World Cup Math

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This curriculum unit is recommended for: Math I/Algebra-I, Grade 8-9

Keywords: Ordered pair, coordinate plane, solution, function, intercepts, exponential functions, exponential growth/decay, y-intercept, x-intercept, domain, range, function, input, output, positive, negative, intervals, intercepts, interval notation, slope, rate of change, parallel and perpendicular lines, slope-intercept form, standard form, and point-slope form.

Teaching Standards: See Appendix 1 for teaching standards addressed in this unit.

Synopsis: This lesson is designed for students to understand that linear and exponential equations can be used to model, analyze and solve real world situations. It reveals how the usage of linear and exponential relationships can determine important statistics in sports. Students will understand that functions describe situations where one quantity determines another and these functions can be used to model real-world situations. Students will develop the concepts of domain and range. They explore many examples of linear functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. It is my goal that the learning outcome will enable students to work collaboratively to develop strategies to distinguish the differences between the two types of relationships. The usage of graphs, calculators, and other manipulative will assist the student in interpreting exactly what a problem is inquiring and how to better solve it. This lesson will demonstrate confidence in students as mathematical thinkers, believing that they can learn and achieve high standards in mathematics in addition to, allowing students to realize that their effort and persistence play a vital role in their success. By integrating math into the real world, students can see how math strategies can be used to explain and interpret their favorite sport and recreational activity.

I plan to teach this unit during the coming year to 34 students in Math I/Grade 8.

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Monica Echols

Introduction

Math has a lot in common with sports and games (doing as much as you can while still following certain rules to try and accomplish a goal). I believe math is to learning as what endurance and strength training is to sports, the basis that enables you to excel in the specialty of your choice. You cannot become a major sports star without being strong and having good cardiovascular ability. You cannot become a business leader within your job or excel in your profession unless you can think smart, in addition to, thinking critically. Learning the proper math skills will make things easier to allow you to do that. As a math teacher, I'm always searching for ideas to better engage my students in the learning process. I believe this seminar class will be a great opportunity to teach mathematics with a focus on a real-life sporting event in order to reach my students and make mathematics come alive. Finding ways to incorporate sports in math lessons can go a long way towards eliminating math phobia by giving students a fun and interesting activity to learn math skills.[10]

The past three weeks have shown much that is positive about the rise of soccer in the States, in particular the sell-out crowds and impressive television ratings, mainly on the Spanish language Univision. Television viewership of club and international soccer in the U.S. is at an all-time high, with major sports networks regularly covering games in some fashion and several other channels dedicated mostly or entirely to the sport. In addition to matches, these channels provide news programs and other information. The rise of these media outlets means that soccer fans in the United States now have near constant access to programming about the sport. There has been increased television coverage of soccer in the United States. In addition to increased coverage from the traditional media, the U.S. has several national networks devoted mostly or completely to covering the sport.[4]

Rationale

This lesson will correspond to the North Carolina Common Core Standards in that students use strategies to strengthen their knowledge by working with and understanding functions in more complex methods. This lesson will help all students solve problems and make connections between mathematics and the real world. For mathematics to be relevant to all students, they must see how it relates to their lives outside the classroom. This is definitely one way to motivate and interest students in making the connections. By integrating math into the real world, students can see how math strategies can be used to explain and interpret their favorite sport and/or recreational activity. This lesson also demonstrates confidence in students as mathematical thinkers, believing that they can learn and achieve high standards in mathematics in addition to, allowing students to realize that their effort and persistence play a vital role in their success. This lesson is designed for my Algebra-I class where, students will understand that linear and exponential equations can be used to model, analyze and solve real world situations. It reveals how the usage of linear and exponential relationships can determine important statistics in sports. Students will understand that functions describe situations where one quantity determines another and these functions can be used to model real-world situations. It is my goal that the learning outcome will enable students to work collaboratively to develop strategies to distinguish the differences between the two types of relationships. The usage of graphs, calculators, and other manipulative will assist the student in interpreting exactly what a problem is inquiring and how to better solve it.[3]

The History

In the past few years there's been considerable growth in American Soccer. The popularity of soccer in the U.S. has been growing since the 1960's and 1970's, and received a significant boost when the United States hosted the 1994 World Cup. In 1967 there were 100,000 people playing soccer in the US; by 1984, that number had grown to over 4 million. Girls' high school soccer experienced tremendous growth in playing numbers throughout the 1970s and 1980s—from 10,000 in 1976, to 41,000 in 1980, to 122,000 in 1990. The 1970s and 1980s saw increased popularity of the college game. Women's college soccer received a significant boost in 1972 with the passage of Title IX, which mandated equal funding for women's athletic programs, leading to colleges forming NCAA sanctioned women's varsity teams.[8]

A match between Saint Louis University and SIU Edwardsville drew a college record of 22,512 fans to Busch Stadium on October 30, 1980. By 1984, more colleges played soccer (532) than American football (505). The soccer matches for the 1984 Summer Olympics were well attended. Five matches drew over 75,000 fans, and two soccer matches at the Rose Bowl stadium in Pasadena, California, drew over 100,000 fans. These high attendance figures were one factor that FIFA took into consideration in 1988 when deciding to award the 1994 World Cup to the United States.[8]

As of 2009, there were more soccer players (18 million) in the United States than in any other country, making soccer the fastest growing team sport in the states. Entering 2013, the U.S. Men's National Team played in six consecutive FIFA World Cups and advanced to the quarterfinals at the 2002 event. U.S. Soccer is a world leader in women's soccer at every level and the U.S. Women's National Team has won two FIFA Women's World Cups and four Olympic Gold Medals- an accomplishment that no other country on men's and women's side has reached in Olympic competition.[8]

The Gold Cup has become as much a festival of Latin football in the United States as it is the continental championship and it has been an entertaining event. The driving force behind most of the numbers has been the Mexican-American community and they ended the tournament with the biggest smiles after victory in the final. While the marketing men in the States are happy to embrace the Mexican national team as a business opportunity, for fans of the U.S. national team the tournament has revealed some uncomfortable truths.[8]

Population

My name is Monica Echols and I'm an eighth grade mathematics teacher at Francis Bradley Middle School. Based upon prior End-of-Grade tests in mathematics, these students have demonstrated grade-level proficiency by scoring at or above Achievement Level III. This accomplishment qualified them to take Algebra I in middle school for a high school credit. There are thirty-three eighth grade students, ages thirteen to fourteen years old in this Algebra I (Honors) class. Thirteen of which are male and twenty that are female. I've seen many examples of their talents, for example: football, soccer, band, dance, basketball, track, volleyball just to name a few and have long been impressed by their diligence and work ethic. They are a group of highly motivated, positive self-starters as well as team players who are eager to learn. Each student brings a common ground to the class as well as essential differences this allows the classroom to be a good fit for each individual. By incorporating their prior experiences, culture, personality, and gender, it allows learning to be more effective. Through the knowledge of their personal preferences, it assists me in understanding and predicting their performance. This also helps me restructure the situation to facilitate more student participation.

Francis Bradley Middle School has a diverse student and staff population. Presently, we have 1153 students enrolled— 31% African-American students, 56% White students, 9% Hispanic, 2% Asian and about 2% of our students come from other races and ethnicities. In the last five years our school has continued to become more and more diverse. Although we consider all of our students to be talented in some way, we have approximately 16.5% of our students in the program for the gifted. We have five classes of self-contained exceptional children (three Autistic classrooms, and two SAC classrooms). There are approximately 120 total exceptional children serviced by nine exceptional children's teachers. Bradley Middle School also serves about 35 LEP students and these students are accommodated through a well-established ESL program in which students are either enrolled in an ESL class or monitored and supported if they are mainstreamed in regular education classes.

This curriculum unit is being designed to investigate how soccer has grown as a sport over the last 50 years in the United States. Students will determine what kind of growth, if any, the sport has experienced (linear or exponential).

The Instructional Content

The background of instructional content for my curriculum unit will be a presentation of the concepts that I intend to cover.

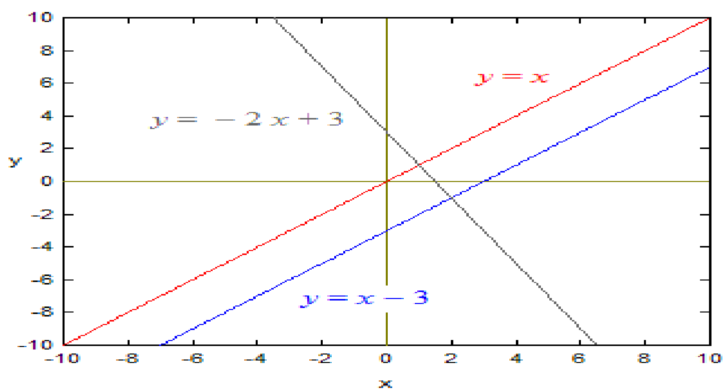
This lesson corresponds to the North Carolina Common Core Standards in that students use strategies to strengthen their knowledge by working with and understanding functions in more complex methods. F.IF.1 allows students to understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. With standard F.IF.2, students use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. Students will learn that for a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship(F.IF.4). Standard F.IF.5 relates the domain of a function while F.IF.6 calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Students will also graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases in standard F.IF.7. They will compare properties of two functions each represented in a different way (F.IF.9 algebraically, graphically, numerically in tables, or by verbal descriptions). Standard F.LE.1 distinguish between situations that can be modeled with linear functions and with exponential functions. In this unit with standard F.LE.2, students construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). Lastly in F.LE.3 they observe using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly while in F.LE.5 the interpretation of the parameters in a linear or exponential function in terms of a context increasing linearly.

Growth: Linear vs. Exponential Relationships.

It is very important to understand relationships between two variables to gain the precise conclusion from a statistical analysis. The relationship between two variables determines how the right conclusions are reached. Without an understanding of this, you can fall into many drawbacks that accompany statistical analysis and will suggest wrong results from your data. A linear relationship is one where increasing or decreasing a variable a number of times will cause a corresponding increase or decrease a number of times in the other variable as well. For example: for a given material, if the volume of the material is doubled, its weight will also double. This is a type of linear relationship. If the volume is increased 10 times, the weight will also increase by the same factor.[6]

A linear function is a relation (a set of ordered pairs) in which each element of the range is paired with exactly one element of the domain. This means that an x-coordinate cannot be repeated. To check if a graph is a function, make sure the graph does not touch any vertical line more than once. This is called the vertical line test. Equations can be written using function notation. Solve the equation for the dependent variable, and then replace that variable with f (independent variable), such as f(x). To graph a linear equation, first find coordinates pairs that make the statement true. Then plot the points and draw a line through the points. Another graphing method is find the x-and y-intercepts by alternatively replacing x and y with 0. Graph these points, and then draw the line that contains these two points. All of the ordered pairs that lie on the line are solutions of the equation.[1]

Linear functions are written in the form: $y = m x + b$, where m and b are constants. A typical use for linear functions is converting from one quantity or set of units to another. Graphs of these functions are **straight lines**. m is the slope and b is the y intercept. If m is positive then the line rises to the right and if m is negative then the line falls to the right.[1]



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Given two points (x_1, y_1) and (x_2, y_2), the formula for the slope of the straight line going through these two points is:

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

where the subscripts merely indicate that you have a "first" point (whose coordinates are subscripted with a "1") and a "second" point (whose coordinates are subscripted with a "2"); that is, the subscripts indicate nothing more than the fact that you have two points to work with. Note that the point you pick as the "first" one is irrelevant; if you pick the other point to be "first", then you get the same value for the slope:[6]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

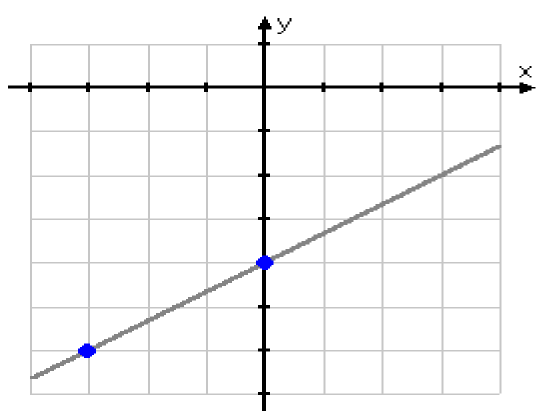
(If you're not sure that the two formulas above give exactly the same values, no matter the pair of points plugged into them, then pick some points and try them out. See what you get.)

The formula for slope is sometimes referred to as "rise over run", because the fraction consists of the "rise" (the change in *y*, going up or down) divided by the "run" (the change in *x*, going from left to the right). If you've ever done roofing, built a staircase, graded landscaping, or installed gutters or outflow piping, you've probably encountered this "rise over run" concept. The point is that slope tells you how much *y* is changing for every so much that *x* is changing. Pictures can be helpful, so let's look at the line $y = (\,^2/_3\,)x - 4$; we'll compute the slope, and draw the line.[6]

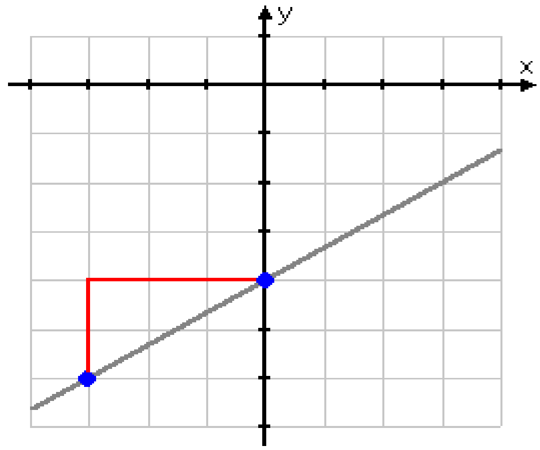
To find points from the line equation, we have to pick values for one of the variables, and then compute the corresponding value of the other variable. If, say, $x = -3$, then $y = (\,^2/_3\,)(-3) - 4 = -2 - 4 = -6$, so the point (–3, –6) is on the line. If $x = 0$, then $y = (\,^2/_3\,)(0) - 4 = 0 - 4 = -4$, so the point (0, –4) is on the line. Now that we have two points on the line, we can find the slope of that line from the slope formula:

$$m = \frac{(-4)-(-6)}{(0)-(-3)} = \frac{2}{3}$$

Let's look at these two points on the graph:[6]

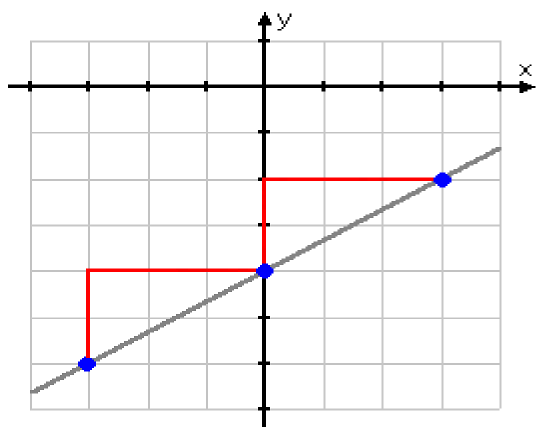


In stair-stepping up from the first point to the second point, our "path" can be viewed as forming a right triangle:[6]



The distance between the y-values of the two points (that is, the height of the triangle) is the "$y_2 - y_1$" part of the slope formula. The distance between the x-values (that is, the length of the triangle) is the "$x_2 - x_1$" part of the slope formula. Note that the slope is $^{2}/_{3}$, or "two over three". To go from the first point to the second, we went "two up and three over". This relationship between the slope of a line and pairs of points on that line is always true.[6]

To get to the "next" point, we can go up another two (to $y = -2$), and over to the right another three (to $x = 3$):

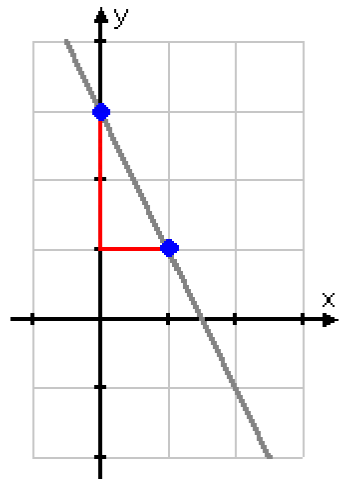


With these three points, we can graph the line $y = (\ ^{2}/_{3}\ )x - 4$.[6]

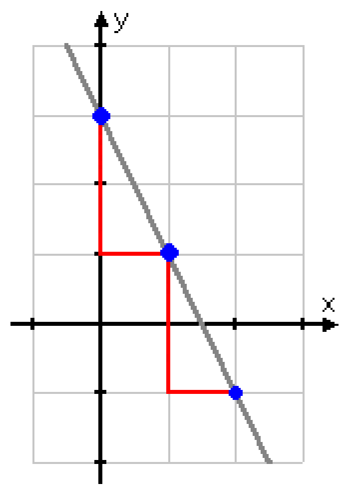
(If you're not sure of that last point, then put 3 in for x, and verify that you get -2 for y.)

Let's try another line equation: $y = -2x + 3$. We've learned that the number on x is the slope, so $m = -2$ for this line. If, say, $x = 0$, then $y = -2(0) + 3 = 0 + 3 = 3$. Then the point (0, 3) is on the line. With this information, we can find more points on the line. First, though, you might want to convert the slope value to fractional form, so you can more easily do the "up and over" thing. Any number is a fraction if you put it over "1", so, in this case, it is more useful to say that the slope is $m = \ ^{-2}/_{1}$. That means that we will be going "down two and over one" for each new point.[6]

We'll start at the point we found above, and then go down two and over one to get to the next point:[6]



Go down another two, and over another one, to get to the "next" point on $y = -2x + 3$:[6]



Then the point (2, –1) is also on this line; with three points, we can graph the line.

Straight-line equations, or "linear" equations, graph as straight lines, and have simple variable expressions with no exponents on them. If you see an equation with only *x* and *y* — as opposed to, say x^2 or *sqrt*(*y*) — then you're dealing with a straight-line equation.[7]

There are different types of "standard" formats for straight lines; the particular "standard" format your book refers to may differ from that used in some other books. The various "standard" forms are often holdovers from a few centuries ago, when mathematicians couldn't handle very complicated equations, so they tended to obsess about the simple cases. Nowadays, you likely needn't worry too much about the "standard" forms; this lesson will only cover the more-helpful forms.[7]

I think the most useful form of straight-line equations is the "slope-intercept" form: $y = mx + b$. This is called the slope-intercept form because "m" is the slope and "b" gives the y-intercept. (For a review of how this equation is used for graphing, look at slope and graphing.)[7]

I like slope-intercept form the best. It is in the form "y=", which makes it easiest to plug into, either for graphing or doing word problems. Just plug in your x-value; the equation is already solved for y. Also, this is the only format you can plug into your (nowadays obligatory) graphing calculator; you *have* to have a "y=" format to use a graphing utility. But the best part about the slope-intercept form is that you can read off the slope and the intercept right from the equation. This is great for graphing, and can be quite useful for word problems.[7]

Common exercises will give you some pieces of information about a line, and you will have to come up with the equation of the line. How do you do that? You plug in whatever they give you, and solve for whatever you need, like this:[9]

Find the equation of the straight line that has slope $m = 4$ and passes through the point: (–1, –6). Okay, they've given me the value of the slope; in this case, $m = 4$. Also, in giving me a point on the line, they have given me an x-value and a y-value for this line: $x = -1$ and $y = -6$.[9]

In the slope-intercept form of a straight line, I have y, m, x, and b. So the only thing I don't have so far is a value for is b (which gives me the y-intercept). Then all I need to do is plug in what they gave me for the slope and the x and y from this particular point, and then solve for b:[9]

$$y = mx + b$$
$$(-6) = (4)(-1) + b$$
$$-6 = -4 + b$$
$$-2 = b$$

Then the line equation must be "$y = 4x - 2$".

What if they don't give you the slope?

Find the equation of the line that passes through the points (–2, 4) and (1, 2). Well, if I have two points on a straight line, I can always *find* the slope; that's what the slope formula is for.

$$m = \frac{(4)-(2)}{(-2)-(1)} = \frac{2}{-3} = -\frac{2}{3}$$

Now I have the slope and *two* points. I know I can find the equation (by solving first for "*b*") if I have a point and the slope. So I need to pick one of the points (it doesn't matter which one), and use it to solve
for *b*. Using the point (–2,4)

$y = mx + b$
$4 = (-{}^2/_3)(-2) + b$
$4 = {}^4/_3 + b$
$4 - {}^4/_3 = b$
$12/3 - {}^4/_3 = b$
$b = 8/3$

...so $y = (-{}^2/_3)\, x + {}^8/_3$.

On the other hand, if I use the point (1,
2), I get:

$y = mx + b$
$2 = (-{}^2/_3)(1) + b$
$2 = -{}^2/_3 + b$
$2 + {}^2/_3 = b$
${}^6/_3 + {}^2/_3 = b$
$b = {}^8/_3$

So it doesn't matter which point I choose. Either way, the answer is the same:

$y = (-{}^2/_3)x + {}^8/_3$

As you can see, once you have the slope, it doesn't matter which point you use in order to find the line equation. The answer will work out the same either way.

To summarize how to write a linear equation using the slope-interception form you

1. Identify the slope, m. This can be done by calculating the slope between two known points of the line using the slope formula.
2. Find the y-intercept. This can be done by substituting the slope and the coordinates of a point (x, y) on the line in the slope-intercept formula and then solve for b.[3]

The other format for straight-line equations is called the "*point-slope*" form. For this one, they give you a point (x_1, y_1) and a slope m, and have you plug it into this formula:

$$y - y_1 = m(x - x_1)$$

Don't let the subscripts scare you. They are just intended to indicate the point they give you. You have the generic "x" and generic "y" that are always in your equation, and then you have the specific x and y from the point they gave you; the specific x and y are what is subscripted in the formula. Here's how you use the point-slope formula:[3]

Find the equation of the straight line that has slope m = 4 and passes through the point (–1, –6). Let's see how the process works with the point-slope formula.

They've given me m = 4, x_1 = –1, and y_1 = –6. I'll plug these values into the point-slope form, and solve for "y=":

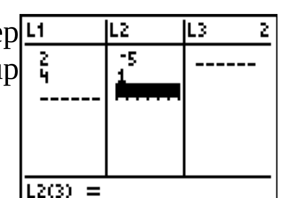
$$y - y_1 = m(x - x_1)$$
$$y - (-6) = (4)(x - (-1))$$
$$y + 6 = 4(x + 1)$$
$$y + 6 = 4x + 4$$
$$y = 4x + 4 - 6$$
$$y = 4x - 2$$

This matches the result I got when I plugged into the slope-intercept form. This shows that it really doesn't matter which method you use (unless the text or teacher specifies). You can get the same answer either way, so use whichever method works more comfortably for you. Once you've got both m and b you can just put them in the equation at their respective position.[3]

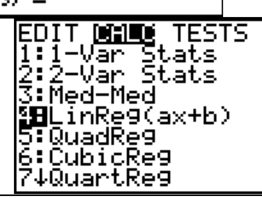
<u>Entering Data into a TI-83 or TI-84</u>: Data is stored in ***Lists*** on the calculator. Locate and press the **STAT** button on the calculator. Choose **EDIT**. The calculator will display the first three of six lists (columns) for entering data. Simply type your data and press **ENTER.** Use your arrow keys to move between lists.

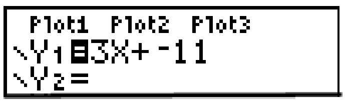
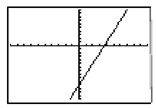
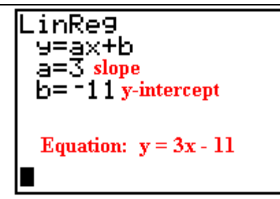
<u>Calculator Solution</u>: The Statistics features of the graphing calculator can be used to write the equations of straight lines given points.

1. Enter the points into lists **L1** and **L2**. Be careful to keep the *x* and corresponding *y* coordinates lined up horizontally.



2. Choose STAT. Arrow to the right to **CALC** at the top of the screen. Choose **#4: LinReg(a*x*+b).**

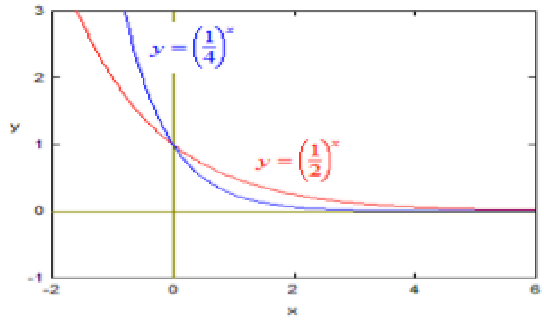


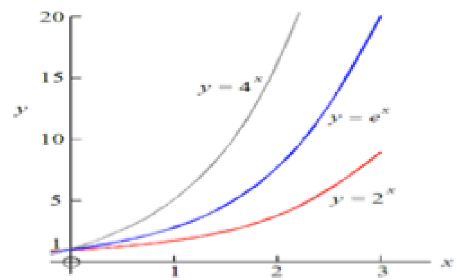
3. On the home screen you will need to tell the calculator which lists contain your points. If you also wish to quickly graph your new line, include **Y1**. **L1** and **L2** are above the numbers **1** and **2** on the calculator. To get **Y1:** Choose **VARS** → **Y-VARS,** **#1Function.** Choose **Y1**	 The Y1 is only needed if you wish to also see the graph of the line.
4. Hitting **ENTER** will show the screen at the right. "a" is the slope, and "b" is the y-intercept. If you hit y = , you will see the equation for the line. Hitting **GRAPH** will show the graph.  	[5]

Like linear relationships, exponential relationships describe a specific pattern of change between a dependent and independent variable. While in linear relationships, y changes by a fixed absolute amount for each change in x; in exponential relationships y changes by a fixed relative amount for each change in x. Quantities related exponentially increase at an increasing rate, producing rapid growth. When noticing and describing patterns between variables, you will look at the value of the independent variable(x), and the dependent variable (y), and the rate of change in the dependent variable relative to the independent variable.[2]

An exponential function has a variable as an exponent. These functions are written in the form: $y = a\,b^{x}$, where x is in an exponent (not in the base as was the case for power functions) and a and b are constants. The base of the exponent must be greater than 0, but not equal to 1. The base cannot equal 1 since 1 to any power equals 1. One way to graph an exponential function is to use ordered pairs. If the base is greater than 1(then the result is exponential growth), the graph rises faster and faster as the x values increase. If the base is less than 1(then the result is exponential decay), the graph falls more slowly as the x values increase.[2]

The graphs of exponential functions can be translated by numbers other than the base and exponent. The y-intercept is changed if a constant is multiplied by original expression translates the graph up or down depending on whether the constant is positive or negative.





Two ways to identify exponential functions are to look at the graph and to look for a pattern in the data. In the data, domain values at regular intervals have corresponding range values that have a common factor, not a common difference.[2]

Exponential functions look somewhat similar to functions you have seen before, in that they involve exponents, but there is a big difference, in that the variable is now the power, rather than the base. Previously, you have dealt with such functions as $f(x) = x^2$, where the variable x was the base and the number 2 was the power. In the case of exponentials, however, you will be dealing with functions such as $g(x) = 2^x$, where the base is the fixed number, and the power is the variable.[3]

Let's look more closely at the function $g(x) = 2^x$. To evaluate this function, we operate as usual, picking values of x, plugging them in, and simplifying for the answers. But to evaluate 2^x, we need to remember how exponents work. In particular, we need to remember that negative exponents mean "put the base on the other side of the fraction line".[3]

So, while positive x-values give us values like these:

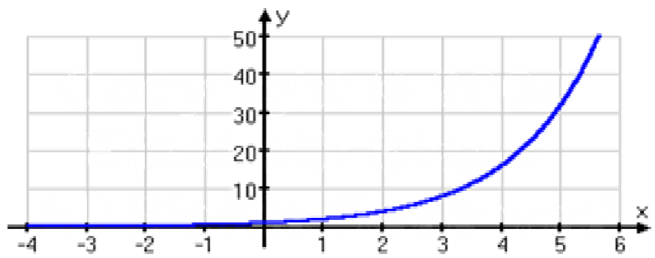
x	$y = 2^x$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$
5	$2^5 = 32$

...negative *x*-values give us values like these:

x	$y = 2^x$
-1	$2^{-1} = 1/2^1 = \frac{1}{2} = 0.5$
-2	$2^{-2} = 1/2^2 = \frac{1}{4} = 0.25$
-3	$2^{-3} = 1/2^3 = \frac{1}{8} = 0.125$
-4	$2^{-4} = 1/2^4 = \frac{1}{16} \approx 0.06$
-5	$2^{-5} = 1/2^5 = \frac{1}{32} \approx 0.03$

Putting together the "reasonable" (nicely able to graph) points, this is our T-chart:

x	$y = 2^x$
-2	$2^{-2} = 1/2^2 = \frac{1}{4} = 0.25$
-1	$2^{-1} = 1/2^1 = \frac{1}{2} = 0.5$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$



You should expect exponentials to look like this. That is, they start small-very small, so small that they are practically indistinguishable from "y= 0", which is the x-axis- and then, once they start growing, they grow faster and faster, so fast that they shoot right up through the top of your graph.[3]

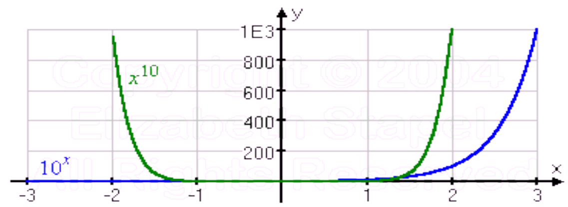
You should also expect that your T-chart will not have many useful plot points. For instance, for x = 4 and x = 5, the y-values were too big, and for just about all the negative x-values, the *y*-values were too small to see, so you would just draw the line right along the top of the *x*-axis.[3]

Note also that my axis scales do not match. The scale on the *x*-axis is much wider than the scale on the *y*-axis; the scale on the *y*-axis is compressed, compared with that of the *x*-axis. You will probably find this technique useful when graphing exponentials, because of the way that they grow so quickly. You will find a few T-chart points, and then, with your knowledge of the general appearance of exponentials, you'll do your graph, with the left-hand portion of the graph usually running right along the x-axis.[3]

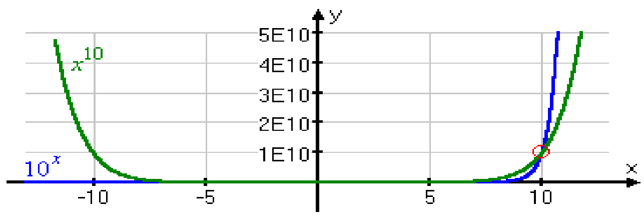
You may have heard of the term "exponential growth". This "starting slow, but then growing faster and faster all the time" growth is what they are referring to. Specifically, our function $g(x)$ above doubled each time we incremented x. That is, when x was increased by 1 over what it had been, y increased to twice what it had been. This is the definition of exponential growth: that there is a consistent fixed period over which the function will double (or triple, or quadruple, etc; the point is that the change is always a fixed proportion). So if you hear somebody claiming that the world population is doubling every thirty years, you know he is claiming exponential growth.[3]

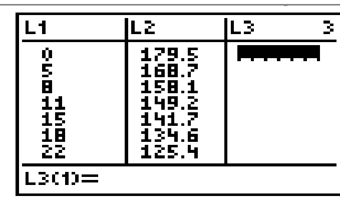
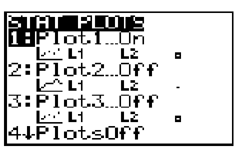
Exponential growth is "bigger" and "faster" than polynomial growth. This means that, no matter what the degree is on a given polynomial, a given exponential function will eventually be bigger than the polynomial. Even though the exponential function may start out really, really small, it will eventually overtake the growth of the polynomial, since it doubles all the time.[10]

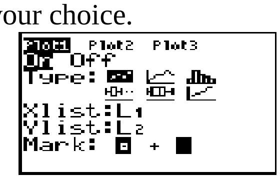
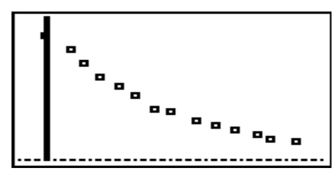
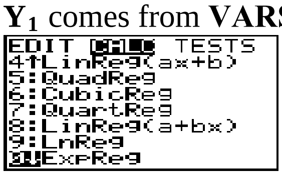
For instance, x^{10} seems much "bigger" than 10^x, and initially it is:

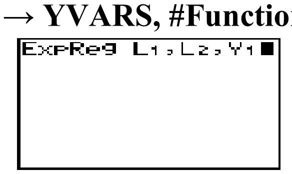
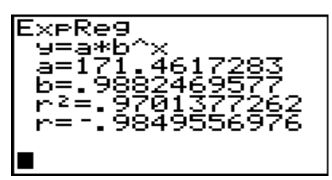


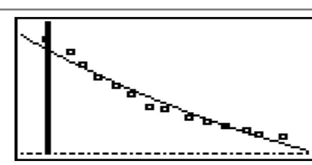
But eventually 10^x (in blue below) catches up and overtakes x^{10} (at the red circle below, where x is ten and y is ten billion), and it's "bigger" than x^{10} forever after:[3]



<table>
<tr>
<td><u>Entering Data into a TI-83 or TI-84</u>
1. Enter the points into L1 and L2.</td>
<td>
</td>
</tr>
<tr>
<td>2. Create a scatter plot of the data. Go to STATPLOT (2nd Y=) and choose the first plot. Turn the plot ON, set the icon to Scatter Plot (the first one), set Xlist to L1 and Ylist to L2 (assuming that is where you stored the data), and select a Mark of your choice.



</td>
<td></td>
</tr>
<tr>
<td>3. Choose Exponential Regression Model.
Press STAT, arrow right to CALC, and arrow down to 0: ExpReg. Hit ENTER. When ExpReg appears on the home screen, type the parameters L1, L2, Y1. The Y1 will put the equation into Y= for you.
Y_1 comes from VARS → YVARS, #Function, Y_1



</td>
<td>

The exponential regression equation is
$y = (171.462) \cdot 0.988^x$
(answer to part a)</td>
</tr>
<tr>
<td>Step 4. Graph the Exponential Regression Equation from Y1. ZOOM #9 ZoomStat to see the graph.[5]</td>
<td></td>
</tr>
</table>

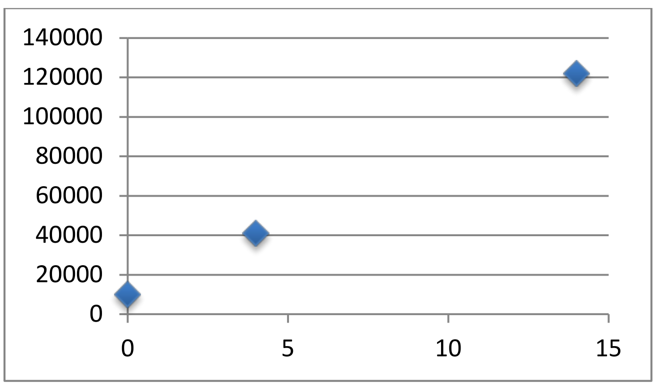
Unit Activities:

Activity 1: Growing Popularity, but how fast?

Students are always asking question, “When will I ever use this stuff”? Here is a great opportunity to prove how this concept has many practical applications in the world beyond school. This activity will get students excited about learning and practicing math. I will ask students to look at statistics about sport participation or spectator statistics obtained from soccer teams’ websites. They will speculate about what the sport might look like in 50 years if the growth rate continues.

For example, take the earlier data. Girls’ high school soccer grew from 10,000 in 1976 to 41,000 in 1980 to 122,000 in 1990. Does this behave more like linear or exponential growth?

First, we can plot the data. Even with 3 points, what do we see?



This appears to be linear rather than exponential. Yet, this only emphasizes a graphical sense of these concepts, which is valuable but not sufficient for my class. Let’s see the difference in linear and exponential growth with this example. The growth was quick but is it quick enough to be exponential?

Suppose the growth rate is linear. What is the slope? To answer this, we can take the slope between 1976 and 1980. This is 7750. What does this mean? For each year, we expect 7,750 more high school girls to be playing soccer. If that trend continued, how many would have been playing in 1990? To do this, we want the equation for this line. This is easiest if we make 1976 year 0. Then, our first point is (0, 10000) and the second point is (4,41000). Then the equation for the line is $y = 7750x + 10000$. Did we do this right? Remember, two points define a line so we need only to put in the values for 1980. This would be $x = 4$ and indeed, we get $y = 41{,}000$. So, we are ready. Let’s put in $x =$

14, which corresponds to 1990. Here, we get y = 118,500. Note, the actual value was 122,000, which is pretty close.

One could argue we were close to 4,000 off. Maybe this was the beginning of the acceleration we see in exponential growth. To aid in this, let's consider this as an exponential growth problem. Not, in 4 years, the number of high school girls playing soccer grew to 41,000. So, in 4 years, we saw the amount essentially quadruple. Let's assume this. Then, in 1980, we would expect 160,000. Wait, this already surpassed the growth in 1990 and we are still a decade away! Still, let's continue to see how fast this growth is. In 1984, we expect 640,000 and then in 1992 we have 2,560,000. Of particular interest, what would the number be for 2012? It doesn't take much work to find it would be over 2 and a half billion. The world population is just over 7 billion. This would mean in 2016, the entire world population is high school girls playing soccer! Clearly, this model of growth didn't work!

Note, this idea can also be extended to investigate a country's population growth. Further, students can look at the ratio of popularity in a sport and compare it to the overall population growth. They can then postulate on whether the growth of the sport is most likely due to increased interest in the sport or overall population growth.

Activity 2: Kicking around a linear and exponential

In this activity, we will get a sense the magnitude of the exponent in an exponential and slope of a line has on the resulting function. We do this in the context of the speed of a soccer ball. Clearly, when you kick a soccer ball, it slows down. We will model this with linear and exponential functions. To begin, students will talk about these two types of functions in groups. What are advantages do they see of modeling the velocity of a ball with each type of function. This type of thinking is often part of the scientific process. What do you expect from what you are considering? You may or may not be right but such thinking gives you a starting place and a proposition to test. We will take two models of the velocity of a soccer ball. One will be linear and the other will be exponential. We alter the magnitude of the exponent in the exponential function. Then we will alter the magnitude of the slope. In both cases, what impact would this have on the velocity of the ball?

Now, the velocity of a soccer ball may not actually be either of these functions. Yet, modeling the exact formulas that determine the flight of a soccer ball is an involved question. In fact, such research was performed by the University of Sheffield's Sports Engineering Research Group and Fluent Europe. They were interested in whether the shape and surface of the soccer ball, as well as its initial orientation, play a fundamental role in the ball's trajectory through the air. They found it does. So, every World Cup, a new ball with a pattern developed and released just for that tournament is created. The

University of Sheffield's research showed that each World Cup is different. This is true for many reasons but with that research, we knew it was because of the ball!

Now, professional soccer players kick a soccer ball at an initial velocity of 70 miles per hour. For what we are about to do, it will be helpful to have this express as inches per second. This can be interesting for students to approximate. If you are on the highway going 70 miles per hour, how many inches per second are you traveling? It turns out, which one can confirm, that 70 miles per hour equals 1232 inches per second.

Suppose the velocity follows the function $v(t) = 1232e^{-t}$, where t is in seconds. First, note that this means that at time zero you are traveling 1232 inches per second, which is what we want. When will this function equal 0, meaning no movement? This is an important question as it underscores how easily a question can be posed. Note, y(t) can never equal zero. So, clearly this can't actually equal 0. Instead, let's say the instant before a ball stops, is it traveling .25 inches per second. That's very, very slow. So, we want to solve $0.25 = 1232e^{-t}$. This can be solved using algebra techniques. We find that t is about 8.5 seconds. Now, we just made that exponent up. Let's try another one. What if we try $v(t) = 1232e^{-2t}$? Now what happens? Note, now, the ball will reach 0.25 inches per second in 0.001 seconds! We blink in 0.4 seconds.

Now, we will try this with a linear function. Let's try $y(t) = -x + 1232$. Again, we have a function that is traveling at 1232 inches per second when we begin. How long would this take to reach 0? Clearly, it would take 1232 seconds. That's 20 minutes! That means you kick a ball and it is traveling for 20 minutes. In this case, we need to slow things down. Let's try $y(t) = -2x + 1232$. When do we have $y = 0$? We again, solve the equation and find 1232/2 or about 10 minutes. At this point, students would be asked to find a slope they believe is realistic and compare theirs to others and discuss their differences. This is the nature of modeling. We may not agree as we, in part, may be emphasizing different aspects of the problem.

Activity 3: Going Viral, mathematically

Soccer has grown in popularity. Soccer videos, like many others, can go viral when some amazing or funny feat is achieved. How can things that go viral grow to quickly? Suppose 100,000 people are watching a video every hour. In another case, every time someone watches the catchy video, they are sharing it with 5 people who also watch it. Which can lead to a viral video?

To answer this question, we move away from sports and look at the video by Korean rapper Psy. He rose to world fame with his single Gangam Style. When he debuted his latest single Gentleman yesterday on YouTube, it was viewed approximately 20 million times. How can this be possible?

Let's assume that 5 people watch the video and email 5 people who then watch. Those people each email 5 people who watch it. How long would it take it to each 20 million views? This type of growth can be modeled by saying that $y(t) = 5^t$ people watch the video at time t. When t = 1, then 5 people are watching. At t = 2, 25 people are watching. If you work this out, when t = 11, you are over 20 million. Take a second a think about this, this means if you keep up the rate of $y(t) = 5^t$, then in 11 steps you have 20 million people watching at time t! So, if you let t represent an hour, you will easily reach that viral rate! In the age of the Internet, this is definitely possible! Note, it must be kept up! Notice also, that we really didn't look at the cumulative growth. That is, we could have added up everyone who watched over those 11 steps! You can have your students this, too.

Now, suppose 100,000 people are watching at each time. We can figure out how many cumulative people watched the video by y(t) = 100,000t. To reach 20 million here, we need t = 200. Note, this means you need over 8 days to pull this off. Note, it's a popular video but not like the one that hit 20 million views in one day!

This example underscores the difference between exponential growth and linear growth. At t = 1, linear is doing great! The video looks viral! The video that has exponential growth doesn't. It only has 5 views. Even after 2 hours, it doesn't look like much. But, in the 11th hour, the viral video is being watched by 20 million in that hour. The video with linear growth is being watched still by 100,000 people in that hour.

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EndNotes

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[1]Bennett 2004
[2]Bruno, 1999
[3]Charles, 2004
[4]ESPN, 2012
[5]Hand, 2008
[6]Holliday, 2003
[7]Jahnke, 1991
[8]Sanchis, 2012
[9]Stapel, 2012
[10]Zaccaro, 2005

Teacher Resources

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I used this algebra book for examples of linear functions.

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This website assisted me with the history of soccer.

Stapel, Elizabeth. "Purplemath."Purplemath. http://www.purplemath.com (accessed November 20, 2012)
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Zaccaro, Edward. *Real World Algebra: Understanding the Power of Mathematics*. Bellevue, Iowa: Hickory Grove Press, 2005.
This book was used to help with the real-world applications with linear and exponential functions and the descriptions of specific vocabulary.

Materials Needed for the Curriculum Unit
TI-83/TI-84
Graphing Paper
Access to Google

Appendix 1

- **F.IF.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
- **F.IF.2:** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F.IF.5:**Relate the domain of a function.
- **F.IF.6:** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **F.IF.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.9:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **F.LE.1:** Distinguish between situations that can be modeled with linear functions and with exponential functions.
- **F.LE.2:** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **F.LE.3:** Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly
- **F.LE.5:** Interpret the parameters in a linear or exponential function in terms of a context. increasing linearly

This lesson corresponds to the North Carolina Common Core Standards in that students use strategies to strengthen their knowledge by working with and understanding functions in more complex methods. F.IF.1 allows students to understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. With standard F.IF.2, students use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. Students will learn that for a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship(F.IF.4). Standard F.IF.5 relates the domain of a function while F.IF.6 calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Students will also graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases in standard F.IF.7.

They will compare properties of two functions each represented in a different way (F.IF.9 algebraically, graphically, numerically in tables, or by verbal descriptions). Standard F.LE.1 distinguish between situations that can be modeled with linear functions and with exponential functions. In this unit with standard F.LE.2, students construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). Lastly in F.LE.3 they observe using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly while in F.LE.5 the interpretation of the parameters in a linear or exponential function in terms of a context increasing linearly.