



Math Reasoning and Vocabulary Integration with KenKen®

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Highland Creek Elementary School

This curriculum unit is recommended for:
3rd and 4th Grade Mathematics

Keywords: Math Reasoning, Problem-Solving, KenKen®

Teaching Standards: See [Appendix 1](#) for teaching standards addressed in this unit.

Synopsis: This unit focuses on developing math reasoning and problem solving skills that many students lack when entering fourth grade. My goal is to provide students with activities to increase their ability to think logically and confidently as they are presented with different tasks to solve. This unit will use KenKen® puzzles to enhance mathematical reasoning, especially those involving integer arithmetic. As students engage rigorous activities with KenKen® puzzles, they will develop and enhance their ability to problem solve and use reasoning skills.

I plan to teach this unit during the coming year to 17 students in Fourth Grade.

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Introduction

Math reasoning is defined by The National Council of Teachers of Mathematics as “a productive way of thinking that becomes common in the process of mathematical inquiry and sense making.” Students need to be given the opportunity to explore math while making sense of their conjectures. I have experienced that students are successful with basic computation of all four operations. However, the moment these operations are embedded into puzzles or word problems, students are less likely to be successful. The creation of the unit is to help students better understand and use the core operations as they practice math reasoning to solve puzzles and word problems this should increase student engagement and add rigor to daily instruction.

Background Information

Currently, I teach fourth grade math, science and health at Highland Creek Elementary School. Our school is the true essence of a neighborhood school nestled on the rolling hills of a community golf course. Daily we have students walk and bike to school. The average bus ride for our students is about five minutes from school to home. The majority of our students come to school prepared to learn. As of last year, Highland Creek was one of the largest elementary schools in our district. The 2015-2016 school year welcomed a decrease in numbers, as our relief school was opened. The ethnic diversity of school is 46 % white, 40% African- American, 3% Multi-racial, 6% Hispanic and 5% Asian. This is a vast difference in diversity since the schools inception in 2008. We have homogeneous Talent Development classes at each grade level starting in second grade. Students in grades three through five are taught in departmental teaching pairs. My personal observation is that due to the recent decline in reading proficiency in our district and nation, many students are falling behind in mathematics concepts as the instructional need for additional reading instruction has increased between grades three and fourth. Read to Achieve initiatives require that by third grade students are no longer learning to read but reading to learn. This requires that students are given additional support to master reading prior to entering fourth grade. Students that have reading deficiencies also display math reasoning delays and enter fourth grade below benchmark.

I teach two blocks of mathematics, science and health daily. My third block of math is comprised of students who need additional support with identified standards based on district or unit math assessments. Block one is a more advanced group of 29 learners that are considered on grade level according to state assessments. Block two learners consist of seventeen students blended with Exceptional Children (EC), those certified with learning disabilities and 504 plans. There are twelve boys and five girls in block two. I will instruct my curriculum unit with this group. According to district cycle assessments, North Carolina End-Of-Grade Test and MAP (Measures for Academic Progress) data in reading and math, these students are considered below grade level. Students in block two are diligent workers who need to be taught at slower pace and require additional time to practice skills. However, skill mastery is necessary for these students to successfully transition into the application of core concepts embedded in math reasoning and problem solving. Due to their inability to effectively comprehend, math reasoning and problem

solving is significantly affected. When given instruction that is intentional and rigorous, these students will show growth by the end of the year.

Content Objective

My objective for this unit is to help students acquire reasoning and problem solving skills to improve their understanding and increase their skill in arithmetic. Twenty first century learners need to understand the relevance of the content they are learning. Students will increase their achievement when the learning has purpose. This unit will correlate with Common Core standards for fourth grade and what has been identified as behaviors of mathematically proficient students in arithmetic.

Common Core standards for this unit will include both literacy and math. It is important to integrate as much literacy with math as we continue to develop students in both content areas. The literacy standards used will be more of an application of writing responses in math as students justify and defend their solutions. Literacy standards require students to use accurate spelling and punctuation. Using math journals will connect math and writing standards as students are given ample time to process their learning. KenKen puzzles will challenge students as they combine integer arithmetic and writing.

Chapter four of, *“Putting the Practices into Action”*, explains the effective use of teaching students to create viable arguments in math by using models and vocabulary to justify their mathematical thinking. In this unit, as students work with KenKen puzzles, they will apply many behaviors of mathematically proficient learners. As a result, students should increase their ability to reason and problem solve. Mathematically proficient students demonstrate eight behaviors that have been proven to increase academic performance and achievement. These behaviors are:

1. Make Sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Math Reasoning and Vocabulary Integration

A great way to increase student engagement and math reasoning is through the use of puzzles and games. When given an enjoyable task like a game or puzzle, students’ ability to strategize is increased greatly. My students are more focused when the announcement of a scavenger hunt or

puzzles are introduced as one of the activities for the day or workshop. Teachers should capitalize on these moments as student participation and rigor are increased. Student's ability to problem solve is triggered as they create self-motivated inquiry based learning practices to each presented task.

As the facilitator of instruction, while students are manipulating various puzzles or game based activities, the teacher should use a series of higher order thinking questions. When posing questions, teachers need to navigate students to apply previous and current skills while connecting content specific vocabulary as they process the task. This increases students beyond rote memorization of a vocabulary and concepts to actually applying each. When students apply learning they make a significant association to the purpose of the content. In the article, *Supporting MATH Vocabulary Acquisition*, Bay-Williams and Livers wrote:

“As students explore mathematics through a meaningful context, they are able to derive meaning of abstract mathematics concepts and see that mathematics is an integral part of their world. Although context can support learning, the connection between context and a mathematical idea is not inherent in the mathematics and must be carefully developed by the teacher.”

These authors continue sharing several important questions that mathematics educators need to consider in building lessons that enhance and provide rigorous instruction:

1. Is the context culturally relevant?
2. Is the context familiar or new?
3. Can the context be reused?
4. When do I provide support?

The above questions, when answered, create intentional lesson planning that promotes not only the success of mathematics language, but also the success of mathematics achievement which should be the basis for every lesson taught.

I am learning that many strategies and methods used to teach students who are English Language Learners also work for students who have learning disabilities or simply need instruction at a slower pace. Understanding the importance of mathematics language to develop proficiency with applying skills and concepts is essential to developing and extending concepts from one grade to next. In order for students to construct viable arguments and critique the reasoning of others, predetermining vocabulary terms or concepts that foster rich classroom discussions will ensure that students are deepening their understanding of math.

This unit will combine the use of KenKen puzzles and mathematics vocabulary to increase student math reasoning and application of concepts. The activities have been developed to encourage “talking” and writing in math, as students listen to conjectures and make reasonable judgements on their viability.

KenKen® Puzzles

The creator of KenKen¹ is Tetsuya Miyamoto, a math educator in Japan. He developed KenKen to help his students develop their logic and arithmetic skills. His teaching method and use of KenKen have proven to be very successful with increasing student achievement. During a seminar session, Dr. Harold Reiter of UNC-Charlotte, introduced KenKen puzzles as a way to increase student reasoning and problem solving. This was the one evening I felt mathematically successful. As the lecture continued, I pondered how KenKen puzzles would assist my students with becoming proficient mathematicians while increasing their reasoning skills. Each puzzle varies in its' degree of difficulty as they provide repetition of the addition, subtraction, multiplication and division.

Dr. Reiter explains,

“KenKen® is a puzzle whose solution requires a combination of logic, simple arithmetic and combinatorial skills. The puzzles range in difficulty from very simple to incredibly difficult. Students who get hooked on the puzzle will find themselves practicing addition, subtraction, multiplication and division facts. In the standard $n \times n$ KenKen puzzle, the numbers in each heavily outlined set of squares, called cages, must combine (in any order) to produce the target number in the top corner of the cage using the mathematical operation indicated. Each of the numbers 1 to n must appear in each (horizontal) row and each (vertical) column. A number can be repeated within a cage as long as it is not in the same row or column.”

16×		7+	
2-			4
	12×	2÷	
		2÷	

KenKen puzzles are similar to Sudoku; however, there are some qualities that separate them. Fault lines are shown as heavy lines throughout that separate the puzzle into a pair of rectangles and create smaller more manageable puzzles. I have included this video clip to explain how to solve KenKen puzzles. I would suggest that you complete several KenKen puzzles before sharing with students. This will give you a general idea of expected student behaviors that may

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occur such as frustration, lack of perseverance, or quick solutions that don't work. KenKen puzzles vary in their degree of difficulty, so differentiate as needed to support student learning.

<https://youtu.be/vbmohVTtDgs>

Teaching Strategies

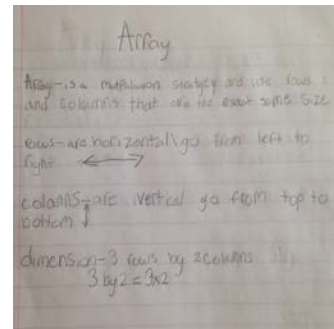
Due to the composition of students in my class, explicit direction instruction and repetition of practice will be prevalent throughout the unit. Students will be given multiple opportunities to practice and grow from each task. My goal is to assist students with being successful at their pace and level of understanding.

Whole Group Modeling

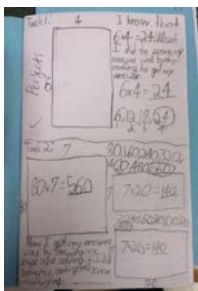
I generally began each lesson with warm up or review that leads directly into to the lesson of the day. Students are able to focus on the given task to get them prepared for task being presented. Depending on the activity, students are engaged with the anticipation of the lesson of the day. Students may be given a question of the day, video or task to begin the lesson engagement.

Math Notebook

For several years, I have used math notebooks with my students to record vocabulary, examples and practice problems for students to have as a reference for completing nightly assignments. I have learned over time that using a three or five subject notebook will span over the entire school year and provide continual support for students as we transition to new concepts. Students often will state that they have forgotten how to perform a math task and I will refer them back to their math notebook. It is a great reference to keeping concepts at student's finger tips and assessment reviews. The notebook builds with each major theme taught in fourth grade, place value, multiplication, divisions, geometry and fractions.



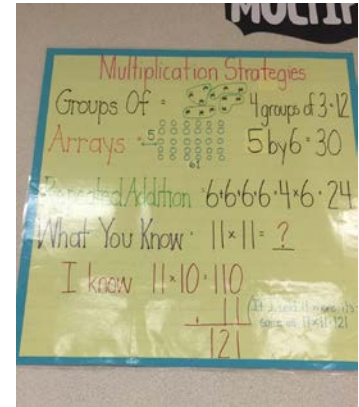
Math Journal



Math journals are an effective tool for students to process their mathematics ideas and explain their responses by using detailed writing. This differs from the math notebook because students are applying math vocabulary as they write about math. Math journals encourage students to slow down and think about the math. Most students think that math is just numbers but as we implement and integrate common core standards in all subject matters, it is important that students incorporate as much writing in math as possible. Students need to see the "writing" in math and what better way to do so than to build a journal.

Anchor Charts

Literacy circles have long used anchors to help frame student thinking as content is being delivered. While promoting literacy in classrooms, teachers are able model their thinking process aloud while recording the content, strategy, cues and guidelines during the learning. Posting anchor charts keeps current learning accessible to students and helps them make connections to prior material. Students are reminded to constantly refer to the charts as they answer questions, explore ideas, or contribute to class discussions and problem-solving. The rules and guidelines will be written on an anchor for students to quickly reference during the KenKen activities. KenKen puzzle of the day will be put chart paper for students with identified vision difficulties can easily access learning.



Math Tool Kit

Students who need support and immediate feedback as lessons progress with difficulty will have access to manipulatives in tubs such as counters, base ten blocks, scratch paper and graph paper to assist with completing the task. Calculators will be included in the tubs for students to use after they have completed the task. The use of calculators must be thoroughly explained and practiced while the lesson is being modeled.

Peer Evaluations

Provide students with guidelines or rubrics to critique each other's work and provide feedback to improve student work products. Students should be thoroughly taught this procedures and how to give and receive critiques. This is a great way to empower students to self-reflect and monitor someone else's work.

Vocabulary Cards

Posting content appropriate vocabulary words and cards throughout the room on word walls or anchor charts. It is important that students have access to content specific vocabulary to help them process new or old learning. Students need to experience more written text in math as much as possible.

Classroom Lessons/Activities

Activity #1 Introducing KenKen puzzles (1-2 days)

Materials: "How to Play" anchor chart, Math Notebook, KenKen Website (puzzle of the day for classrooms) <http://www.kenkenpuzzle.com/teachers/classroom>

Pre Planning: Create a KenKen puzzle of the day free account. KenKen puzzles will be accessible online and compatible for multi-media boards and technology. According to the website, all puzzles are available for free printing.

Session One: When introducing KenKen puzzles it is imperative to thoroughly explain the rules. Ask students if they are familiar with playing Sudoku. Explain that today we will learn a similar puzzle that helps with problem solving. Provide students with “The Rules” worksheet. Create an anchor chart with students listing the rules of play. The anchor chart should include the following:

1. Choose a grid size.
2. Fill in the numbers from 1 to grid size.
3. Do not repeat a number in any row or column.
4. The numbers in each heavily outlined set of squares, called cages, must combine (in any order) to produce the target number in the top corner mathematical operation indicated.
5. Cages with just one square should be filled in with the target number in the top corner.
6. A number can be repeated within a cage as long as it is not in the same row or column.

Start with a 3x3 puzzle to solve. Focus on one cage to model for students while referring back to the anchor chart for the rules. In this two minute video clip, students will be given the basics for completing a puzzle. After showing the video clip, allow students to ask questions as you guide through completing the KenKen puzzle.

<https://www.youtube.com/watch?v=q2Un71kE2Xs>

2-		2
2÷	3÷	
	1-	

Repeat this process until you have solved the puzzle as a class or an individual student has completed the puzzle. If that occurs, allow that student to share how they solved the puzzle. Have students glue the completed puzzle inside their math notes as a reference. Provide students

with a practice puzzle to take home. Have a variety of puzzle options for the math ability of your students. For students who solve their puzzles quickly provide them a 4X 4 or 5 X 5 for practice.

Activity #2 “Cage Designers” (1day)

Materials: 3X3 Completed grid, 3X3 Blank Grid 2 per student, Black markers, Math Notebook,

Session Two: Today’s lesson will begin by solving a 4X4 puzzle as a group. Use the KenKen website to have access to free and printable puzzles. You will notice that as you solve puzzles directly from the web, there are options to play while being timed. The more you engage your class with KenKen, use the timer to increase rigor and atomicity of rules. Once the puzzle has been solved, identify how certain cages were completed and the operations used to complete each cage. Tell students that today they will become “Cage Designers”.

Provide each student with a 3X3 completed grid. Note that each grid is completed with digits one through three on each row and column but are void of any math operations. Model for students how to partition the grid into cages until the grid is complete.

Option A

1	2	3
3	1	2
2	3	1

Option B

1	2	3
3	1	2
2	3	1

Option C

1	2	3
3	1	2
2	3	1

Explain that the operations and target numbers are missing from the each cage. Show students all the options for each operation and target number. Be sure to include options that will not work. Allow students to orally share reasons why or why not the options work. Once you have discussed each cage, show students where to place the target number with its operation.

Option A. Target number is 6; Operation is addition = $1+2+3$

Option B. Target number is 6; Operation is multiplication = $1 \times 2 \times 3$

Option C. Target number is 1; Operation is subtraction = $3-2$

Continue modeling options for each cage above until you work together to completed each cage. Then provide students with a blank 3X3 grid. Have students place the digits 1through 3 in each row and column. Remind them they should not repeat a digit in the same row or column. Then using black markers, students will partition the grid into cages. Students should have 3-4 cages per grid. Explain that students will work to fill each cage that they created with a target number and operation. Have student’s double check their math to ensure it works.

After students have completed filling their 3X3 grid with cages, operations and target numbers, they should glue what is now a puzzle into their math notebooks. Provide them with another blank 3X3 grid to copy their puzzle on. Now have students exchange puzzles with a friend and solve each other's puzzle.

Differentiate for students who may complete the 3X3 grids quickly by providing them with 4X4 grids to complete this activity again.

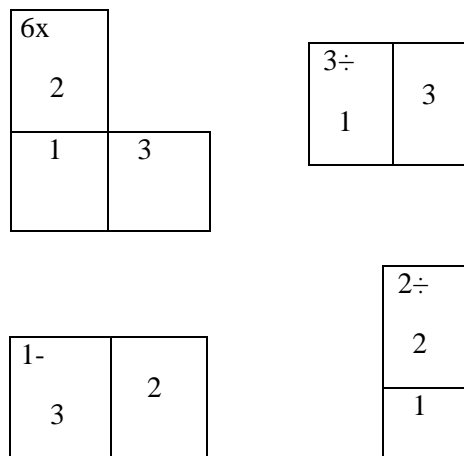
Activity #3 Creating Puzzles (2-3 days)

Materials: Student Groups, Pre-cut Solved Cage sections, Math Notebook, Math Journals, Math Tool Kits

This lesson may take several days depending on your level of students and the mathematical diversity in your class.

Session Three: Today's lesson will begin by solving a 4X4 puzzle as a group. Use the KenKen website to have access to free and printable puzzles. You will notice that as you solve puzzles directly from the web, there are options to play while being timed. The more you engage your class with KenKen, use the timer to increase rigor and atomicity of rules. Once the puzzle has been solved, identify how certain cages were completed and the operations used to complete each cage. Tell students that today they will become "Puzzle Designers".

Use 5 solved 3X3 puzzles and cut along the darkened cage lines. Each 3X3 puzzle will give four cage sections. By cutting 5 different puzzles, there should be 20 cage sections for a class size of 20 students. Provide each student with a pre-cut solved cage section. In the example below, these pre-cut cages are solved. Students should understand that $2 \times 1 \times 3 = 6$; $3 \div 1 = 3$; $3 - 2 = 1$; $2 \div 1 = 2$.

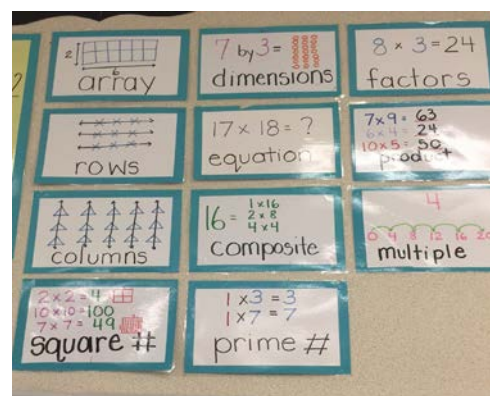
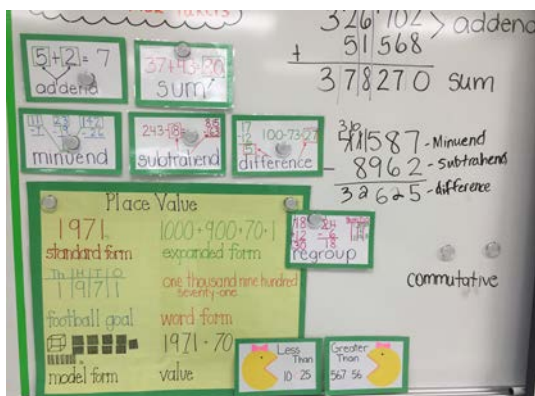


Once each student has a cage section, they must work with the other students at their table to see if their cages can connect. Allow sufficient time for students to try to connect each cage at their tables. As students make a connection, they should remain at that table. Signal to students who have not made a connection to move to another table and work to connect their cage. Students should continue this process until they have found the other three students or puzzle

partners that complete their puzzle. Students must follow all the rules for solving a KenKen puzzle. As students work to find their “puzzle partners”, remind them that the digits 1, 2, and 3 may only occur once in each row and column. Monitor students by listening and providing feedback as they discuss the rationale for their cage pieces to work successfully. Though this may seem as a simple activity, the fact that 5 puzzles are being put together at once will create an environment of rich mathematical conversation and problem solving. Students will need sufficient time to complete all puzzles. They may become frustrated with this process based on their level of fluency with adding, subtraction, multiplying and dividing.

After students have found their “puzzle partners”, have them work as a team to glue the cages together onto construction paper. Students should copy their completed puzzle into their math journals. Then process the activity by explaining how they were able to solve their puzzle by writing in their math journals. Provide students with a list of math terms that are posted on an anchor chart or word wall for easy reference. These words should also be recorded in student math notebooks as a reference when writing. Remind students that mathematicians use appropriate mathematic vocabulary when recording their responses. The vocabulary may include:

Addend, sum, minuend, subtrahend, difference, factor, product, multiple, dividend, divisor, quotient



Anchor charts and vocabulary while teaching.

Vocabulary posted after the unit has been taught.

Reinforce appropriate writing expectations with complete sentences and proper punctuation. Pair students and have their partner evaluate their cage and written response. During the peer evaluations, provide students with a rubric to focus their critique. Each partner should make sure the cage works and the written response is sensible. They should share recommendations and assist each other with making corrections.

Wrap up the lesson by allowing volunteers to share their cage and written responses. Encourage struggling students to share their accomplishments and if they need support when sharing make sure they have a “buddy” beside them for support.

Lesson # 4 KenKen Teachers (1-2 days)

Pair with teacher that does not use KenKen in their classrooms. Allow your students to teach other students how to play KenKen puzzles in a small group. Before sending students out to other classrooms, have them practice their “teaching” skills with a parent as a homework assignment. This will assist each student with making clear statements with a precise set of rules when teaching others. This could be a reward for students or a way to increase the leadership for the students.

Activity 2

3X3 Grid

1	2	3
3	1	2
2	3	1

Activity 2

4X4 Grids

Activity 3

Puzzle 1

3+	4+	
2	1	3
1	8+	1-
	3	2
3	2	1

Puzzle 2

5+		9x
1	2	3
2	3	1
3	2÷	
3	1	2

Puzzle 3

2x 1	2	7+ 3
5+ 2	3	1
3	1	2÷ 2

Puzzle 4

1- 2	3	6+ 1
3÷ 1	2	3
3	2÷ 1	2

Puzzle 5

2- 3	2÷ 1	2
1	6x 2	3
1- 2	3	1

Peer Critique Rubric

Name: _____ Date: _____		
Assignment: _____		
	Yes	No
Does the student show math work that is clear and easy to follow?		
Is your partner's work correct? If not, what's the mistake?		
Can you identify specific math vocabulary used?		
Does your partner use capitalization and punctuation correctly?		
State one thing you liked about your partner's work?		
Name suggestion that your partner should consider?		
Name:		

Appendix 1: Implementing Teaching Strategies

CCSS.MATH.CONTENT.4.OA.A.1- *Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.*

As students enter fourth grade, many have difficulty with mastering multiplication which foundation for most of the yearly instruction. Students need to be proficient with understanding how numbers are multiplied in groups and how equations can be written and interpreted. The use of beginning level KenKen puzzles will help develop fluency with multiplication. As students become more proficient with multiplying single digit numbers, they will increase their reasoning ability with two-digit by one digit multiplication problems.

CCSS.MATH.CONTENT.4.OA.A.2-*Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.*¹

Students have been taught this year, when solving for the unknown variable, the variable can be represented as a letter, symbol or even an empty space. KenKen puzzles have empty spaces for students to reason and problem solve using all operations of multiplying, dividing, adding and subtracting.

CCSS.MATH.CONTENT.4.NBT.B.4-*Fluently add and subtract multi-digit whole numbers using the standard algorithm.*

As students transition into fourth grade, the use of standard algorithm is required with adding and subtracting. KenKen puzzles reinforce this standard as students use a variety of conjectures to test while adding and subtracting to complete each puzzle. Depending on the puzzle represented, students may solve puzzles that are one operation or multiple operations which increases the difficulty to the task.

CCSS.MATH.CONTENT.4.OA.C.5

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

CCSS.ELA-LITERACY.W.4.1.B

Provide reasons that are supported by facts and details

Mathematically proficient students need to make viable arguments when justifying their responses. Using math facts and details of math work to reason and solve problems is supported writing math responses.

Annotated Bibliography

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Kosko, K. (n.d.). What Students Say About Their Mathematical Thinking When They Listen. *Sch Sci Math School Science and Mathematics*, 214-223. This article shares the importance of allowing students to have math conversations while completing a task or solving a problem. It also encourages students to be able to effectively listen to each other and value the thoughts of others as they work cooperatively.

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Will Shortz Presents Kenken by the Seaside 100 Easy to Hard Logic Puzzles That Make You Smarter. (2009). Griffin. Will Shortz introduces this KenKen puzzle book by sharing about the creator of KenKen and how he used the puzzles with his students. It also explains how to play KenKen followed by 100 puzzles from easy to difficult to solve.