for students
- Coordinate plane stamp for students' notebooks
- Poster paper

Bibliography

Reiter, Harold. "Understanding Fundamental Ideas in Mathematics at a Deep Level." Harold Reiter's Home Page. Accessed October 31, 015. http://math2.uncc.edu/~hbreiter/.
This seminar's leader has maintained the most resourceful website for math teachers k-12 for math lessons, problems, and challenging activities.

https://www.engageny.org/sites/default/files/resource/attachments/g7-m1-teacher-materials.pdf.
A great resource for planning lessons according to common core standards.

Crawford, Mathew, and Mathew Crawford. *Solutions Manual: Introduction to Number Theory*. 2nd ed. Alpine, CA: AoPS, 2008.

"Flawless Layout Logo, the Golden Section Proportion and the Fibonacci Series Formula - Flawless Layout." Flawless Layout. Accessed October 31, 2015. http://www.flawlesslayout.com/the-fibonacci-series-formula-and-the-golden-section-proportion/.
This website should be assigned to students to investigate the proportional relationship of the golden section and the Fibonacci series within the numbers.

"Grade 8 Mathematics Module 4." Grade 8 Mathematics Module 4. Accessed October 31, 2015.
Another resource for planning the daily lessons, activities, and assessments.

Humphreys, Cathy, and Ruth E. Parker. *Making Number Talks Matter: Developing Mathematical Practices and Deepening Understanding, Grades 4-10.*

"Intermediate Rubrics." Intermediate Rubrics. Accessed October 31, 2015. http://wvde.state.wv.us/teach21/IntermediateRubrics.html.
Rubrics sample documents are ready for teachers use.

"K-12 Math Projects: About Project-Based Learning." K-12 Math Projects: About Project-Based Learning. Accessed October 31, 2015.

http://www.ct4me.net/math_projects.htm.
A great website with lots information on project-based learning. It leads to wide variety of workshops and professional development ideas.

Moynihan, Christine. *Math Sense: The Look, Sound, and Feel of Effective Instruction*.
Every math teacher should read this book during the first years of teaching. It offers options to develop abstract reasoning.

"ORC." ORC. Accessed October 31, 2015.

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Polya, G., and Jeremy Kilpatrick. *The Stanford Mathematics Problem Book: With Hints and Solutions*. Ann Arbor, Mich.: University Microfilms International, 1991.

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Ray, Max. *Powerful Problem Solving: Activities for Sense Making with the Mathematical Practices.*

"Self-Reflection on Project Work | Project Based Learning | BIE." Self-Reflection on Project Work | Project Based Learning | BIE. Accessed October 31, 2015. http://bie.org/object/document/self_reflection_on_project_work.
The sample of the students' self- reflection after the project completion.

"The Great Math Mystery." PBS. Accessed October 31, 2015. http://www.pbs.org/wgbh/nova/physics/great-math-mystery.html.
A great resource to use in the classroom where students will be able to learn how much mathematics is relevant to the whole world around us.

Lya, George, and John H. Conway. *How to Solve It: A New Aspect of Mathematical Method*. Expanded Princeton Science Library ed. Princeton [N.J.: Princeton University Press, 2004.