Circumventing Place Value Issues with Gifted Children

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This curriculum unit is recommended for:
Mathematics, Grades 3, 4 & 5

Keywords: place value, base ten, alternative base systems

Teaching Standards: See Appendix 1 for teaching standards addressed in this unit. (Insert a hyperlink to Appendix 1 where you've stated your unit's main standards. For directions on how to insert a hyperlink, see Fellows Handbook, p. 29.)

Synopsis: The aim of this unit is to facilitate the development of the number sense of place value with fourth grade students, but may also be used with third and fifth grade students. This is accomplished by examining the progression of strategies for teaching place value. The unit focuses on mathematics, but also considers components of writing and character development (through the area of perseverance). Students will explore the workings of the base two and base five number systems before transitioning to base ten. The non-base ten number systems will aid students in understanding the mechanics of place value without having to initially understand the arithmetic. This unit provides teachers with numerous activities to teach these mathematical concepts - strategies employed will include Exploding and Fusing Dots, base ten blocks, expanded form and area models. Wide ranging blooms levels are used, including the higher level of application and creating

I plan to teach this unit during the coming year to 24 students in fourth grade mathematics. I will also share this unit with the four other teachers on my grade level as well as the third and fifth grade teachers for use as appropriate for their students.

Circumventing Place Value Issues with Gifted Children

Muriel Shuman

Introduction

How can we as teachers insure that students have a thorough understanding of numbers and their place in our number system? How can we insure that when they arrive in fourth and fifth grade that they can tell us that a "seven in the thousands place" is one hundred times more than a "seven in the tens place"? How can we insure that the pattern of decimal place value makes sense to them? We want our students to look at their answers to mathematics problems and be able to evaluate the answer as being reasonable or not. Without a true and deep understanding of place value in our children, none of these things will be "in place" – *pun intended*! In fact, Roger Howe states,

> "The most important thing a student of arithmetic can learn is to think in terms of the pieces".[1]

School & Student Background

I teach fourth grade at an elementary school comprised of around 630 students. My school is very unique, in that we have four separate programs making up our overall student population. Our school consists of extremely high performing students (the Horizons program), academically gifted students (our Talent Development (TD) program), classes for K-2 students who have shown potential to be in our TD program (the Learning Immersion students), and our "Academy" students who come largely from our surrounding neighborhood. We have recently been deemed a "Title One" school, which means that at least 75% of our student population is at poverty level. Our school racial makeup is very diverse and we have a high level of parent involvement, especially within our Horizon's and TD programs.

I teach a fourth grade Talent Development class. In order to be in the TD program, my students have to have been identified as "Academically and Intellectually Gifted" within either a Literacy track or a Math track. Most of my students are higher achieving in Literacy than they are in Math, although they still are grade level or above in Mathematics. Within my class I have ten boys and thirteen girls. I teach Mathematics every day for between one and one and one-half hours. My classroom is very close to a

"One-to-One" technology class as most of my students bring a Chrome Book or an iPad to school every day. The few students who do not bring a device are able to use one of the school Chromebooks when needed.

Objectives

This unit is designed to be taught in chunks, woven with the associated standards throughout the first quarter of the school year, and reviewed whenever possible throughout the year. Since the unit deals with Place Value and Number systems, the idea is to use the strategies mentioned here to lay a foundation for the understanding of our decimal number system at the beginning of the school year, and additionally for the strategies to come into play when the basic operations of arithmetic (addition, multiplication, subtraction, division) are taught later in the quarter. Roger Howe states that,

> "In schools throughout the world, learning whole number arithmetic means studying the decimal system."[2]

The unit is also intended to include Language Arts, especially in the area of Writing, and the Arts, aided by the use of Technology. Although I have designed the unit for use in my fourth grade class, my hope is that some of the strategies will be utilized in earlier grades (especially third), so that some of the foundational concepts are taught earlier and need to only be reinforced in fourth and fifth grade.

My concerns, which lead me to developing this unit, are that students arrive in fourth grade without a conceptual understanding of number sense as demonstrated in their lack of understanding of place value and the base ten model. This lack of understanding seems to catch up with them in fourth grade when they are asked to think beyond basic math skills as they work with large numbers, manipulating with all operations, and especially with decimals. Compounding this deficit, the parents of my students rush to teach the children the traditional algorithm for all operations (because they believe this will help their child to succeed in mathematics). Unfortunately, once the student "learns" this algorithm, they tend to use the algorithm, relying on a pattern of steps to solve a problem, but with no understanding of what those steps really mean. I want my students to be able to use the algorithm as a way to stay organized, but understand why the algorithm works the way that it does. They should also be able to examine their own work and their answers and determine if the answer is reasonable.

Place Value for whole and partial (decimal) numbers is then the Mathematics emphasis of this unit. As stated earlier, and bears repeating, the concepts of place value are so foundational that Roger Howe believes that "the most important thing a student of arithmetic can learn is to think in terms of the pieces." [1]

Content Objectives

The main Mathematical objectives of this unit are for students to be able to recognize that in a whole number, a digit in one place can represent a value of ten times what the same digit would be if located in the place to its right. The students will be able to use this understanding to express and manipulate numbers that are written in expanded form, represented by Base Ten blocks, and modeled with "Exploding and Fusing Dots". Once these objectives are met, students will be able to utilize this deeper understanding of place value when preforming all four Math operations (addition, subtraction, multiplication and division).

In addition to the conceptual, mathematical processes that this unit explores, there is a 'character' element that I wish to address with my students. The Common Core State Standards Initiative has put forth eight standards for mathematical practice. The first standard states, "Make sense of problems and persevere in solving them".[3] I see a pronounced lack of desire and ability to persevere when my students come to a challenge that they cannot solve immediately. Gifted students especially, tend to be perfectionistic[4] and reluctant to repeat the steps for a problem, hence, checking work and trying other strategies are traits that must be pushed for and developed within the student.[5] Dr. Harold Reiter stresses the importance of helping students learn to "tolerate confusion".[6] It is our job as teachers of Gifted children, to help them build skills where they will keep on trying, risking failure, and pushing through this confusion.

Other objectives of this unit include the students having such a complete understanding that they can use not only models and drawings, but words as well to describe their strategies and methods of solving tasks. This will be done through regular Math Journal prompts, meeting the Common Core standards of writing explanatory texts and conveying ideas clearly. The use of Technology and Art to aid these explanations will come into play here.

The specific student objectives for this unit include:

- Students will be able to convert base ten block representations by using powers of ten, i.e., they will accurately model 2,347 using thousands cubes, hundreds flats, tens rods and ones' blocks and then will be able to model the same number without using any thousands cubes.
- Students will demonstrate basic understanding of place value with in Base 10 number system as shown by telling the relative value of digits within a number,

i.e., comparing the value of the 4 in the number 473, to the value of the 4 in the number 4032.

- Students will demonstrate the operations of addition, subtraction, division by utilizing expanded form, exploding & fusing dots and the standard algorithm, and be able to explain where the values of specific digits exist in each strategy
- Students will thoroughly explain the place value system in writing, including giving examples, drawings and words to explain

Concepts & Strategies

Children develop an understanding of concepts through experiences with real things rather than symbols.[7] Most third, fourth and fifth graders fall into a stage termed by Piaget as the Concrete Operations stage.[8] He said that students in this stage cannot fully understand Math until they get to put their hands on the tangible Math items (blocks, cubes, 3-D objects). Some fourth or fifth grade students may have reached the next developmental stage after the Concrete Operations stage, that of Formal Operations. It is here that children develop the ability to think abstractly, and the ability to manipulate written symbols with no hands-on practice. According to Bobby Ojose,

> "The importance of hands-on activities cannot be overemphasized at this stage (concrete). These activities provide students an avenue to make abstract ideas concrete, allowing them to get their hands on mathematical ideas and concepts as useful tools for solving problems."[9]

These real life experiences are invaluable to the children, but unfortunately, many students come through elementary school having only experienced Math through paper and pencil (a representational or abstract model). In order to remedy this, the first exposure that a child gets in learning place value should be with objects i.e., counting out toothpicks and making tens, and groups of tens. This usually happens in second or third grade at the latest. The next tangible exposure of building ten groups of tens to make hundreds should happen with Base Ten Blocks, but again, unfortunately this is often skipped. When these tangible items are not used with the student who is in the concrete stage, and instead the teacher introduces new concepts with paper and pencil the student is left trying to establish their understanding without a basis or foundation laid down in their brain.

Because of the need to concretely build a child's understanding, and not assume that our students are at the higher cognitive level, it is important to first instruct place value concepts utilizing manipulatives such as Base Ten Blocks, Digi Blocks or even simple things such as toothpicks and snap cubes. Once a child's concrete understanding in in place, the teacher can transition to the next level using a representational model.

Representational models include pictures, graphics, and number models such as expanded form. The representational model used in this unit, in addition to Expanded Form, is a strategy called, "Exploding Dots". This strategy has been published and promoted largely by James Tanton,[10] but I am not sure if he was really the first to use the method. In my class we use the term "Exploding and Fusing Dots" for this process. The strategy involves teaching place value by using a representational model of dots. This strategy will be explained further in the section on "Math Activities".

The students will first actively explore numbers in a "system" other than base ten. They will work with base two and base five and will be asked to use an alternative base to represent addition and subtraction. This will allow them to get a general understanding of how place value works. When considering this teaching of alternative base-number systems, consider this quote by Dr. Harold Reiter:

> "Learning arithmetic in another base is like learning another language, it forces you to consider the form and the patterns and the inner workings of the 'language'. When students master place value of another base system, they then have a greater understanding of how our customary base-ten system operates." [11]

The students will therefore actively explore other base systems and much of this will be with the aid of "Exploding and Fusing Dots", but the students will also be asked to utilize other ways to model the use of alternative base-number systems.

Math Activities

There are many ways to attack teaching place value, and doing so in a manner than provides enough conceptual foundation for the students who still struggle with the abstract.

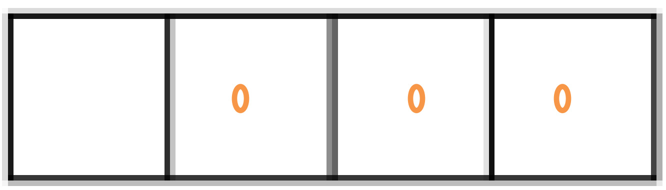
The basic progression that I followed with will be spelled out is as follows:

- Base 2 – place value (utilizing Exploding & Fusing Dots)
- Base 5 – place value (utilizing Exploding and fusing dots)
- Base 10 – place value (utilizing Exploding & Fusing dots, but quickly progressing to Base 10 blocks).
- Base 10 – place value extension – powers of ten and decimals
- The students' own creation of rules for addition, subtraction and division in base system of their own choosing.

Phase 1 of Instruction

When introducing the strategy of "Exploding & Fusing Dots", I did not want my students to make the connection immediately to the base ten number system, but instead, I wanted them to understand the strategy and the process first, and be able to link that to ANY base number system. Because of this, I began by teaching the dot system from a base two or Binary number perspective.

The Exploding Dot method teaches students to follow a rule that dot-boxes can only hold so many dots. Once you get too many dots in a box (based on the rule), they must explode, disappear and become one dot in the box to their immediate left (or "explode & fuse" as we called it in my class). So for instance with the 1←2 system, none of the boxes can hold 2 dots; they can hold zero dots or one dot only.
This 1←2 rule system is actually the Binary or Base Two system, but it is not necessary for students to know this. An example of this process is to change the Base Ten number "seven" into Base 2, instead of having seven dots in the rightmost box, you instead have one in the fourth box from the left (a value of 'one'), one in the third box from the left (a value of 2), one in the second box from the left (a value of 4), which gives 1 + 2 + 4 = 7.

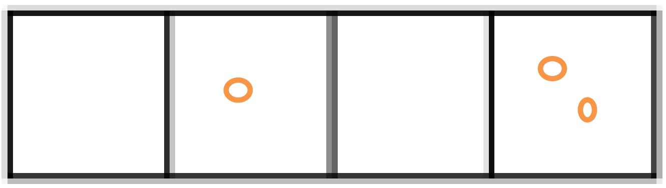


Value of boxes = *4* *2* *1*

After working with this system some, we progressed to the base 5 "Exploding and Fusing Dots" system, using the 1 ← 5 rule: Whenever there are five dots in any one box they "explode and fuse" to become a dot in the next box to their left. We spent quite a bit more time in the 1←5 rule system (base 5), because the extra digits of 2, 3, & 4 gave more of a chance for students' understanding than the base 2 system (the Binary or 1←2 rule system being much more abstract and hard to work with). Working with the base 5 system took approximately three class periods (this would vary with different classes of students) and included first finding the "secret code" for the base 10 number.

An example of finding this code for a base 10 number is as follows. Since the first box on the right stands for 5^0 (or 1), and the second box from the right stands for 5^1 (or 5) and the third box from the right stands for 5^2 (or 25), then for the number 27, the "secret code" would be 1-0-2 or 25+0+2 = 27. After we worked for a while with finding the Base-5-secret code for a given Base 10 number, then we worked backwards which involved finding the Base 10 number when first given the "secret code". This backwards work on the 1←5 rule system enabled the students to construct the base 5 "place value" by recognizing the value of the dots as going up by multiples of 5 for each "place".

Finding Base-5 “secret code” for the Base 10 number twenty seven (27):



Value of boxes = 5^{3} 5^{2} 5^{1} 5^{0}

(125) (25) (5) (1)

The reverse of this was working backwards, beginning with the code of 1-1-3-0, the students knew this represented (1 x $5^{3)}$ + (1 x 5^{2}) + (3 x 5^{1}) + (0 x 5^{0}), which is the same as 125 + 25 +15 = 165.

Phase 2 Instruction

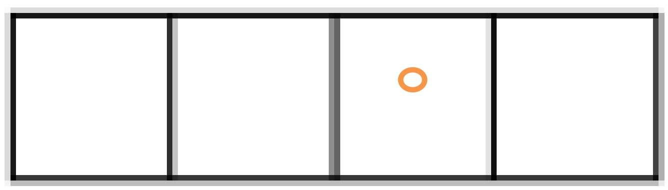
Phase 2 transitions the use of the bases two and base five to our customary base ten system. Once the 1 ←5 rule was mastered by most of the students, the “Exploding and Fusing Dots” model was introduced to the students, this time using the 1← 10 rule. Students very quickly realized that this new rule, using our previous model, was the “Regular” number system (Base 10 / decimal system) in disguise. Once again, the rule was taught by giving a regular number and having students devise the “secret code” by using the dot boxes. They quickly realized that a code for any number, i.e., 14, utilized the same numerals, 1 - 4 to represent the number of dots in the boxes.

The true “ah-ha” moment for the students when first learning this comes as you give the students the code (i.e., 2-4-1) and have them recognize that the two is in the 10^{2} or hundreds box, the 4 is in the 10^{1}, or tens box and the 1 is in the 10^{0} or ones box, giving of course, (2 x 100) + (4 x 10) + (1 x 10^{0}), or the base ten number 241.

For the students who have trouble with this transitional method of representing numbers, it is important to have our typical base ten blocks handy. The base ten blocks are brought forth to reinforce the fact that if there is one dot in the hundreds space, then that is the same as having one hundreds flat as we build our numbers. The base ten blocks help to mirror what is happening in the Exploding & Fusing Dot model.

The use of the Exploding and Fusing Dot model in conjunction with traditional base ten blocks is especially powerful when you discuss with children “powers of ten”. The Common Core Math standard 4.NBT.1 says that students should be able to recognize that a digit in one place represents ten times what it represents in the place to its right.

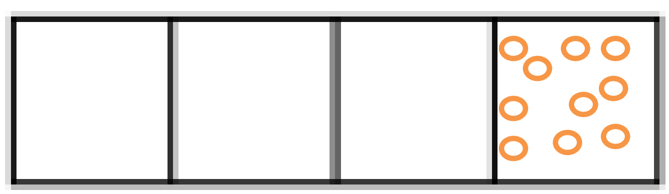
And this is often one of the hardest things for students to truly grasp, i.e., if I have 450 that is not only 4 hundreds and 5 tens, but it is also 45 tens. If we break up four hundreds' flats into forty tens rods, and/or we take 4 dots in the 10^2 dot spot and explode them to make 40 dots in the 10^1 dot spot, then students see that the power of ten works going to the place to its right and as well as to the left (which is easier for them to understand). This is easily seen with the Exploding and Fusing Dot model, in that every "one dot" in the hundreds spot can be exploded into ten dots in the tens block.



Value of boxes = 10^3 10^2 10^1 10^0

(1000) (100) (10) (1)

Becomes …



Value of boxes = 10^3 10^2 10^1 10^0

(1000) (100) (10) (1)

*** NOTE: This EXTENSION can be used here*:

Once the students have a pretty good handle on naming and using numbers in the bases 2, 5 and 10, they are given the next challenge. This task is especially important to build perseverance of our gifted students. They are told to select a different base number system and corresponding rule (i.e., base 4 would be the 1 $\leftarrow$ 4 rule system), and write out the numbers 1 through 50 in the base they choose and then are asked to explore addition and subtraction within this base. Students can also be challenged at this point to see if they can represent multiplication or division in the base of their choosing.

** Alternatively, this extension can be given after the further instruction in expanded form (Phase 3), depending on the class of students.

Phase 3 Instruction

The natural transition from the base ten models and the Exploding and Fusing Dot model is into expanded form. This can almost be taught at the same time, but it really is helpful for the students to not have to worry about manipulating the numbers while gaining understanding of place value through base ten blocks and the dot model.

Since expanded form is just actually a way of applying numbers to the dots and blocks, it comes pretty natural to the students. Roger Howe mentions …

> That the basic principle of base ten notation is to decompose any whole number into special numbers, its base ten pieces.[12]

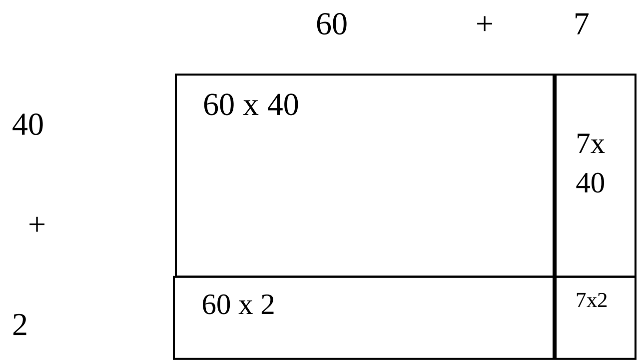
A quick transition to the expanded form model helps those few students who have not yet had their own personal 'ah-ha' moment to see for themselves the place value and decimal system in action.

Once we are able to do this successfully the decomposed numbers (expanded form) easily lends itself to manipulation within the math operations of addition, subtraction, multiplication and division. Expanded form with multiplication becomes the "Area Model" or the "Box Method", which many teachers do not even mention, rushing ahead to teach the standard algorithm. However, the Area Model is beautiful in that the numbers are manipulated within their places, keeping the places straight, which sometimes is hard for students with the traditional algorithm.

The process of using the area model is shown here for 42 x 67:

- 42 written in expanded form becomes 40 + 2
- 67 written in expanded form becomes 60 + 7

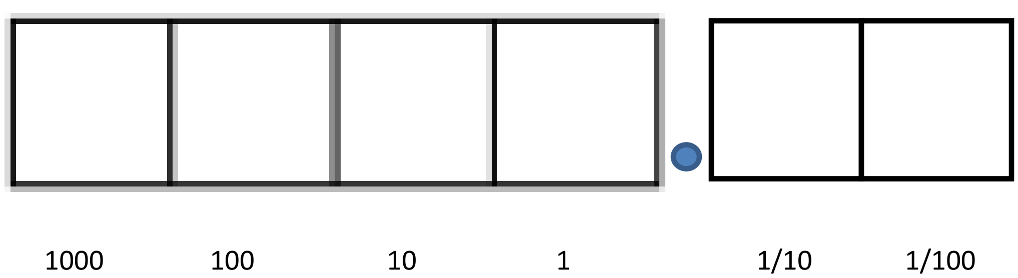
When these are arranged in an area model it looks like this:



The students end up with four "partial Products" whose sum gives them the final product for 67 x 42 or (2400 + 280 + 120 + 14 = 2814).

Phase 4 Instruction

The real beauty of the Exploding and Fusing Dot method comes when introducing and explaining decimals. Students are taught that each time they move to the left on the place value chart (or on the dot model), they are going up by a power of ten (multiplying) and that going to the right on the chart they go down (divide) by a power of ten, but this is not intuitive for them until they conceptually understand place value. The Dot model is of immense value here.



If we know that each box going to the right goes down by a power of ten (or is divided by ten), and if we use a decimal point to designate going from whole numbers to fractional parts, then the box to the right of the decimal point must have a place value of 1 ÷ 10, or 1/10 ("tenths"). If this is true, then ten dots in the box to the right of the decimal point would have to "Explode and Fuse" into the box on the left giving one dot there. Likewise the box to the right of the tenths box must be 1/10 ÷ 10 or 1/100 ("hundredths"). The boxes also help students to equate the decimals to their fractional equivalent. So 0.2, which has a 2 (or 2 dots) in the first box to the right of the decimal must equate to 2/10 and so on.

Phase 5 Instruction

The culminating activity for this unit is to have the students pick a number base system to showcase their understanding of place value. (They can use base-2 or base-5, but since these were used in instruction, they should be encouraged to pick a different base.) They will use this different system to show counting, and then they will demonstrate the use of the four operations – to the extent of their degree of mastery. This activity, that uses the higher order thinking skills of application, synthesis and creation, forces the students to truly digest the material. This is especially important for our gifted students who need to be further challenged in order to move above their place of comfort and current understanding. The students will be encouraged to showcase their work in a creative and artistic manner, utilizing either traditional art mediums of paint, markers and paper or that of technology.

Appendix 1: Implementing Teaching Standards

CCSS.Math.Content.4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division*.

CCSS.MATH.CONTENT.4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. *For example reading and writing a number in word, standard and expanded form and comparing numbers using place value understanding and equality and inequality symbols.*

CCSS.MATH.CONTENT.4.NBT.B.4_Fluently add and subtract multi-digit whole numbers using the standard algorithm. *Adding and subtracting numbers using foundation of place value understanding.*

CCSS.MATH.CONTENT.4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. *It is really important that students can "see" multiplication with models that are based on place value and that lead to the algorithm.*

CCSS.MATH.CONTENT.4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. *Likewise, it is so important here in laying a foundation with various models before trying to introduce the algorithm*.

CCSS.ELA-LITERACY.W.4.2
Write informative/explanatory texts to examine a topic and convey ideas and information clearly. *Students will use mathematics information and express their ideas and thinking with writing journal entries and explanations*.

List of Materials for Classroom Use

- Activity sheets for each phase of instruction (see Appendix 2)
- Base Ten blocks or Digi blocks
- Other manipulatives to model groups of ten
- Graph paper (handy for aligning numbers)
- Creation "Apps" , two handy easy to use ones are "ShowMe" and "Educreations"

Student Resources

- http://www.amathsdictionaryforkids.com An excellent site to build a student mathematics vocabulary, that includes definitions, examples, activities, and practice.
- http://khanacademy.org Sal Khan has put together thousands of videos and activities in many subjects and the site is especially helpful for mathematics students. Students can search for resources to help their understanding and/or teachers can assign practice sets.
- https://www.mathplayground.com Site contains arcade games, regular math games, word problems and probably the most helpful, a section on "Common Core". Within the Common Core site there are games organized by standard.
- https://frontrowed.com This site is set up for either assigned or adaptive math practice and is valuable for students to practice concepts after the teacher has introduced them. Specifically in regard to this unit, the section on "Base Ten", and levels 24-30 cover place value concepts.

Teacher Resources

- https://illuminations.nctm.org/ Online site with standards-based resources for teachers and students. There are interactive tools as well as lesson plans and activities.
- https://www.nms.org/resources/freelessons.aspx This is the National Math & Science Initiative site. There are multiple resources here including this one that relates specifically to place value: "Introduction to Decimals", listed under "Free Sample Elementary Lessons".
- https://frontrowed.com This site is set up for either assigned or adaptive math practice and is valuable for students to practice concepts after the teacher has introduced them. Specifically in regard to this unit, the section on "Base Ten", and levels 24-30 cover place value concepts.

- Developing Number Concepts. Place Value, Multiplication, and Division. Kathy Richardson. Dale Seymour Publications. 1999.
- http://www.learnillion.com This is an open, cloud-based curriculum that has full curriculum (K-8) lessons, as well as a mathematics video lesson library. The video teaching on Learn Zillion is conceptually presented, and is an invaluable resource for any math teacher.
- https://youtu.be/vOJ0Mlc3Mto This is the overview video by Jim Tanton on "Exploding Dots". A good supplement to the pdf (listed next) for the teacher who is trying to understand the Exploding Dot process for use with her students.
- http://gdaymath.com/lessons/explodingdots/1-1-base-machines/ The Jim Tanton site with a multitude of courses and specifically this one, on exploding dots.
- http://math2.uncc.edu/~hbreiter/CTI2015 Site by Dr. Harold Reiter, Professor of 2015 FALL CTI course. This site has a wealth of number related resources.

Appendix 2 – Student Activities & Instructions

Phase 1

1. Within the following boxes, create the code for the number 17 using the 1 ← 2 rule.

2. Given the Secret code of 1-1-0-0 in the 1 ← 2 rule system, draw this out in the boxes below and determine the original base ten number. Add an explanation for your answer.

3. Within the following boxes, create the code for the number 78 using the 1 ← 5 rule.

4. Given the Secret Code of 3-4-2 in the 1 ← 5 rule system, draw this out in the boxes below and determine the original base ten number. Prove that your Code equals the original number using the value for each box.

Phase 2

1. Within the following boxes, draw dots and create the code for the number 123 using the 1 ← 10 rule.

2. Given the secret code of 5-2-6-7 in the 1 ← 10 rule system, draw the dots and determine the original number.

3. Draw the Exploding and Fusing Dot pattern for 451 using the 1 ← 10 rule system.

Now, assume you are unable to use the first or second box from the left. Draw the new pattern for 451 by exploding the four dots in the "Hundreds" box into the "Tens" box.

Explain why the two boxes from problem #3 represent the same number.

Phase 3

1. Expand the numbers below using expanded form. Here is an example:

5,467 = 5,000 + 400 + 60 + 7

5,467 = (5 x 1,000) + (4 x 100) + (6 x 10) + (7 x 1)

Expand:

a. 5,743

b. 8,094

c. 658

d. 9,400

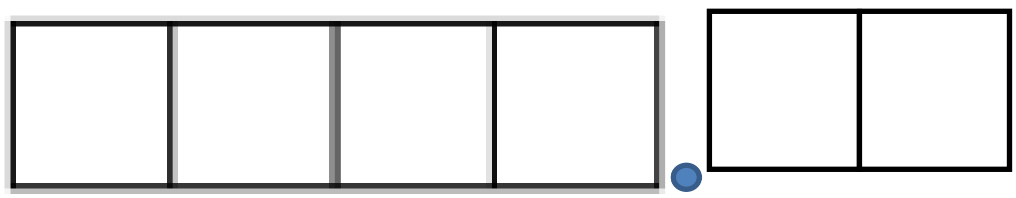
2. Utilize Expanded form to add these two numbers: 4,599 + 3,670 = n

3. Utilize Expanded form to subtract these two numbers: 5,600 – 3,214 = n

4. Utilize your knowledge of expanded form to multiply these two numbers with the area model. 46 x 39 = n

Phase 4

1. Use your knowledge of Exploding and Fusing Dots in the $1 \leftarrow 10$ system to express the number 532.08 in dot boxes.



2. Knowing that each box in the $1 \leftarrow 10$ system can only hold 9 dots before they must explode and fuse, and concluding that if you have 2 dots in a box of a certain value then it is 2 out of 10 possible, show the above number (532.08) in fractional (mixed number) form.

Explain why you know that the dot box representation for 532.08 is indeed equal to the fractional (mixed number) form above.

Phase 5

1. Apply all of the knowledge that you have regarding number base-systems, place value, dot boxes and expanded form, and pick a number base-system to showcase your understanding. (Try to pick a number system other than base 2, base 5 or base 10).

- Count in your base system up to 50.
- Show the dot box representation for 2 different base-ten numbers in your base system.
- Begin with a secret code sequence of numbers in your base system and translate them into base ten.
- Try to show each of the following operations using your base system (you do not have to use dot boxes): addition, subtraction, multiplication and dividion.
- Create a fractional number representation (similar to decimals in base-ten) that shows how you would represent parts of a whole in your base

** Showcase your understanding in an artistic rendering (poster, picture book, anchor chart) or utilizing technology (you could use an app like Google Slides, Showme, or Educreations).

Bibliography for Teachers

Common Core Navigator, Math 4. "Common Core Navigator." Accessed September 5, 2015. http://learnzillion.com/common_core/math/4 Resource for Common Core standards and objectives.

Gday Math . "Exploding Dots. 1.1 Base Machines". Accessed September 12, 2015. http://gdaymath.com/lessons/explodingdots/1-1-base-machines/. Explanation of Jim Tanton's Exploding Dot method of explaining place value to students.

Howe, Roger and Susanna Epp. "Taking Place Value Seriously: Arithmetic, Estimation, And Algebra." 2008, Accessed September 5, 2015. http://www.maa.org.pmet/resources/PlaceValue_RV1.pdf . This article was the foundational article for my unit. Howe described what I see in my classroom on a daily basis. Students, their teachers and their parents must deeply understand the ideas of place value as presented in this article.

Ojose, Bobby. "Applying Piaget's Theory of Cognitive Development to Mathematics Instruction." *The Mathematics Educator* 18.1(2008): 26-30. This article takes The basics of Piaget's developmental theories and applies them to mathematics Instruction.

Peters, D. "Coping 101: Building Persistence and Resilience in Gifted Children." *California Association for the Gifted*. 43.4(2012). Accessed September 5, 2015. http://www.davidsongifted.org/db/Articles_id_10772.aspx. Gifted children have special "quirks" that must be taken into account when instructing them in the classroom.

Richardson, Kathy. *Developing Number Concepts. Place Value, Multiplication and Division.* New Jersey: Dale Seymore Publications, 1999. Good teachers guide for teaching multiplication and division with conceptual methods.

Stepanek, Jennifer. "Meeting the Needs of Gifted Students: Differentiating Mathematics and Science Instruction." Eric Institute of Education Services. AccessedSeptember 26, 2015. http://files.eric.ed.gov/fulltext/ED444306.pdf.

Understanding Fundamental Ideas in Mathematics at a Deep Level. Howe, Roger. "The Most Important Thing for Your Child to learn about Arithmetic." Accessed September 5, 2015. http://math2.uncc.edu/~hbreiter/CTI2015/MostImportantrev2.pdf This site has many great resources for general place value and foundational mathematics understanding.

Notes

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[1] Roger Howe. “The Most Important Thing for Your Child to learn about Arithmetic.” *Understanding Fundamental Ideas in Mathematics at a Deep Level*. Accessed September 5, 2015. http://math2.uncc.edu/~hbreiter/CTI2015/MostImportantrev2.pdf

[2] see [1] above

[3] Common Core State Standards Initiative. “Standards for Mathematical Practice”. accessed October 15, 2015 www.corestandards.org/math/practice

[4] Peters, D. “Coping 101: Building Persistence and Resilience in Gifted Children.” *Califirnia Association for the Gifted.* 43.4 (2012). Accessed September 5, 2015. http://www.davidsongifted.org/db/Articles_id_10772.aspx .

[5] see [4] above

[6] Reiter, Harold . Private Conversation, Charlotte, NC. August 21, 2015

[7] Kathy Richardson. *Developing Number Concepts. Place Value, Multiplication and Division. (*New Jersey: Dale Seymore Publications, 1999.), xiii

[8] Bobby Ojose, “Applying Piaget’s Theory of Cognitive Development to Mathematics Instruction.” *The Mathematics Educator.* 18.1 (2008): 26-30

[9] see 8 above

[10] James Tanton, “Exploding Dots. 1.1 Base Machines”. Gday Math . “Accessed September 12, 2015. http://gdaymath.com/lessons/explodingdots/1-1-base-machines/.

[11] Reiter, Harold . Lecture, Fundamentals of Mathematics, Charlotte Teachers Institute. September 24, 2015

[12] Howe, Roger and Susanna Epp. “Taking Place Value Seriously: Arithmetic, Estimation, And Algebra.” 2008, http://www.maa.org.pmet/resources/PlaceValue_RV1.pdf . Accessed September 5, 2015.