Fractals: Beauty and Art in Science and Mathematics

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This curriculum unit is recommended for:
Fifth and Sixth Grade Math and Science Courses

Keywords: fractals, Mandelbrot, Siepinski, Koch

Teaching Standards: See Appendix 1 for teaching standards addressed in this unit.

Synopsis: The topic of my unit is fractals and as a 6th grade math and science teacher, I was initially challenged to find ways to have this topic blend with both disciplines. As I learned more and more each day about Mandelbrot and the theory and formula for fractals that he developed in 1979, the integration of this concept into math and science seemed organic – it is already math, science, and art combined.
I plan to implement a six-part unit (Introduction to Fractals, Fractals in Geometry, Fractals in Science and Nature, Science in the Fine Arts, Interdisciplinary Integration of Fractals) using Common Core standards and objectives, in addition to incorporating the six levels of Bloom's Taxonomy throughout the lessons.
As a teacher and a life-long student, in addition to my thirst for knowledge, I have a love of the arts – literary, theatrical, and visual. Also, as someone who enjoys and appreciates nature, exploring these in unexpected and non-traditional ways through mandated curriculum this has been an opportunity to enhance my life culturally and rejuvenate my spirit.

I plan to teach this unit during the coming year in to **30** *students in* **6th grade Math and Science Courses**.

Fractals: Beauty and Art in Nature and Geometry

Melanie Mowry

Content Background

My curriculum unit is built around the subject areas of science and mathematics. The focal point of my study is the beauty of fractals found in art and nature, discovered in the field of science by Benoit Mandelbrot in 1979. Mandelbrot's academic background, which extends from Europe to the United States is impressive. He was born in Warsaw in 1924 and moved to Paris when he was twelve years old. [1] He received his undergraduate degree at the Ecole Polytechnique, from 1945-1947; his Master of Science in Aerosciences in 1948; and his Ph.D. in Mathematics in 1952. [2] He traveled in Princeton, Geneva and Paris as a visiting scientist and then, in 1958, moved to the United States. [3] While in the United States, he worked at IBM as a research staff member and IBM Fellow (1974), before taking a position at Harvard University as professor of Mathematics (1984). [4] It was here, by integrating and implementing his extensive and impressive knowledge in science and mathematics, that he was led to his discovery and development of the concept of the Mandelbrot Set and fractals in 1979.

The word "fractal" comes from the Latin word *fractus*, meaning "fractured" or "broken." This is to say that the edges of a fractal bent or twisted. [5 Benoit] Mandelbrot is responsible for creating and using the term "fractal" to describe the self-similarity in the concept, unlike previous mathematicians who described this phenomenon as abnormal. [6] Fractals, in addition to being aesthetically pleasing to most, is unrequitedly rooted in many disciplines of mathematics. There is an unmistakable intertwining of geometry, algebra, linear algebra, analysis, measure theory and many more. [7]

Rationale

> "Why is geometry often described as cold and dry? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline or a tree. Clouds are not spheres, mountains are not cones, coastlines are not circles and bark is not smooth, nor does lightning travel in a straight line**……**Nature exhibits not simply a higher degree, but an altogether different level of complexity. The number of distinct number of patterns is for all purposes infinite.
>
> The existence of these patterns challenges us to study those forms that Euclid leaves aside from being formless, to investigate the morphology o the amorphous. Mathematicians have distained this challenge, however, and have increasingly chosen to flee from nature by devising theories unrelated to anything we see or feel." [8] (Benoit Mandelbrot)

The integration of this concept into math and science seems organic, as it is already math, science and art combined. Organic also applies, as fractals subsist of biological and mathematical components. In addition to implementing this six-part unit using standards and objectives, I also intend to incorporate the six levels of Bloom's Taxonomy throughout the lessons. Part I will be an introduction to fractals and research into the biography and work of Benoit Mandelbrot. Parts II, III and IV will include three tiers: Knowledge, Application and Evaluation. Part V will be a reflection on the entire unit. The approach I plan to implement in my unit will in five parts.

Fractal Unit in Five Sections

Part I: Introduction to Fractals
Part II: Fractals in Geometry
Part III: Fractals in Science and Nature
Part IV: Fractals in the Fine Arts
Part V: Exploration of the Integration of Fractals

Part One – Introduction to Fractals

Most students are hooked by visual media, which includes the digital world, as it is so prevalent in their lives already. The first part of my unit is three-fold and includes video, literature and note-taking. The first day of the unit will be a viewing of a clip from a seminar by Mandelbrot himself, entitled Fractals and the Art of Roughness [9] and Fractals: Hunting the Hidden Dimension. [10] This episode of a television series follows the story of innovative mathematicians who delved more deeply into Mandelbrot's discovery from the sixties and seventies and made it relevant to today's world - applicable to everything from heart machines, Saturn's rings and college level mathematics. For this reason, I felt this would be a great introduction to the study of fractals for grade six. (CCSS.ELA-Literacy.RST.6-8.9 - Compare and contrast the information gained from experiments, simulations, video, or multimedia sources with that gained from reading a text on the same topic.)

In addition to the video, my plan is to read aloud from a wonderful, age-appropriate text entitled The Adventures of Penrose, The Mathematical Cat. [11] Including various topics of math, including one entitled "Penrose meets the Fractal Dragon," [12] this book ventures into the whimsy that can be a part of math and science, all while serving educational purposes. Each four-description and lesson is followed by a set of topic related problems to solve. Reading the book aloud and working the problems will be the culmination of the literature aspect of the introduction.

Finally, the students will be shown a more detailed definition, such as the one described and defined by Marywood University.

Definition of a fractal

"A fractal is "a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole," a property called self-similarity. The term fractal was coined by Benoit Mandelbrot in 1975 and was derived from the Latin fractus meaning "broken" or "fractured." A mathematical fractal is based on an equation that undergoes iteration, a form of feedback based on recursion. A fractal often has the following features:

- It has a fine structure at arbitrarily small scales.
- It is too irregular to be easily described in traditional Euclidean geometric language.
- It is self-similar (at least approximately or stochastically).
- It has a Hausdorff dimension which is greater than its topological dimension (although this requirement is not met by space-filling curves such as the Hilbert curve)
- It has a simple and recursive definition." [13]

They will also be given a description of fractals and the life and work of its discoverer, Benoit Mandelbrot, via a power point presentation. They will begin a journal in a composition notebook, which they will begin by taking notes on the presentation. (CCSS.ELA-Literacy.RST.6-8.2 - Determine the central ideas or conclusions of a text; provide an accurate summary of the text distinct from prior knowledge or opinions.)

Lesson Plan I: Introduction to Fractals

Vocabulary: fractals, Benoit Mandelbrot

Time Allotment: 90 minutes

Materials: Spiral bound notebooks, pencils

Essential Question: "What is a fractal?"

Common Core Essential Standards: CCSS.ELA-Literacy.RST.6-8.9; CCSS.ELA-Literacy.RST.6-8.2

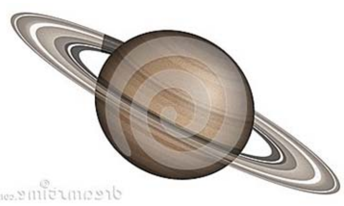
Introduction: The students will be shown clips of Benoit Mandelbrot's seminar Fractals: Hunting the Hidden Dimension, produced by NOVA/PBS in 2008 as a preliminary guide to the understanding of the basics of fractals. Although the level of the seminar is academically advanced, the content of the clip is appropriate and the students will also be giving the opportunity to put a face with a mathematical concept.

Content: The students will be guided through the fundamentals of fractals and how they appear in the world. The students will be asked to have a spiral bound notebook dedicated to this unit. The various notes and written products will be recorded in this notebook. The content of the lesson will center on the idea that a fractal begins with an object, like a triangle or point, which is constantly being altered following a mathematical rule an infinite amount of times. The will be introduced to the idea that the rule can be described in words or formulas.

Summary: The story, "The Shapes Convention," from the book Fractals, googols and other mathematical tales, by Theoni Pappas, will be read aloud to the students.

Independent Practice: Students will be asked to summarize the idea of fractals in a three to five sentence paragraph in their Fractal Journal.

Examples of the Fractals in Today's World




Part Two – Fractals in Geometry

In Part Two, the students will be shown and learn about how fractals occur in mathematics, specifically geometry (knowledge), recreate Sierpinski's Triangle [14] and Koch's Snowflake [15] and create their own fractals (application) and reflect on the implications of fractals in architecture, engineering and graphic design (evaluation.)

First, students will be shown in geometry how fractals can be produced and how they occur. There are several mathematicians throughout history who have touched on fractals. Cantor (1845-1918), a German mathematician responsible set theory; [16] Blaise Pascal (1623-1662), a great scientist and mathematician, who was thought to be the discoverer of Pascal's triangle. [17] It was in fact a Chinese version had already been published in 1303; [18] Giuseppe Peano (1858-1932) and David Hilbert (1862-1943), who were both noted in history for their work with curves. [19]

> "If I have seen farther than others, it is because I have stood on the shoulders of giants." [20] (Sir Isaac Newton)

These mathematicians and scientists delved deep in their studies and for the purposes of a sixth grade unit, we will only cover the two most commonly recognizable works and discoverers. These are Waclaw Sierpinski (1882-1969), who developed the "Sierpinski gasket" [21] and Helge von Koch, who introduced what is now known as the "Koch curve." [22] There are several video clips which explain and demonstrate how these concepts are produced. Before beginning a guided construction of their own, the students will watch "The Case of the Missing Fractals" [23] and answer the questions from the quiz afterwards and "Fractals." [24]

Next, students will be shown the slides that accompany the book, Fractals in the Classroom by Heinz-Otto Peitgen, which allow the students to see stationary fractals, as opposed to animations. I feel this will be beneficial as the students begin to construct their own. The first construction will be of the Sierpinski gasket, or as we are going to focus on, the Sierpinski Triangle. Students will be given two different, contrasting colors of construction paper and be asked to trace and cut out a triangle from the template provided. They will also be given a handout of six stages of constructing a Sierpinski triangle. They will measure the center triangle, which is exactly one-fourth of the original triangle and cut it out on the second color of construction paper. (CCSS.Math.Content.6.G.A.4 - Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.)

The class will be divided into two groups: the first group will be students who can work independently using the original template and handout, and the second group will be comprised of a group of students who need additional support in the form of my direct assistance. After completion, the students will share and then the separate groups will work on one of two projects: the independent group will attempt the construction of the Sierpinski Carpet and the second group will continue to work with me on either another triangle or a guided carpet (depending on level of success.) This series of steps will also be followed with the creation of the Koch curve (guided and independent groups) and the Koch snowflake or carpet (independent groups.)

Finally, the students will be allowed to investigate from an assortment of library books from various institutions and in the computer lab, the way fractals and architecture are related. They will also be allowed to access the computer program, FRAX [25] or Coolmath.com, [26] if available, due to district-wide controls of internet program accessibility. If available, they will also be allowed to print their creations. (TT.1.3 – Select appropriate technology tools to present data and information effectively, as in multimedia, audio and visual recording, online collaboration tools, etc.)

Lesson II: Fractals in Geometry

Vocabulary: Sierpinski's Triangle, Siepinski's Carpet, Koch Curve, Koch Snowflake

Time Allotment: 270 minutes

Materials: Spiral bound notebooks, pencils, rulers, scissors, colored paper, templates

Essential Question: "How do fractals occur in geometry?"

Common Core Essential Standards: CCSS.Math.Content.6.G.A.4; CCSS.TT.1.3

Introduction: The students will be introduced to this lesson by examining the works of Cantor, Pascal, Peano and Hilbert, Sierpinski and Koch and specifically on how their studies influenced the work of Mandelbrot and fractals. The students will take notes and include the link that each mathematician had to Mandelbrot.

Content: After viewing a slide show which accompanies Fractals in the Classroom, [27] the students be asked to concentrate on two of the most familiar examples of fractals occurring in a more elementary form. These include Sierpinski's Triangle and Koch's Snowflake. The content of the lesson will include video animations of Sierpinski's Triangle [28] and Koch's Snowflake, [29] including how these are made and illustrations of each one.

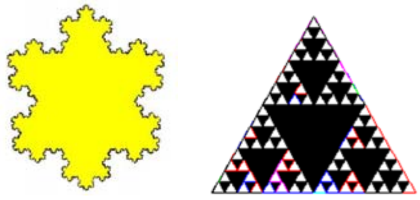
Guided Practice: The students will be given two, contrasting colors of construction paper, handouts of the six steps of the Sierpinski Triangle, scissors, rulers and a template. Students will be guided through the steps of constructing the beginnings of the triangles by tracing the template provided on one color of construction paper and cutting out the triangle created. The students will then divide into two groups.

Differentiation: The students will work in one of two groups, based on the level of understanding and execution of the project. Group one will continue to work with guided assistance and group two will work independently on the project. After completion, the students will share and then the separate groups will work on one of two projects: the independent group will attempt the construction of the Sierpinski Carpet and the second group will continue to work with me on either another triangle or a guided carpet. This series of steps will also be followed with the creation of the Koch curve (guided and independent groups) and the Koch snowflake or carpet (independent groups.)

Summary: The students will be asked to share their products with the class and what they have learned about fractals as it relates to mathematics. A copy of the handout and samples of their products will be affixed to the pages of their Fractal Journals.

Independent Practice: The students will be encouraged, but not required, to investigate the creation of their own fractals through computer programs such as FRAX or Coolmath. The computer lab will be reserved for the following day for the continuation of their investigations or, for students without internet or computer access, an opportunity to explore these programs.

Examples of Fractals in Mathematics



Part Three – Fractals in Science and Nature

Perhaps one of the most fascinating aspects of fractals is how they appear in nature. For Part Three, the students will again begin with the knowledge phase and through guided discovery, will learn how fractals naturally occur in science. They will apply hands-on lab of the dissection and observation with the naked eye and through a microscope, of samples of cauliflower, broccoli, ferns and sea-shells. Finally, they will evaluate their findings by sketching and highlighting the fractals they observed.

To begin, the lab partners will be provided with microscopes, two pieces of sketching paper and samples of broccoli, cauliflower, ferns and seashells. They will start off the lab by folding the pieces of their sketching paper in to four sections. In the left hand boxes on each of the two sheets, they will title and sketch the samples as they appear to the naked eye. They will make observations and share with their partner different types of fractals they see, if any. Underneath each sketch, they will describe their findings and predict by hypothesizing what they will find under the microscope for each sample. (CCSS.ELA-Literacy.RST.6-8.3 - Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.) (CCSS.ELA-Literacy.RST.6-8.8 - Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.) (CCSS.ELA-Literacy.RST.6-8.9 - Compare and contrast the information gained from experiments, simulations, video, or multimedia sources with what was gained from reading a text on the same topic.)

Next, the students will place each item in turn under the microscope and sketch what they see in the corresponding boxes on the right side of the two pieces of sketch paper. Again, they will describe their findings and determine and write about whether or not their hypothesis was correct. One way to evaluate their findings will be to emphasize the portion of the whole that was discovered to be the fractal in each of their samples with multicolored highlighters.

The last step of this process will also be aesthetic, as in other parts. Students will be allowed to explore and investigate through multi-media, including computers, textbooks, library books and an outdoor exploration, to find examples of fractals in science and nature. A few excellent resources are *Elegant Chaos,* [30] *The Beauty of Fractals: Six Different Views,* [31] and *The Beauty of Fractals – Images of Complex Dynamical Systems.* [32] (CCSS.ELA-Literacy.RST.6-8.2 Determine the central ideas or conclusions of a text; provide an accurate summary of the text distinct from prior knowledge or opinions.)

Lesson III: Fractals in Science and Nature

Vocabulary: fractals, recurring patterns, hypothesis, the scientific process

Time Allotment: 180 minutes

Materials: Spiral bound notebooks, pencils, multicolored highlighters, sketch paper/graph paper, microscopes, and small samples of ferns, sea-shells, broccoli, and cauliflower

Essential Question: “How do fractals occur in science and nature?”

Common Core Essential Standards: CCSS.ELA-Literacy.RST.6-8.3; CCSS.ELA-Literacy.RST.6-8.8; CCSS.ELA-Literacy.RST.6-8.8; CCSS.ELA-Literacy.RST.6-8.2

Introduction: The students will be introduced to this lesson by reviewing the mechanics and operation of the microscope.

Content: The students will be asked to select a lab partner and will be given all of the materials necessary for the experiment. Each student will divide both pieces of sketching paper into four sections or boxes. These will be used for writing and drawing their findings. The students will sketch in the four boxes on the left hand sides of the paper what the samples look like without the aid of magnification. Independently, they will write down in words in their journals what they have observed and make note of any fractals they may notice. At this point, they will also formulate a hypothesis about what they may observe after magnification. The students will then take turns looking at the samples under the microscope and in the corresponding right boxes, will again draw what they observe. They will record their findings in their journals and determine if their hypothesis is correct.

Guided Practice: The students will be asked to guide their partners through their discovery process and share thoughts, conjectures, sketches, and conclusions about the experiment.

Summary: The students will be asked to share their discoveries and/or drawings with the class and what they have learned about fractals as it relates to science and nature. Their sketches and findings will be included and affixed to the pages of their Fractal Journals.

Independent Practice: The students, through various media, will look for and describe, make a copy, provide a sample of, or sketch an example of a fractal in nature or science that was not previously discussed.

Examples of fractals in science and nature

Part Four – Fractals in the Fine Arts

Also fascinating is how fractals can be translated into art, music, poetry – and many other fine arts. Part IV will be a cultural approach to fractals as students move through the three tiers. They will be exposed to paintings, sketches, sculptures and musical and lyrical pieces that include fractal elements. They will learn how artists in various mediums develop and create fractals and integrate them into works of art and will also create their own artistic product from any artistic discipline. After presenting these to the class, they will identify the fractal component, provide feedback and ask questions about the other students' products.

If possible, I would truly love to take the students on a field trip to the Mint Museum. However, with that being a slim possibility, I will attempt to bring the museum to them. I intend to prepare a power point presentation that will include works from Yale, a phenomenal collection of fractal art from one of our affiliated universities, Yale University. [33] (CCSS.6.V.2.3 Understand that original imagery is a means of self-expression used to communicate ideas and feelings.) The power point will also include excerpts from *Mathematics and Music: From Pythagoras to Fractals,* [34] (CCSS.6.ML.1.3 - Recognize expressive elements, such as dynamics, timbre, blending, and phrasing, of music), Raymond Flood and Robin Wilson and their website on fractals, [35] and a fractals in poetry activity. [36]

Lesson IV: Fractals in the Fine Arts

Vocabulary: integration, Fine Arts

Time Allotment: 90 minutes/three days for independent project

Materials: Spiral bound notebooks, pencils, PowerPoint presentation

Essential Question: "How do fractals occur in art, music and literature?"

Common Core Essential Standards: CCSS.6.V.2.3; CCSS.6.ML.1.3

Introduction: The students will be introduced to the lesson by listening to the instrumental composition "Dueling Banjos." [37]

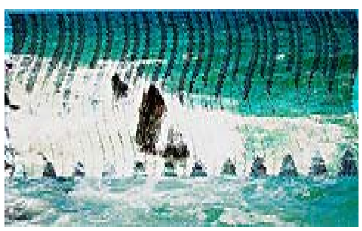
Content: The students will be led in a discussion on how the musical piece listened to in the introduction relates to fractals. The discussion will continue as the students follow along through a Power Point presentation of collections of paintings, sketches, musical compositions and musical and poetic lyrics. The students will continue in their journals with note taking and examples from each medium.

Guided Practice/Independent Practice: The students will assigned a project that will include integrating fractals into an original artistic creation from any medium. They may choose to work independently, with a partner or in a group of no more than four people.

Summary: The students will be asked to share their artistic products with the class on the due date and what they have learned about fractals as it relates to the fine arts. The original, a copy or a description of their piece will be included and/or affixed to their journal.

Examples of Fractals in Art

Examples of Fractals in Music



Examples of Fractals in Poetry

The Sail of Ulysses (Canto I)

If **knowledge** and thing **known**
are one
So that to a man is to be
That man, to **know**
a place is to be
That place, and it seems to come to that;
And if to **know** one man is to **know** all
And if one's sense of a single spot
Is what one **knows** of the universe,
Then **knowledge** is the only life,
The only sun of the only day,
The only access to true ease,
The deep comfort of the world and fate [38]

My intention with this part of the unit is to introduce the students to the beauty of fractals and in doing so, hopefully cultivate a love of the arts as well. After viewing the power point presentation, we will spend five minutes in silence, reflecting their perceptions of beauty in various mediums of the fine arts. I will ask the students to then write a paragraph describing their thoughts and feelings about the things they saw and heard. Finally, for this portion of the activity, the students will choose from the various forms in the fine arts discipline, from the ones given in the power point as examples or any other example in the fine arts, and create their own masterpiece of fractals. This will be a take-home project and on the due date, students will present their products to the class and identify the fractals in their own pieces. The culmination of this unit will be providing positive feedback and asking questions about their classmates' finished work.

Part Five – Exploration of Interdisciplinary Integration of Fractals

As the unit comes to a close, the students will summarize and will be asked to choose a method of reflection (essay, poem, song, oral presentation, poster, etc.) on how fractals occur in various places in their core class. They will contemplate and communicate through a chosen medium how fractals can be considered artistic and academic and how fractals occur and impact the world around us.

Lesson V: Fractals in the World

Vocabulary: integration

Time Allotment: 90 minutes

Materials: Spiral bound notebooks, pencils, media center

Essential Question: "How do fractals occur in other ways around us?"

Common Core Essential Standards: 6.V.2.3

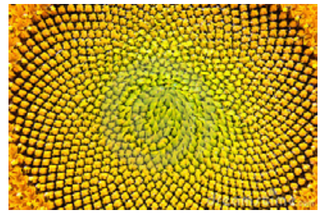
Introduction: The students will be introduced to the lesson by a discussion of the variety of other ways fractals can occur in the world (ex: architecture.)

Content: The students will be led in a discussion on how fractals can occur anywhere, including their everyday lives at school.

Guided Practice/Independent Practice: As a culminating activity and as the conclusion of the unit, the students will asked to choose any medium and using the knowledge gained in from the unit, reflect in a one page essay and produce a piece that illustrates their understanding of fractals on an academic and aesthetic level.

Summary: The students will be asked to share their reflection and products with the class on the due date and what they have learned about fractals as it relates to the world around them. The original, a copy or a description of their piece and their written reflection will be included and/or affixed to their journals.

Examples of Fractals in the World around Us



Conclusion

I truly enjoyed the experience of putting together this unit. I have learned so much more than I thought possible about the concept of fractals. The exploration of this new concept to me has enhanced my knowledge and appreciation for the subjects I teach. This experience also solidifies my connections with importance of integrating education and the arts and I believe that the marriage of fractals and nature, science, the fine arts and mathematics is simply organic. I am thankful for this opportunity.

Appendix 1: Implementing Teaching Standards

CCSS.ELA-Literacy.RST.6-8.9 - Compare and contrast the information gained from experiments, simulations, video, or multimedia sources with that gained from reading a text on the same topic. I will implement this standard in the Lesson I introduction as the students watch a seminar clip via the internet and reading from notes about Mandelbrot.

CCSS.ELA-Literacy.RST.6-8.2 - Determine the central ideas or conclusions of a text; provide an accurate summary of the text distinct from prior knowledge or opinions. This standard will be addressed in Lesson I as the students read, listen and take notes in a spiral-bound notebook known as their Fractal Journal. They will also implement this standard in Lesson III when they record the reflections about the lab experience in their journals as well.

CCSS.Math.Content.6.G.A.4 - Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. The students will be engaged in constructing the Sierpinski Triangle and Koch Curve in Lesson II and thus will be integrating this standard into their products.

CCSS.TT.1.3 – Select appropriate technology tools to present data and information effectively, as in multimedia, audio and visual recording, online collaboration tools, etc. In Lesson II, as independent practice, the students will use a computer program to create their own fractal.

CCSS.ELA-Literacy.RST.6-8.3 - Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. As students perform their lab experiments with samples of fractals in nature with microscopes in Lesson III, they will be expected to follow the scientific process.

CCSS.ELA-Literacy.RST.6-8.8 - Distinguish among facts, reasoned judgment based on research findings, and speculation in a text. In Lesson III, students will make a hypothesis about what they expect to find in their experiments and share the reflections and observations written in their journals with their lab partners.

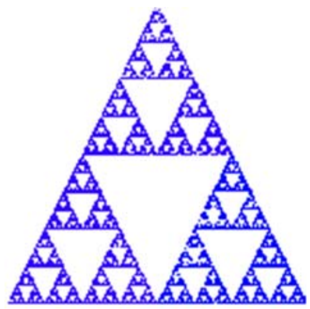
CCSS.6.V.2.3 - Understand that original imagery is a means of self-expression used to communicate ideas and feelings. This standard will be addressed in Lesson IV; the students will view a power point presentation which includes many pieces of art from many different mediums. If resources permit, the students will also take a field trip to the Mint Museum to experience art first hand.

CCWSS.6.ML.1.3 - Recognize expressive elements, such as dynamics, timbre, blending, and phrasing, of music. In the introduction of Lesson IV, students will experience the musical composition, “Dueling Banjos,” and will analyze how music may contain fractals, too.

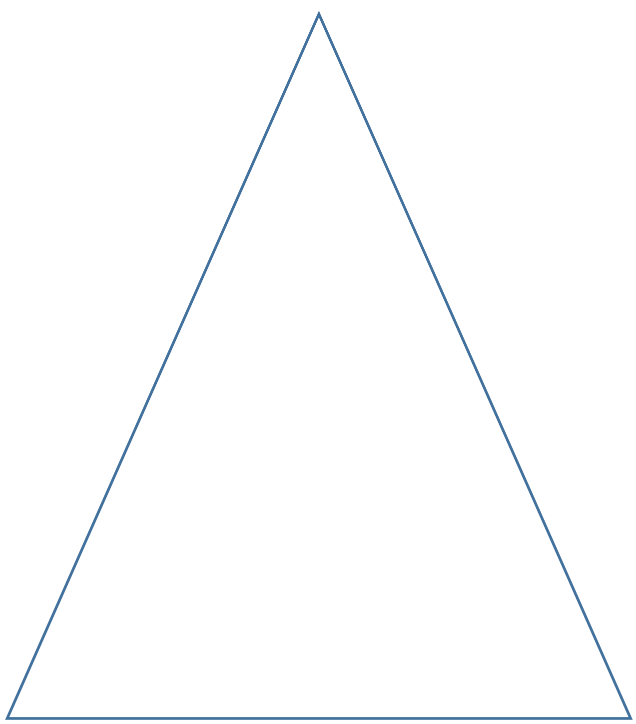
Materials for Classroom Use

<u>Original Handout for the Sierpinski Triangle Project</u>

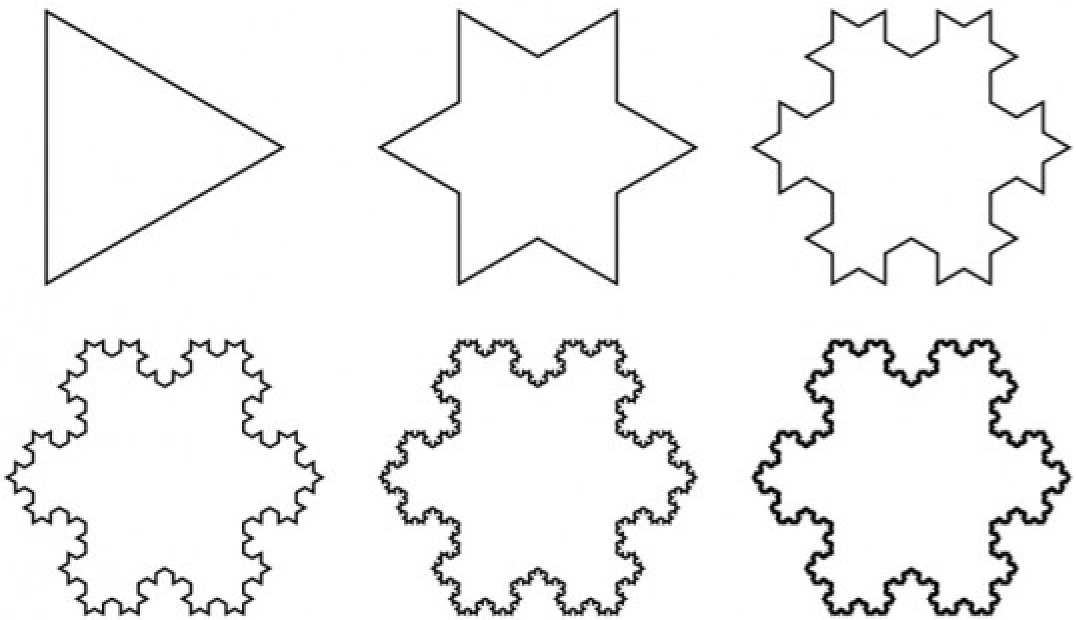
The Sierpinski Triangle



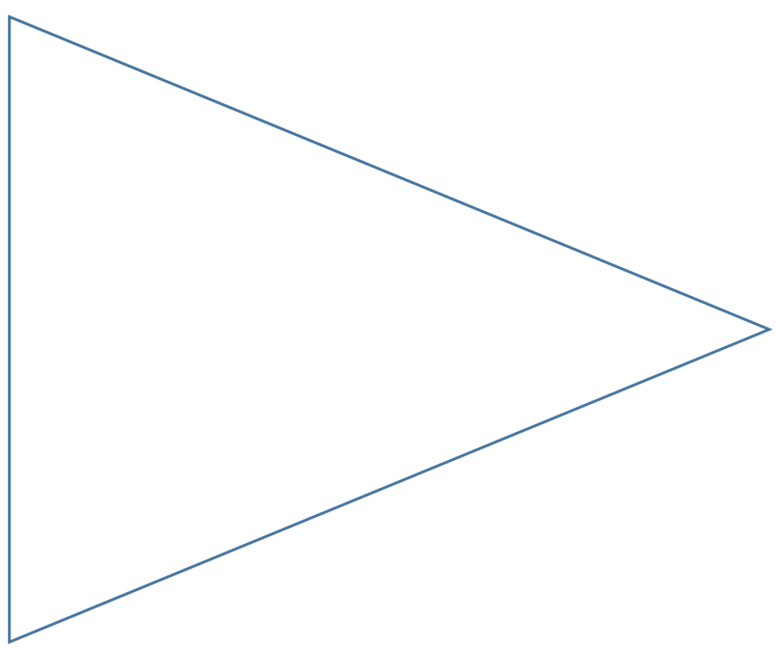
TEMPLATE



Original Handout for Koch's Snowflake Project



TEMPLATE



Bibliography for Teachers

Sprott, Julien Clinton. *Elegant Chaos-Algebraically Simple Chaotic Flows.* Singapore: World Scientific Publishing Co., 2010. This is a very complex book of algebraically formulas as they relate to fractals. I chose this book for the pictorial example of a fractal on page fourteen.

Helmburg, Gilbert. *Getting Acquainted with Fractals.* Berlin, Germany: Walter de Gruyter GmbH & Co., 2007. I chose this book for the inordinate amount of beautiful, colorful graphics illustrating a multitude of various fractals. The algebraic formulas for the formation of fractals is too complex for grade six; however, the colored illustrations are excellent resources. The sequencing of the Koch Curve illustrated in multi-colored format on pages twelve-seventeen is especially easy to follow.

Petigen, Heinz-Otto, Hartmut Jurgens, and Dietmar Saupe. *Fractals for the Classroom.* New York: Hamilton Printing Co., 1992. This book is an excellent resource, in both the writing and pictures. Fractals from all facets of the world and mathematical drawing board are presented in a reader-friendly format and the pictures and illustrations are exquisite.

Peitgen, Heinz-Otto and P. H. Richter. *The Beauty of Fractals-Images of Complex, Dynamical Systems.* Berlin, Heidelberg: Springer-Verlag, 1986. A beautifully illustrated and photographed book, complete with one hundred, eighty four figures, many in color. Another excellent resource for pictorial examples of fractals.

Gulick, Denny and Jon Scott. *The Beauty of Fractals-Six Different Views.* Washington, DC: The Mathematical Association of America, 2010. This resource provides the reader with a view of fractals from the world of science and nature. Illustrations of fractals in plant leaves and ferns and the plant cycle (page eight and ten.) An extremely photogenic view a mountain ridge beside a lake, entitled "Fractal Mountains" is especially awe inspiring.

Pappas, Theoni. *Fractals, googols and other mathematical tales.* California: World Wide Publishing/Tetra, 1993. This light-hearted children's book is entertaining, as well as educational. The book follows the antics of Penrose, the mathematical cat, as he explores mathematical concepts, including, but not limited to, decimals, fractals and tangrams. The story, "The Shapes Convention" is an amusing tale about fractals.

Lauwerier, Hans. *Fractals-Endlessly Repeated Geometrical Figures.* Princeton, New Jersey: Princeton University Press, 1991. This is a well-written book geared toward a wide audience, which is rich with illustrations and color pictures. According to the author in his preface, 'Everyone will find something to his liking."

Feder, Jens. *Fractals.* New York: Plenum Press, 1988. This is a complex, formula ridden and driven work, on an academic level much higher than grade six. I chose this book primarily for two pages: page sixteen, the six step process of constructing triadic Koch curve; and color plate pullouts from the sleeve on the back side of the front cover (in particular, the one entitled "fractal landscapes presented as clouds.")

Barnsley, Michael. *Fractals Everywhere.* San Diego, California: Academic Press Inc., 1988. This book contains many color plates of fractals in nature, as well as in art. It also magnifies the pictures in order to better visualize the parts of the whole.

Crownover, Richard M. *Introduction to Fractals and Chaos.* Boston, Massachusetts: Jones and Bartlett Publishers, 1995. I chose this book and after a brief review of the text, I found that it, too, is far too advanced for most middle school students; however, I found the most beautiful color plate of the Mandelbrot set I have seen to date and, for this reason, I kept this book in my list of resources.

Fauvel, John, Raymond Flood, and Robin Wilson. *Music and Mathematics-From Pythagoras to Fractals*. Oxford, New York: Oxford University Press, 2003. This book shows how music and mathematics are integrated and how fractals can be present in the fine arts. It is not an easy read, but there are sections that are very beneficial when exploring inter-disciplinary integration.

Peitgen, Heinz-Otto, Hartmut Jurgens, Dietmar Saupe, Evan Maletsky, Terry Perciante, and Lee Yunker. *Fractals for the Classroom-Strategic Activities Volume One.* New York: Springer-Verlag New York Inc., 1991. I chose this book for the stages of the Sierpinski's Triangle on pages twenty-four through twenty-eight. In addition, there is a comparison of Pascal's triangle and Sierpinski's Triangle on page twenty-nine. I am also including this book in my list of resources for the slides that are included within the pages.

Reading List for Students

Pappas, Theoni. *Fractals, googols and other mathematical tales.* California: World Wide Publishing/Tetra, 1993. This light-hearted children's book is entertaining, as well as educational. The book follows the antics of Penrose, the mathematical cat, as he explores mathematical concepts, including, but not limited to, decimals, fractals and tangrams. The story, "The Shapes Convention" is an amusing tale about fractals.

Helmburg, Gilbert. *Getting Acquainted with Fractals.* Berlin, Germany: Walter de Gruyter GmbH & Co., 2007. I chose this book for the inordinate amount of beautiful, colorful graphics illustrating a multitude of various fractals. The algebraic formulas for the formation of fractals is too complex for grade six; however, the colored illustrations are excellent resources. The sequencing of the Koch Curve illustrated in multi-colored format on pages twelve-seventeen is especially easy to follow.

Peitgen, Heinz-Otto, Hartmut Jurgens, Dietmar Saupe, Evan Maletsky, Terry Perciante, and Lee Yunker. *Fractals for the Classroom-Strategic Activities Volume One.* New York: Springer-Verlag New York Inc., 1991. I chose this book for the stages of the Sierpinski's Triangle on pages twenty-four through twenty-eight. In addition, there is a comparison of Pascal's triangle and Sierpinski's Triangle on page twenty-nine. I am also including this book in my list of resources for the slides that are included within the pages.

Campbell, Susan C. and Richard P. Campbell. *Mysterious Patterns-Finding Fractals in Nature.* Homesdale, Pennsylvania: Boyds Mills Press, 2014. This book for grades two through five is a fascinating photographic journey through fractals in nature. It is visually engaging and one of the few age-appropriate books on fractals to be found.

Falconer, Kevin. *Fractals-A Very Short Introduction.* United Kingdom: Oxford University Press, 2013. This elementary introduction to fractals is very student-friendly and includes dozens of beautifully illustrated and photographed fractals. Falconer does an excellent job of taking a complex concept and adapting it to make the information accessible to sixth graders.

Lesmoir-Gordon, Nigel, Will Rood and Ralph Edney. *Fractals-A Graphic Guide.* London: Icon Books Ltd., 2013. This book describes the basic concepts of fractals in the style of a comic book/graphic novel, as the title suggest. For this reason, I feel it will be most appealing to students and as a bonus, it covers a great deal of material about fractals.

Notes

[1]Peitgen, Heinz-Otto and P. H. Richter. *The Beauty of Fractals-Images of Complex, Dynamical Systems.* Berlin, Heidelberg: Springer-Verlag, 1986.
[2]Peitgen, Heinz-Otto and P. H. Richter. *The Beauty of Fractals-Images of Complex, Dynamical Systems.* Berlin, Heidelberg: Springer-Verlag, 1986.
[3]Peitgen, Heinz-Otto and P. H. Richter. *The Beauty of Fractals-Images of Complex, Dynamical Systems.* Berlin, Heidelberg: Springer-Verlag, 1986.
[4] Peitgen, Heinz-Otto and P. H. Richter. *The Beauty of Fractals-Images of Complex, Dynamical Systems.* Berlin, Heidelberg: Springer-Verlag, 1986.
[5]Gulick, Denny and Jon Scott. *The Beauty of Fractals-Six Different Views.* Washington, DC: The Mathematical Association of America, 2010.
[6]*Fractal Explorer*. http://www.fractal-explorer.com/benmandelbrot.html.
[7]Helmburg, Gilbert. *Getting Acquainted with Fractals.* Berlin, Germany: Walter de Gruyter GmbH & Co., 2007.
[8]Peitgen, Heinz-Otto and P. H. Richter. *The Beauty of Fractals-Images of Complex, Dynamical Systems.* Berlin, Heidelberg: Springer-Verlag, 1986.

[9]*Benoit Mandelbrot: Fractals and the art of roughness.* http://www.ted.com/talks/benoit_mandelbrot_fractals_the_art_of_roughness?language

[10]NOVA. *Hunting the Hidden Dimension.* http://www.pbs.org/wgbh/nova/physics/hunting-hidden-dimension.html.

[11]Pappas, Theoni. *Fractals, googols and other mathematical tales.* California: World Wide Publishing/Tetra, 1993.

[12]Pappas, Theoni. *Fractals, googols and other mathematical tales.* California: World Wide Publishing/Tetra, 1993. .

[13]*Mathematics: About Fractals.* http://www.marywood.edu/math/fractals.html.

[14]Feder, Jens. *Fractals.* New York: Plenum Press, 1988.

[15]Feder, Jens. *Fractals.* New York: Plenum Press, 1988.

[16]Crownover, Richard M. *Introduction to Fractals and Chaos.* Boston, Massachusetts: Jones and Bartlett Publishers, 1995.

[17]Crownover, Richard M. *Introduction to Fractals and Chaos.* Boston, Massachusetts: Jones and Bartlett Publishers, 1995.

[18]Crownover, Richard M. *Introduction to Fractals and Chaos.* Boston, Massachusetts: Jones and Bartlett Publishers, 1995.

[19]Crownover, Richard M. *Introduction to Fractals and Chaos.* Boston, Massachusetts: Jones and Bartlett Publishers, 1995.

[20]Crownover, Richard M. *Introduction to Fractals and Chaos.* Boston, Massachusetts: Jones and Bartlett Publishers, 1995.

[21]Lauwerier, Hans. *Fractals-Endlessly Repeated Geometrical Figures.* Princeton, New Jersey: Princeton University Press, 1991.

[22]Lauwerier, Hans. *Fractals-Endlessly Repeated Geometrical Figures.* Princeton, New Jersey: Princeton University Press, 1991.

[23]Rosenthal, Alex and Zaidan, George. *"The Case of the Missing Fractals."* TedEd Original. http://ed.ted.com/lessons/the-case-of-the-missing-fractals-alex-rosenthal-and-george-zaidan

[24]Phislord Mathes. *Fractals.* https://www.youtube.com/watch?v=C4UK1hCnCEI

[25]Weiss, Ben, Krause, Kai and Beddard, Tom. *FRAX: Immerse, Create, Inspire.* http://fract.al/.

[26]Coolmath. *Fractal Generator.* http://www.coolmath.com/fractals/fractalgenerators/generator1/index.html

[27]Petigen, Heinz-Otto, Hartmut Jurgens, and Dietmar Saupe. *Fractals for the Classroom.* New York: Hamilton Printing Co., 1992.

[28]Fractals: World of Mathematics. *Fractals and Dimensions.* http://world.mathigon.org/Fractals.html

[29]Fractals: World of Mathematics. *Fractals and Dimensions.* http://world.mathigon.org/Fractals.html

[30]Sprott, Julien Clinton. Elegant Chaos-Algebraically Simple Chaotic Flows. Singapore: World Scientific Publishing Co., 2010.

[31]Gulick, Denny and Jon Scott. The Beauty of Fractals-Six Different Views. Washington, DC: The Mathematical Association of America, 2010.
[32]Peitgen, Heinz-Otto and P. H. Richter. The Beauty of Fractals-Images of Complex, Dynamical Systems. Berlin, Heidelberg: Springer-Verlag, 1986
[33]Yale University. Fractals. http://classes.yale.edu/fractals/Panorama/Art/ArtIntro/Art.html.
[34]Fauvel, John, Raymond Flood, and Robin Wilson. Music and Mathematics-From Pythagoras to Fractals. Oxford, New York: Oxford University Press, 2003.
[35]Raymond Flood and Robin Wilson. *Fractal Music.* http://www.tursiops.cc/fm/#whatis.
[36]Colby-Sawyer College. *Fractals in Poetry Project.* http://www.colby-sawyer.edu/assets/pdf/Fractals-in-Poetry-Activity.pdf
[37]Arthur "Guitar Boogie" Smith. *Feudin' Banjos.* Warner Brothers Records, 1972.
[38]Stevens, Wallace, Kermode, Frank, and Richardson, Joan. *Wallace Stevens: Collected Poetry and Prose.* Library of America, 1997.