Fundamentals of Mathematics: How Do Numbers Make Sense?

by Lisa Clark Ashworth, 2015 CTI Fellow
Barringer Academic Center

This curriculum unit is recommended for:
Mathematics and Language Arts in Grades 6-8

Keywords: mathematical problem solving, numbers, statistics, algebra, algorithms, probability

Teaching Standards: See Appendix 1 for teaching standards addressed in this unit.

Synopsis: This unit will be conducted with a mixed group of fourth and fifth grade students in a math class of seventeen students who are operating at two or more grade levels above fifth grade in their math activities. These students will learn many of their math concepts through inquiry-based activities in the classroom. They will gain a better understanding of the why and how math concepts have evolved and how they work in their everyday lives. The students will combine their current classwork of Pearson's *Connected Mathematics Grade 7* with new games and activities that correlate with the Math Common Core Standards for grades 6-8. This majority of this unit will take place over the first nine weeks of the school year; however, some of the competition activities will take place over a period of five months in the school year. The main purpose of this unit is to provide activities for students that provide a basic understanding of the fundamentals of mathematics as they connect to pre-algebra concepts.
I plan to teach this unit during the coming year in to 15 students in Horizons 4th and 5th grade, Mathematics and English Language Arts, grades 6-8.

Fundamentals of Mathematics: How Do Numbers Make Sense?

Don't Memorize! Analyze and Synthesize!

Lisa Clark Ashworth

Rationale

Mathematics can be defined as "the abstract science of number, quantity, and space."[i] Teachers often refer to mathematics as simply a study of concepts that deal with numbers. For many people who grew up in the twentieth century, the learning of mathematical concepts centered around the memorization of math facts that dealt with concepts such as place value, formulas for area and volume as well as the operations of addition, subtraction, multiplication and division. Often times in today's world, parents are signing their children up for auxiliary math classes that still focus on the memorization of math facts and formulas. While it is important for students to know their computational math facts such as multiplication, division, addition, and subtraction facts, there is an even greater need for students to also have conceptual knowledge about how mathematical concepts work in problem solving. For example, what happens when we (our students) multiply or divide fractions? What do we do with the denominators and numerators? How does this affect the product or quotient of this operation? Why should we believe the formulas of geometry if we are not given a rationale or proof for the formula? Why do have the Pythagorean Theorem? How are formulas used for in our everyday life? On one hand, we ask students to be skeptical about believing ideas that they are simply told to believe, so we should not blame them when they ask for a proof of a formula or algorithm. When students understand the "why" behind these mathematical concepts, this process will lead to greater problem solving ability.

The math curriculum for this instructional unit is closely tied to the seventh grade common core standards. In the area of math, students will investigate concepts ranging from analyzing number sense, to solving algebraic equations, to calculating ratios and percents, to calculating surface area, volume and perimeter of polygons. These mathematical concepts are part of the *Common Core Process Standards*[ii] of mathematics standards. This middle school math curriculum unit is based on the first four standards. The purpose of these standards is to have students gain a basic understanding of why and how certain mathematical concepts are used and interpreted in their everyday lives. By introducing students to the "how" and "why" behind mathematical concepts, this will give them a better foundation to further their math education in future school years as

well as strengthen their understanding of the real world application of these math concepts.

Introduction

Barringer Academic Center is an elementary school in Charlotte, N.C., for students in kindergarten through fifth grade. The students have a wide range of abilities. There are students who perform at a basic academic level all the way up to those who perform at a world class academic level. There are two levels of gifted education curriculum at Barringer. One program is a talent development program containing six gifted inclusion classes for third through fifth graders. The other gifted program, called the Horizons program, is a kindergarten through fifth grade academic program for students classified as highly gifted. The curriculum for the Horizons students is accelerated two to four years above their current grade level. The students participating in the fundamentals of mathematics curriculum unit are in the third through fifth grade of the Horizons program and a few students from the fifth grade talent development program. These students use advanced math curriculum which includes pre-algebra, algebra, and geometry concepts. The language arts curriculum is also advanced, and the oldest students in fifth grade read middle school level novels and write persuasive essays and other middle school level compositions. These students study and master many advanced concepts in fifth through ninth grade math, language arts and social studies curriculum.

Classroom Background

As one of four Horizons teachers at Barringer, I am responsible for twelve to fifteen students who are working in three different math textbooks with pre-algebra and geometry concepts that range from sixth through eighth grade level. These students are from third through fifth grade classrooms in the Horizons program. They received pre-assessments at the beginning of the school year in order to group them with similar students in their ability range. I am responsible for individualizing and differentiating each student's curriculum to fit his/her abilities. Some of the students are operating at two years above their current grade level, while others are four to five years above grade level. Essentially, my goal is to help students achieve a personalized learning situation within the classroom that best fits their needs.

Within my Horizons math classroom, I often have to plan for individual or paired math instruction for my students. I introduce new concepts in groups of two, three, or four students with oral instruction using a textbook, Smart Board, and manipulatives. Following this introduction to new concepts, students are "set free" to work at their own pace through the investigations of the Pearson Connected Mathematics[iii] textbooks. Students demonstrate their understanding through classwork investigations, peer tutoring, verbal observations, as well as formal assessments. Because students are working at their own pace, they need time and experiences with hands-on activities in order to explore the new concepts they are learning. It is also important that they have opportunities to create

new ideas and applications using these math concepts. They also need the opportunity to share and present these new ideas and applications within the classroom.

By incorporating creative problem solving activities and analysis into my mathematics' curriculum, students will have the opportunity to explore fun, creative ways to gain a better understanding of some middle school math concepts. Students will have the opportunity to see how math concepts such as algorithms, arithmetic, statistics, probability, fractions, ratios, tables, graphs as well as formulas that involve area and volume originate and apply to their everyday world. Specifically with arithmetic, students will be able to correlate the area model to their understanding of working with polynomials. In addition, by incorporating "writing and speaking" in math activities into their language arts curriculum, students will have the opportunity to extend and improve their language arts skills through their math conceptual understanding.

Content Background

Roger Howe, a mathematician, wrote an article in 2008 about the critical things that children should learn about arithmetic in order to have a strong understanding of mathematics. In this article he emphatically states, "the most important thing a student of arithmetic can learn is to think in terms of pieces."[iv] His argument for a basis of mathematical understanding is that students need to first learn place value of numbers by dividing the numbers into base ten pieces. Howe believes that students will have a better understanding of performing algorithms or operations if they break a number up into its base ten pieces. For example, an addition problem such as:

$$567 + 234 = 500 + 60 + 7 + 200 + 30 + 4 =$$

$$= 500 + 200 + 60 + 30 + 7 + 4 =$$

$$= 700 + 90 + 11 = 801$$

In this problem, students break apart the hundreds, tens, and ones places and then regroup them before adding them together. Students can also do this process for other operations such as subtraction, multiplication and division. Howe feels that students will be able to grasp how to perform these operations if they are grouped by size first instead merely looking at the number as a series of digits. This fundamental math unit will begin with a lesson on breaking up numbers in algorithms into base ten pieces. This beginning lesson, based on Roger Howe's article, will provide a basic mathematical background for the other activities in this unit.

Mathematical Content Objectives

The first common core standard that students will explore in this unit is (CCSS MP1) "make sense of problems and persevere in solving them." In the seventh and eighth

grade curriculum, students are asked to organize algebraic expressions to write and solve equations as well as analyze and build tables and graphs. Students will apply these skills in real world math problems especially in the units called *Variables and Patterns* and *Say it with Symbols*. These two units encourage students to analyze problems and find the patterns that help them "persevere" to solve problems. During this section of the unit, students will explore patterning concepts through activities such as examining palindromes. In addition to patterns, students will learn how to construct equations using variables and symbols in solving problems in *Math Olympiad, AMC 8 and 10* Competitions as well as in their middle school math real world practice problems. Overall, students will learn to identify correspondences between different approaches to solving problems.

The second process standard is (CCSS MP2) to "reason abstractly and quantitatively." Students will learn how to contextualize and decontextualize problems using and analyzing symbols to solve problems. Two primary units in seventh grade math that use this standard are *Comparing and Scaling* and *Stretching and Shrinking.* In these units, students construct ratios, proportions, and percents, compare shapes and compute scale factors as they apply to real world situations. This standard demonstrates to students how to apply these mathematical concepts to their everyday lives, as well as how they relate to the common core standard called *Model with Mathematics*.

Model with Mathematics (CCSS MP4) is where students will apply math to solve problems in everyday life, society, and the workplace. They will apply proportional reasoning to plan a school event or analyze a problem in the community. Students will delve into how functions are used to describe how one quantity depends on another. The goal of learning this standard is not to just incorporate mathematics, but to encourage students to use their writing, speaking and math skills to complete a Project Based Learning assignment where students use multiple core curriculum skills in creative problem solving. Students will be given a choice of projects to complete in this section of the curriculum unit.

The third common core processing standard (CCSS MP3) which encourages students to "construct viable arguments and critique the reasoning of others," complements the problem based learning assignment involved with *Model with Mathematics*. Students of all grade levels and abilities can learn from reading, listening, or constructing arguments and deciding if they make sense. Students will gain a better understanding of mathematical concepts by asking questions to clarify their arguments.

A common mathematical thread that I would like my students to gain a good understanding of is place value. I would like my students to better understand our base number system by completing activities that demonstrate the difference between rational and irrational numbers. I believe that a strong foundation of place value is a good way to make the distinction between rational versus irrational numbers. Students need to understand the properties of operations in order to practice and comprehend the order of

operations. Some students struggle with understanding how to order positive and negative integers. A common misunderstanding of students is the number sense behind adding, subtracting, multiplying and dividing negative numbers. By students gaining a better understanding of number sense and place value, they can apply this understanding to many common middle school math concepts of finding patterns, making predictions, and even extending predictions through solving linear equations and analyzing models.

The main objective of this mathematics curriculum unit is to help students gain a better understanding of basic principles behind common middle school math concepts. It will combine this mathematical understanding with research skills, writing skills, and speaking and presentation skills. It will provide the differentiation needed to meet the varying needs of developing creative problem solving skills in my students. Students will be able to explore the real world mathematical concepts involved in various activities while they are able to develop creative ideas and projects. This understanding of the fundamentals of mathematics will help students further develop their creative problem solving skills which is a necessary skill for success in life.

Writing and Speech Content Objectives

This unit will also help students improve their argumentative writing skills as they analyze and formulate their own opinions in their mathematical problem solving. Students will be asked to clearly compose a written argument based on their analysis of math problems. Students will use their written arguments in class discussions and in preparation for a culminating math project board for a math fair.

Classroom Strategies

This mathematical curriculum unit should take approximately four to six weeks to teach. It will be spread throughout the school year. It is important for students to have lots of hands on activities in order to adequately grasp the concepts associated with problem solving, statistics, probability, and number sense. Students also need hands on activities to understand the algorithm of going from a repeating decimal to a fraction. In addition to solving math problems, participating in class discussions, they will also have fun with math games and tricks. They will be provided with creative problems solving activities that will explore the math in terms of a variety of real world applications of mathematics. The first strategy in the teaching of this unit will be instruction of understanding place value in terms of “base ten pieces”. The second major strategy in the teaching of this unit will offering hands-on instruction opportunities. Students will conduct and participate in projects, games, cooperative learning, think-pair-share activities, as well as inquiry based instruction using creative problem solving. By utilizing background knowledge and multiple hands-on experiences, students will be able to connect the applicable concepts of math and language with their own experiences of mathematics in the real world.

Students will develop stronger critical thinking and problem solving skills as they progress through this unit. A critical aspect of problem solving is being able to give clear directions and steps involved in utilizing algorithms in math activities. The writing strategies in this math unit will help students improve their writing skills. I want students to increase their ability to succinctly summarize and analyze though the problem solving they attempt in mathematics. They will also use their writing, drawing, and speaking skills to help explain or illustrate a math concept that utilizes algorithms and problem solving concepts.

Classroom Activities

Place Value

Place value will be the first mathematical concept that students will be introduced to in this unit. Place value is one of the fundamental concepts in arithmetic. Dr. Harold Reiter, professor of mathematics at the University of North Carolina at Charlotte explains the fundamentals of place value in terms of scientific notation and the distribution property:

> The place value interpretation of 4273 is 4000 + 200 + 70 + 3, which is a sum of multiples of powers of 10. The relevant powers of 10 are $10^3 = 1000$, $10^2 = 100$, $10^1 = 1$. Each one has a coefficient or multiplier, 4, 2, 7, and 3 respectively. Thus $4 \cdot 10^3$, $2 \cdot 10^2$, $7 \cdot 10^1$, and $3 \cdot 10^o$ are multiples of powers of 10 and therefore 4273 is a sum of multiples of powers of 10. So place value notation means decimal notation in this case. Each of the addends in the expanded forms of a number will be called a single-place number. Thus for example $4 \cdot$
>
> 10^3 is a single place number. Once we learn how to do arithmetic with single place numbers, we can use that knowledge along with the distribution property of multiplication over addition, to do arithmetic with decimal numbers in general. This represents a key virtue of place value: it enables arithmetic computation. The fact that the basic arithmetic operations can be efficiently performed by effectively teachable algorithms was the reason that the place value system, which was only introduced into Europe in the late middle ages, supplanted the well entrenched system of Roman numerals.[v]

When teaching students about place value, it is important to show them multiple ways of organizing numbers to understand place value. For example, a teacher can demonstrate multiple examples of expressing place value through showing the distribution and commutative properties of multiplication and addition as well as using exponents and powers of 10. The following is an example problem of this place value math instruction:

> Find the product of $23 \cdot 41$, and recognize their place value number: of 23 =20 + 3 and 41 = 40 + 1. Then

$23 \cdot 41 = (20 + 3) \cdot (40 + 1)$

(1) $= (20 + 3)\ 40 + (20 + 3)^1$

(2) $= 20 \cdot 40 + 3 \cdot 40 + 20 \cdot 1 + 3 \cdot 1$

(3) $= 2 \cdot 10 \cdot 4 \cdot 10 + 3 \cdot 4 \cdot 10 + 2 \cdot 10 \cdot 1 + 3 \cdot 1$

(4) $= 8 \cdot 10^2 + 12 \cdot 10 + 2 \cdot 10 + 3 \cdot 10^o$

(5) $= 8 \cdot 10^2 + (10 + 2) \cdot 10 + 2 \cdot 10 + 3 \cdot 10^o$

(6) $= 9 \cdot 10^2 + 1 \cdot 10^2\ 2 \cdot 10 + 3 \cdot 10^o$

(7) $= 9 \cdot 10^2 + 4 \cdot 10 + 3 \cdot 10^o$

(8) $= 943,$

Where, we have used the distribution property of multiplication over addition in (1), (2), and (6); commutativity of multiplication and addition in (3) and (6); and place value notation in (6), (7), and (8).

	40	1
20	20 x 40 = 800	20 x 1 = 20
3	3 x 40 = 120	3 x 1 = 3

The box (above) illustrates how the distribution property works through this illustration of the "Area Model" for multiplication.[vi]

The previous explanations on place value are from Dr. Harold Reiter's website, and the following activities were designed by Dr. Harold Reiter. These activities help explain why digits that have a positive integer have different meanings depending on their position.

Example 1: Pick a three digit number. Multiply it by 7. Then multiply your answer by 11, and finally multiply by 13. Students should then explain why they got that answer.

Example 2: Next, consider the following problem. Find a four-digit number $\underline{abcd}$ which is reversed when multiplied by 9. In other words, find a,b,c, and d such that $9 \times \underline{abcd} = \underline{dcba}$.

Solution: The solution method is to reason digit by digit. First note that a = 1 since otherwise $9 \times \underline{abcd} \geq 9 \times 2000 = 18000$, which is a five-digit

number. Since a = 1, it follows that d = 9. Now the equation takes the following form: 9 x 1bc9 = 9cbl . This can be expressed as follows:

9 x (1009 + 100b + 10c) = 9001 + 100c + 10b

Next the number 9 is distributed

9081 + 900b + 90c = 9001 + 100c + 10b then goes to:

80 + 890b = 10c.

Since the right side 10c is at most 90 (c is a digit) , we can conclude that b = 0, and hence c = 8. Therefore abcd = 1089 is the only such number.

Example 3. The amazing number 1089. Start with your favorite two-digit number, reverse it and then subtract the smaller from the larger. Now pick a three-digit number like 742. Reverse it to get 247 and subtract the smaller of the two from the larger. Here we get 742 – 247 = 495. Now take the answer and reverse it to get 594, then add these two to get 495 + 594 = 1089. Finally, compute the product of 1089 and 9. You get 1089 x 9 = 9801. Isn't that odd, multiplying by 9 had the effect of reversing the number.

Here's the rationale for always getting 1089. First note, assuming a > c that $\underline{abc}$ - $\underline{cba}$ = 100(a – c) + c – a = 99(a – c) = 99d. Now suppose 99d = $\underline{u9v}$. Why is the middle digit of 99d always a 9? Is there a connection between u and v ? For d = 1 , we have udv = 099, so u = 0 and v = 9. For d = 2, u9v = 198. Again we see that u + v = 9. Check out what we get for each of d = 3,…,9. In each case u + v = 9. Thus u9v + v9u = 100(u + v) + v + u = 900 + 18 + 9 = 1089. [vii]

These place value activities are very helpful to use with students who are beginning the 7th grade math curriculum. The first unit in the *Connected Math Series* is entitled "Variables and Patterns."[viii] This unit incorporates the concepts of place value, basic algebra and arithmetic.

Place Value, Arithmetic, and Algebra

Repeated Subtraction

To see how to use repeated subtraction, first make a list of all the integer powers of 5 that are not bigger than the number we are given. In the case of 283, we need the powers 50 = 1*;* 51 = 5*;* 52 = 25*;* and 53 = 125. Next repeatedly subtract the largest power of 5 that is less than or equal to the *current number* (which changes during the process). So we have 283 = 125+158. At this point our current number becomes 158 and we repeat the

process. Then 283 = 125 + 158 = 125 + 125 + 33, and our current number is 33. Repeating the process on 33 gives 33 = 25 + 8 and incorporating that in the above gives 283 = 2 · 125 + 25 + 8 = 2 · 125 + 1 · 25 + 8. Continuing this with 8 leads to 283 = 2 · 53 + 1 · 52 + 1 · 51 + 3 · 50, which is a sum of multiples of powers of 5, just what we want. Thus 283 = 21135, just as we saw above. Repeated subtraction has two advantages over the repeated division method. First, it is closely related to the definition, hence it leads to a better conceptualization. Second, it can be used in other situations when repeated division cannot, as in the case of Fibonacci representation.[ix]

Repeated Division

The repeated division method requires that we repeatedly divide the given integer by base 5 and record the remainder at each stage. First we divide 283 by 5 to get 283 ÷ 5 = 56:6. We can interpret this as 283 = 5 · 56 + 3, so the *quotient* is 56 and the *remainder* is 3. Notice that the remainder can never exceed 5 since in such a case the quotient would have been larger. Next divide the quotient by 5 and record the new quotient and the remainder. Thus 56 = 5 · 11+1. Repeat the process with the new quotient 11 = 5 · 2+1 and finally, 2 = 5 · 0 + 2. Next write the remainders in reverse order, 2*;* 1*;* 1*;* and 3 to get 21135 as the base 5 representation of 283. You'll see why the order must be reversed in the following example.

Example 1. Repeated Division To see why 283 = 21135, we can repeatedly replace each quotient with its value obtained during the division process. Thus

$$
\begin{aligned}
283 &= 5 \cdot 56 + 3 \\
&= 5(5 \cdot 11 + 1) + 3 \\
&= 5(5(5 \cdot 2 + 1) + 1) + 3 \\
&= 5(5 \cdot 5 \cdot 2 + 5 \cdot 1 + 1) + 3 \\
&= 5 \cdot 5 \cdot 5 \cdot 2 + 5 \cdot 5 \cdot 1 + 5 \cdot 1 + 3 \\
&= 2 \cdot 53 + 1 \cdot 52 + 1 \cdot 51 + 3 \cdot 50 \\
&= 21135
\end{aligned}
$$

The advantage of repeated division is that it is computationally more efficient. Also, the method of justification can be applied in other situations (synthetic division and Euclidean algorithm). When we get to the section on fusing dots, you'll see in yet another way why it makes sense to record the remainders upon division by *b*.[x]

These two activities, repeated addition and repeated division, are particularly helpful in explaining the background concepts of algebra, arithmetic, and place value within the

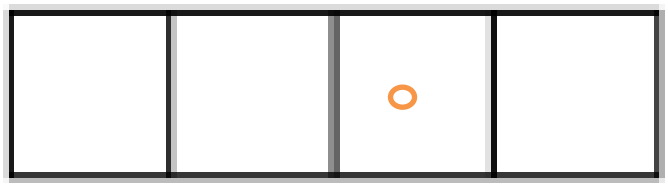
units of *Variables and Patterns* (Introducing Algebra), *Accentuate the Negative* (Integers and Rational Numbers), and *Moving Straight Ahead (Linear Relationships).*[xi]

Decimals, Exponents and Base 10

Fusing Dots Activity

The following activities are good ways of explaining how we work with decimals, fractions, exponents and Base 10. These activities are helpful for working with the middle school math units of *Variables and Patterns* (Algebra*), Accentuate the Negative* (arithmetic), and *Moving Straight Ahead* (Linear Relationships). These activities are also from Dr. Reiter's website.

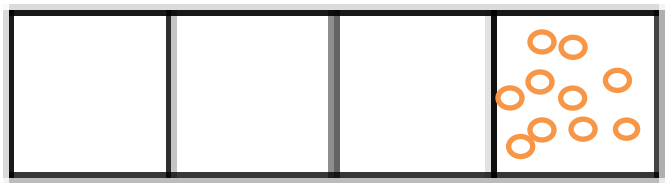
> Many thanks to Jim Tanton for the idea of exploding dots. We're going to explore several machines that enable us to represent positive integers and some other real numbers is some odd ways. Initially, we're given a two-way infinite tape with empty squares, with a heavy line (a bar) at one place on the tape: ⋅ ⋅ ⋅ : : :
> To represent a number *n*, we put *n* dots in the square just to the left of the bar, and let the machine go to work. This square, also called a box, is called the *unit box*. The following is an example using powers of 10:



Value of boxes = 10^3 10^2 10^1 10^0

(1000) (100) (10) (1)

Becomes …



Value of boxes = 10^3 10^2 10^1 10^0

(1000) (100) (10) (1)

The *Fusing Dots* activities are great methods to have students understand the concepts of decimals and exponents in a box format. Students will be able to visually work through the operations of multiplication and division using decimals and exponents as they group or "fuse" the dots together to represent groups of numbers such as ten or six. Fusing dots in groups of ten or six will help students gain a good understanding of the concepts of working with Base 10 and Base 6. Here is a link to this activity: http://math2.uncc.edu/~hbreiter/CTI2015/eFusingDots.pdf

Ratios, Proportions, and Three-Dimensional Measurement

ZomeTools Activity

In this activity, students will construct two-dimensional and three-dimensional figures and compare ratios and proportions that involve side-lengths, angles, and shapes. They will explore how to calculate and understand geometric formulas involving surface area and volume. These activities correlate with the seventh grade math units of *Stretching and Shrinking, Comparing and Scaling, and Filling and Wrapping*. These units contain real world problems involving three-dimensional measurement, understanding similarity, and ratios and proportions.

Teachers will need to gain access to *ZomeTools*[xii] for this activity. They can be purchased directly from the website http://www.zometool.com/products/design-3.html or from *Amazon.com*. *ZomeTool* kits consist of various length and color plastic struts and white balls with small openings in the shapes of rectangles, triangles, squares, and pentagons. Students will be asked to construct two, three, and four dimensional cubes and polygons with the tools. The teacher can purchase bubble solution or make it up from water and common dish soap. The teacher can also assign the different length and color struts different number values so that students can calculate the surface areas and volumes of the different figures that they create. When the students dip their geometric figure creations into a bubble solution, they can visually examine the full shape and surface area of the figure. They can also better visualize the comparison of side-lengths and angles. While the *ZomeTool* kit is a financial investment, it is an activity that students will enjoy repeating in the classroom. It is a fun way of encouraging creative problem solving in your math curriculum.

Creative Problem Solving Extension Activities

KenKen®[xiii] is a puzzle activity that I would use as a warm-up activity, a closing activity or just a "Fun Problem Solving Friday" activity in my classroom. Again, Dr. Harold Reiter has written an essay on the implementation and benefits of using *KenKen*® in the mathematics classroom to practice the four basic operations of arithmetic: addition, subtraction, multiplication and division. This activity also encourages students to develop mathematical strategies as they solve the puzzles, as well as it helps students

develop perseverance in solving math problems. Here is a link to the *KenKen*® website, http://www.kenkenpuzzle.com/?redirected=1# , to sign up for receiving puzzles that can be printed out or completed online. They are especially fun to solve together on a *SmartBoard*®. The following is a brief excerpt and explanation of *KenKen*® on Dr. Reiter's website:

> *KenKen*® is a puzzle whose solution requires a combination of logic and simple arithmetic and combinatorial skills.1 The puzzles range in difficulty from very simple to incredibly difficult. Students who get hooked on the puzzle will find themselves practicing addition, subtraction, multiplication and division facts. Specifically, for this paper, a standard *KenKen*® puzzle is an $n \times n$ grid divided into *cages* using heavy lines. Each cage has a mathematical clue that consists of a number and one of the four arithmetic operations, $+;\times;-;\div$. A *solution* is an $n \times n$ array of the numbers 1 through n such that no two of the same numbers appear in any row or column, and the clues are satisfied by the numbers in the cells of each cage. A number may be repeated within a cage, provided it is not in the same row or column. Any arrangement of the numbers from 1 to n satisfying the Sudoku-like requirement of non-duplication is called a Latin Square.
>
> Many teachers have found that KenKen has the potential to engage their weakest students, and those students learn two great lessons: first, they practice arithmetic without realizing it, and second, they develop the habit of persevering when they are unable to solve the puzzle immediately.

Students will love solving *KenKen*® puzzles, and teachers will find that students will ask to do them every day! You can also order *KenKen*® books with many different levels of problems to use in the classroom. Many teachers even enjoy solving themselves!!

Math Contests

Math Olympiad

MOEMS®[xiv] or *Mathematical Olympiads for Elementary and Middle Schools* is a mathematical competition for students in grades four through eight. Students are organized in teams of up to thirty-five students, and the competition runs from November to March each school year. Students can practice using copies of former Olympiad problems. They compete once a month with a five problem activity that encourages students to develop problem solving strategies while further their skills of critical and thinking and perseverance. Here is a link to the Math Olympiad website that teachers can use to sign up for their students : http://moems.org/ .

AMC 8 and 10 Contests

The American Mathematics Competitions, AMC 8, 10, and 12 are contests designed for high school students to help students develop strong mathematical strategies. They also encourage students to further develop their higher order critical thinking skills while encouraging them to persevere through problems. These contests are given one in the fall and once in the spring. Students can qualify for the U.S. Math Team through advancing through these competitions. Teachers who can find more information about these competitions from the website: http://www.maa.org/math-competitions .[xv]

Writing and Speech Activities

Final Project: Math Fair

Our school holds a science fair each school year where students research a problem of their choosing that is STEM based. They carry out the scientific method as they create and experiment to explore this problem. While students use mathematical data in their investigation of the problem, their projects are primarily scientifically based. In addition to the science fair, we have decided to hold a Math Fair at our school where the central focus of the Math Fair is a *problem-solving* fair. Unlike a Science Fair, a Math Fair is noncompetitive. The projects are student-centered and problem-based. Students will follow the guidelines and timeline traditionally followed for our Science Fair, in that they will identify a topic, conduct research, identify a problem and develop a hypothesis. The basic difference is that student's focus will be on manipulating concepts and numbers rather than physically experimenting to test their hypotheses (although this may be done).

Students may conduct tests, surveys and investigations that they have designed or they can try to replicate the work of other mathematicians. Students will be encouraged to centralize their focus of how mathematics fits into different careers. Areas of study may include subjects such as economics, architecture, engineering, cooking, interior design, sports, probability as well as traditional math investigations. For example, a student may want to work on understanding The Pythagorean Theorem and then use a data base of American Professional Baseball statistics from previous baseball seasons to test it out. Another example is a student who is exploring how *Euler's Theorem* works in his/her world. Another student may investigate how a chef or a caterer takes recipes and figures out the correct proportions to increase or decrease the amount of food that is prepared for different amounts of people.

Students will create tri-folds to display their findings and report their conclusions with a written paper and oral presentation that includes their research, data, connections to the real world and bibliography. These math projects will be presented within the classroom as well as on display at a math night organized for the week of "∏ Day" (March 14, 2016). This culminating activity will provide an assessment of a student's understanding

of a fundamental mathematical concept as they explain the concept in both oral and written form.

---

[i] *Webster's Online Dictionary*, last accessed on Oct. 29, 2015, http://www.merriam-webster.com/ .
[ii] *Common Core Standards*, last accessed on Oct. 29, 2015, http://www.corestandards.org/Math/ .

[iii] Glenda Lappan et al., *Connected Mathematics 2*: Grade Seven. Boston: Pearson Prentice Hall, 2009.
[iv] Roger Howe, "The Most Important Thing for Your Child to Learn About Arithmetic", Yale National Initiative (2015), accessed Oct. 29, 2015, http://www.teachers.yale.edu/curriculum/viewer/initiative_15.05.03_u .

[v] "Fundamentals of Mathematics", Harold Reiter's Home Page, last accessed Nov. 20, 2015, http://math2.uncc.edu/~hbreiter/CTI2015/ .
[vi] "Fundamentals of Mathematics", Harold Reiter.
[vii] "Fundamentals of Mathematics", Harold Reiter.
[viii] Lappan et al., *Connected Mathematics 2: Grade Seven*.

[ix] "Fundamentals of Mathematics", Harold Reiter.
[x] "Fundamentals of Mathematics", Harold Reiter.
[xi] Lappan et al., *Connected Mathematics 2: Grade Seven*.
[xii] ZomeTools, last accessed on Oct. 29, 2015, http://zometool.com/ .
[xiii] KenKen Puzzles, last accessed on Nov. 20, 2015, http://www.kenkenpuzzle.com/ .
[xiv] Math Olympiad, last accessed on Nov. 20, 2015, http://moems.org/ .
[xv] AMC 8, 10, and 12 Competitions, last accessed on Nov. 20, 2015, http://www.maa.org/math-competitions .

Appendix 1: Implementing Teaching Standards

Common Core Math Process Standards: These standards are used throughout all of the activities in this curriculum unit as well as in the Connected Math textbook work.

CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.

CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.

CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.

CCSS.Math.Practice.MP4 Model with mathematics.

Common Core Math Standards: These are standards used in the *Connected Math* Textbook and all the activities in this curriculum unit.

CCSS.Math.Content.7.RP.A.2 Recognize and represent proportional relationships between quantities.

CCSS.Math.Content.7.RP.A.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

CCSS.Math.Content.7.RP.A.2c Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

CCSS.Math.Content.7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

CCSS.Math.Content.7.EE.B.3 Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Common Core Reading/Writing Standards: These standards will be used in the final project presentation for the Math Fair.

CCSS.ELA-Literacy.W.7.2 Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.

CCSS.ELA-Literacy.SL.7.4 Present claims and findings, emphasizing salient points in a focused, coherent manner with pertinent descriptions, facts, details, and examples; use appropriate eye contact, adequate volume, and clear pronunciation.

CCSS.ELA-Literacy.SL.7.5 Include multimedia components and visual displays in presentations to clarify claims and findings and emphasize salient points.

Bibliography, Resources, and Reading List

"American Mathematics Competitions." American Mathematics Competitions. Accessed November 20, 2015. http://www.maa.org/math-competitions.
This site is for teachers to access and enroll their students in the American Mathematics competitions.

"Bing." Dictionary Webster Mathematics -. Accessed September 20, 2015.
This is a bibliography resource for a definition of math in this unit.

"DAILY CHALLENGEor Select Puzzle Type and DifficultyPuzzles Update at Midnight EST." KenKen Puzzle Official Site. Accessed November 20, 2015. http://www.kenkenpuzzle.com/.
This site is for teachers to access to print puzzles for their students or refer their students to solve them online.

"Harold Reiter." Harold Reiter's Home Page. Accessed November 20, 2015. http://math2.uncc.edu/~hbreiter/.
This site is a wealth of knowledge for teachers to access in order to find pedagogy and a wide variety of math resources to use in any classroom. Harold is a professor of Mathematics at UNC-Charlotte, N.C. Harold Reiter's Home Page is a site that many teachers will love and will refer to many times for help with providing quality resources for the math classroom.

Howe, Roger. "The Most Important Thing for Your Child to Learn About Arithmetic." Lecture, Abstract, March 1, 2015.
This article is a great resource for teachers to read for helping students learn arithmetic. This article is found on Harold Reiter's Home Page.

Lappan, Glenda. *Connected Mathematics 2*. Boston, Mass.: Pearson, 2009.
This is a textbook for seventh grade students that correlates with the Common Core Standards.

"Math Olympiads for Elementary and Middle Schools." Math Olympiads for Elementary and Middle Schools. Accessed November 20, 2015. http://moems.org/.
This site is for teachers to enroll their students in Math Olympiad.

"Standards for Mathematical Practice." | Common Core State Standards Initiative. Accessed November 20, 2015. http://www.corestandards.org/Math/Practice/.
This site is for teacher's to access the Common Core Process Standards and Objective Standards for Mathematics.