Lines
by Catherine Perez

Introduction

The Idea

What is a line and what can I do with one? Before writing this unit I was fairly oblivious to the ways lines are used, especially in the world of art. A line was a straight path between two points. Art was a product of the process of painting, sculpting, drawing, etc. Math was the use of numbers (and only numbers) as a language to make precise descriptions of the way things in our world work. Now I think a little differently. Before preparing this unit, I had never heard of graph theory or topology or platonic solids or knot theory or a man named Euler (pronounced "Oiler"). I had no idea that artists used math to draw things in perspective or that the Escher drawings I was so fascinated by were created using the concepts of symmetry and perspective.

I decided that I would expose my third grade students and colleagues to some of the fascinating things I was learning by writing a unit on lines. It seemed to me that most of the gaps in my mathematical understanding were related to patterns. The way I was able to organize my thoughts for the unit was by thinking about lines. Third graders study lines and even though the math needed to fully understand these concepts is too advanced for them, I decided there was no reason that they could not benefit from an introduction to many of the concepts using simple activities with limited calculations.

Purpose

Through this unit I will teach our North Carolina state curriculum objectives on lines, points, intersecting lines, parallel lines, perpendicular lines, and angles. Then I will relate these terms to new math concepts as I introduce knot theory, graph theory, perception, and symmetry and some of their uses in art. Students will use this knowledge to create their own works of art.

I have designed this unit for use with elementary students in third grade. It includes an assortment of artistic activities to expose students to some new concepts that they might otherwise not study until their middle school or high school years. The concepts are a little abstract at some points, so I recommend that it be used for third grade and up instead of with lower grades where students are still predominantly thinking concretely.

I teach third grade at Torrence Creek Elementary in Huntersville, North Carolina. Our teachers follow the state curriculum closely, but are given freedom to plan a variety of lessons to reach our educational goals. Our literacy curriculum is based on themes and we integrate our units of study in social studies, science, math, and the arts with literacy. I intend to teach this unit in conjunction with our literacy unit on the solar system, our math unit on geometry, our science unit on objects in the sky, and our social studies curriculum on geography. My plans are to integrate the lessons and activities in this unit with our math block, but as it has a heavy art component, we may accomplish some of these activities in our literacy workshop block, so that we have enough time to complete them. Students will be using sketchbooks throughout the year to produce art in all subject areas and units as an overarching effort to heavily integrate art into my classroom curriculum.

This unit first explains background information on the math content and art content involved in the lessons. Then I explain the strategies and classroom activities I will use for teaching the content. Finally, I include resources to use with the classroom activities. These lessons are also designed so that they may be taught individually.

Objectives

Rationale

I think there is a need in my classroom for math instruction that lets children explore foreign ideas in math, as well as to apply math concepts to produce works of art. Elementary education has fortunately moved towards teaching math so that students understand the content of the math involved and away from memorizing a standard algorithm to solve problems. Students are encouraged to find and use a variety of methods to solve a problem. Learning now involves manipulatives and pictorial representations.

Furthermore, as educators we find that students can come up with a mathematical concept on their own before they are taught the term for that concept. Unintentional omission of the process of discovery in math unfortunately starts at an early age. “Mathematics is discovered by looking at examples, noticing patterns, making conjectures, and testing those conjectures. Once discovered, the final results get organized and put in textbooks. The details and the excitement of the discovery are lost.”[1] Some years ago, when I realized that math was like science and that rules could be broken, mathematics became much more enjoyable for me. Beforehand, I never discovered anything for myself in my grade school education and I took everything I was taught as unchangeable truths. I never was given the opportunity to discover anything in math. I was taught directly to understand a set body of knowledge and to memorize anything I didn’t understand.

Fortunately, we adopted the Math Investigations curriculum in our school system this year, so our lessons are built to help students invent their own understanding of new math concepts before standard methods are introduced. I want to take a math scenic detour using ideas that were foreign to me prior to beginning this unit. Two of the math topics that we will scratch the surface of in this unit break my preconceived notions of math. These are topics that don’t require formulas or procedures. One type of math involves knots and the other involves symmetry. They require using math differently from the way we typically think of when trying to solve a problem.

The next element of the unit is to connect lines in math to lines in the world of art by having students explore how they are used in art and to create works of art using the ideas we will be studying. Arts education should be an important component of a child’s education. “Years of research show that it's closely linked to almost everything that we as a nation say we want for our children and demand from our schools: academic achievement, social and emotional development, civic engagement, and equitable opportunity"[2] Certainly, the study of literacy and math alone, to prepare for state testing is not going to produce educated citizens whom I want to shape the world I will retire into. It is important to me to expand the amount and the quality of art education in my classroom.

I am not an artist and so the task of doing this seemed fairly daunting at first. However, the first art textbook I picked up said that lines are one of the elements of art. The rest are shape, form, space, color, value, and texture.[3] Given this solid connection to the basics of art, I felt that this unit was definitely strong in two areas that were key objectives for third graders in North Carolina, lines in math and lines in art.

Background

In this section I provide background information on the subjects that will be taught in the Classroom Activities section.

Geometric Terms and Concepts

Third graders learn the vocabulary and properties of lines, angles, polygons, and polyhedral. Appendix B includes the definitions and names of regular polygons and some three dimensional figures. In our school, teaching methods for these concepts include Geoboards, color tiles in a variety of shapes, string activities, word walls, vocabulary study, Tangrams, and standard lessons in our textbook. I do not go into instruction for these concepts, however, I mention them here because it is important for students to learn this vocabulary in order to have a common language for preparedness for fourth grade and future study.

Hatching Lines

Hatching is a drawing technique that adds the appearance of three dimension to drawings. Hatching lines are a series of lines drawn close together. In general, the closer the lines are together the darker the appearance. Additionally, lines drawn with hard pressure on the pencil appear darker than lines drawn with a lighter touch. Examples of cross hatching, which are intersecting hatching lines, are shown later in Figure 13.

Perspective Drawing

Perspective drawing is a technique that artists use to draw or paint something on a flat surface so that it looks three dimensional. There are four basic rules of perspective.[4] Rule 1: Things in the picture get progressively smaller as they get farther away from the drawing plane (the drawing surface). Rule 2: Things in the picture that are parallel to the drawing plane are drawn accurately. For example, lines that are parallel to each other and parallel to the drawing plane are drawn as parallel lines. Rule 3: Lines that are not parallel to the drawing plan are drawn as lines that converge at a point on the drawing called a vanishing point. Rule 4: If more than one set of parallel lines are not parallel to the drawing plane, the vanishing points are all located on the same line. For our purposes, this will be the horizon line, which is an imaginary line where the sky and ground seem to meet.

One Point Perspective

Figure 1 is an example of one point perspective drawing. The vanishing point is labeled. The horizon line is the horizontal line where the vanishing point is located.

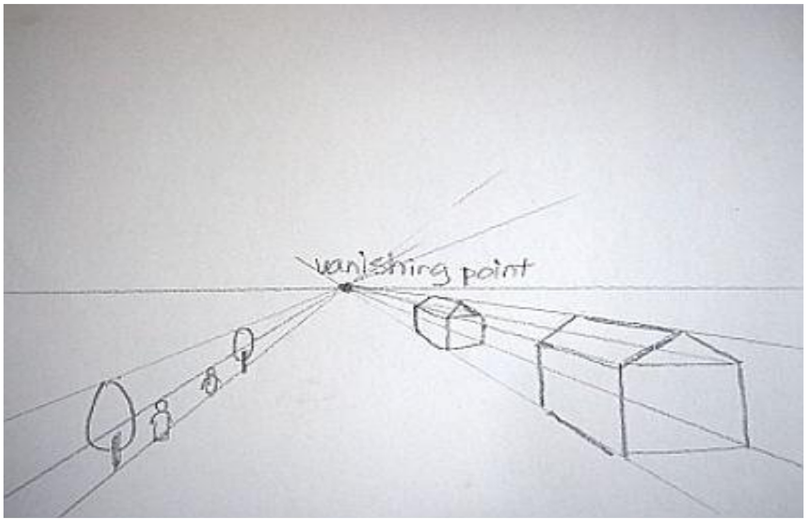


Figure 1. Vanishing Point on the Horizon Line

Another interesting note in one point perspective is that sometimes the objects in the distance also become a little distorted or fuzzy to represent the distance from the picture plane. An example of this is shown in Figure 2.

Figure 2. Example of One Point Perspective with Fuzzy Objects in the Distance

Two Point Perspective

Two point perspective is when a cube or other object has two sets of parallel lines that are also parallel to the drawing plane. The other lines on the object determine the two vanishing points. In Figure 3, notice how these lines are extended to the horizon line and that the point where they intersect the horizon line is the Vanishing Point. Figure 3 also depicts how objects are drawn that are above or below the eye level of the person viewing the painting.

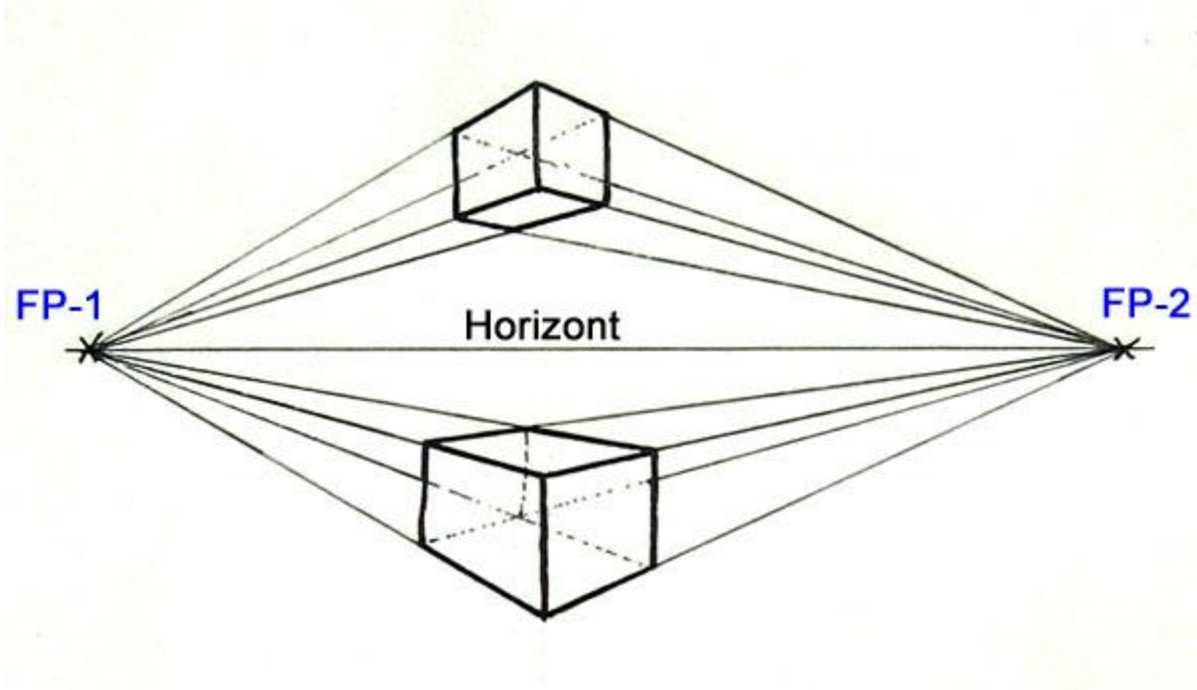


Figure 3. Illustration of Two Point Perspective

Symmetry and Tiling

Symmetry is present in many areas of our lives. Religious symbols are often symmetrical such as the Christian Cross, the Jewish Star of David, the Islamic Crescent, and the Taoist Yin-Yang symbol. Music is often symmetrical, as well as carpets and quilts. Symmetry in math is usually thought of geometrically, using translation symmetry, rotation symmetry, or reflection symmetry. This type of geometric symmetry is the important foundation for artwork that is created using tilings.

In David Farmer's book *Groups and Symmetry*, he first asks the reader to think about an infinitely large grid of points.[5] On this grid, the student is to draw a regular polygon that is formed by connecting grid points. Then the students is to devise a set of "legal moves" for the day, which is basically moving the polygon or the grid in a set way. For example, put a square on the grid and move the square up, right, left, or down by 1 unit. Eventually, after making repeated legal moves there are a few regular polygons that will completely cover the grid without overlapping and without leaving gaps between the polygons. These polygons are the parallelogram, hexagon, and triangles.

The famous artist, M.C. Escher, used this concept in much of his artwork. Figure 4 illustrates how to draw a tiling or tessellation using rectangle grid paper. This type of tessellation uses translations. Basically, any kind of line is drawn from the top left to the bottom left corner. This line is repeated on the right side. A different line is drawn on the bottom of the rectangle, which is repeated on the top of the rectangle. Lastly the rectangular grid lines are erased. The illustrations in Figures 4 come from an excellent website, Tesselations.org. This website has interactive lessons to create tesselation drawings using hexagons and triangles. The triangle is the trickiest because two sides have identical lines drawn between their vertices, but the third side is split is half. This half has two equivalent lines drawn upon each half, but they are rotated 180 degrees from each other.

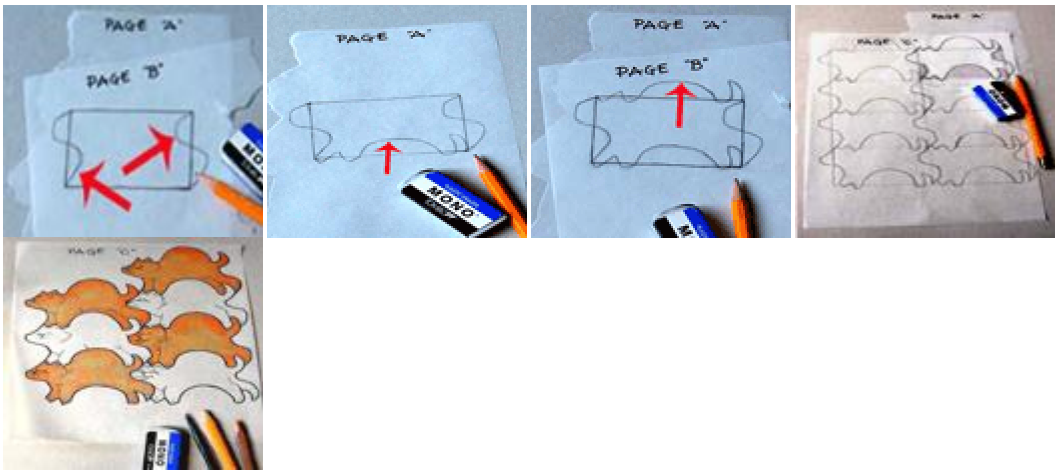


Figure 4. Illustrations of How to Make an Escher Tiling

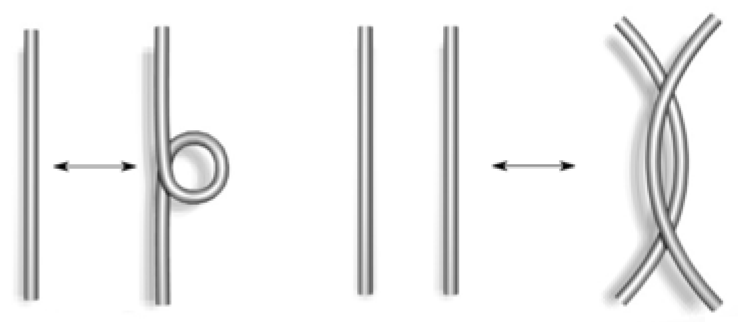
Topology and Knot Theory

Topology is a branch of mathematics that studies the properties of figures that do not change when twisted or stretched but not torn. One example is of a coffee mug and a donut. The donut has one hole in it and if it was made out of a stretchy material it could be stretched and pulled into the shape of a coffee mug, where the hole is in the handle. Distances don’t matter in the study of topology. Knot theory is a part of topology that studies closed loops, knots made in closed loops, and links, which are two or more connected loops. A knot that can be untwisted into a circle is called an unknot. Two knots are equivalent if they can be manipulated to look like each other when spread out on a flat surface.

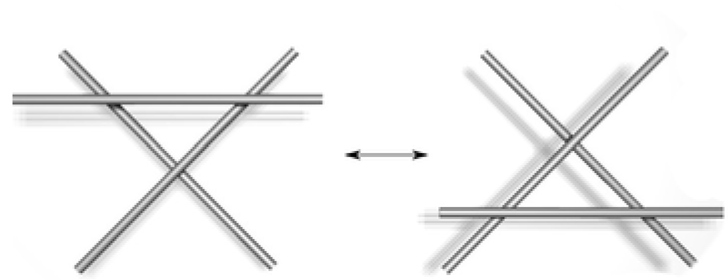
In studying knots, students may look at whether or not a knot is tri-colorable. A knot is tri-colorable if every strand is colored with one color and the entire strand is the same color (the color only changes at an undercrossing, which is where the rope crosses underneath itself). You must use only two or three colors to color the knot, and at every crossing either all 3 colors appear or exactly one color appears. Kurt Reidemeister proved in 1927 that all manipulations on the knot were actually either putting in or taking out left-hand or right-hand kink, sliding a loop over or under another strand, or sliding a strand past a crossing. These moves are called Reidemeister moves (see Figure 5).[6] A knot diagram is a drawing of a knot in which the rope that crosses under another is drawn with a break in the rope and a slight gap on each side of the solid rope that passes over the top. Two knot diagrams are equivalent if there are a series of Reidermeister moves that do not change the tricolorability of the knot.

I. Twist and untwist in either direction.
II. Move one strand completely over another.
III. Move a strand completely over or under a crossing.

Reidemeister moves



Type I Type II



Type III

Figure 5. The Three Types of Reidemeister Moves

Celtic knots are drawn using knot diagrams. To draw a Celtic knot, use a regular polygon to start. Draw a rope crossing the center of each edge of the polygon. The rope that crosses over the top (the unbroken one) should be the one that, when rotated counter-clockwise, passes over the edge of the polygon first. After these crossings are drawn on top of each edge they are connected to one another by drawing lines between the closest, opposite crossing rope ends.

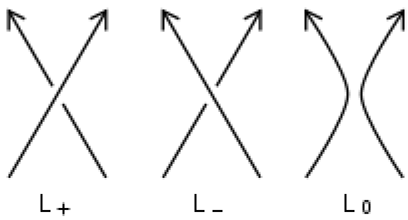


Figure 6. Different Types of Knot Crossings

Topology and Graph Theory

Like knot theory, graph theory is also a branch of topology that evaluates paths between locations, where distance and direction are not considered.

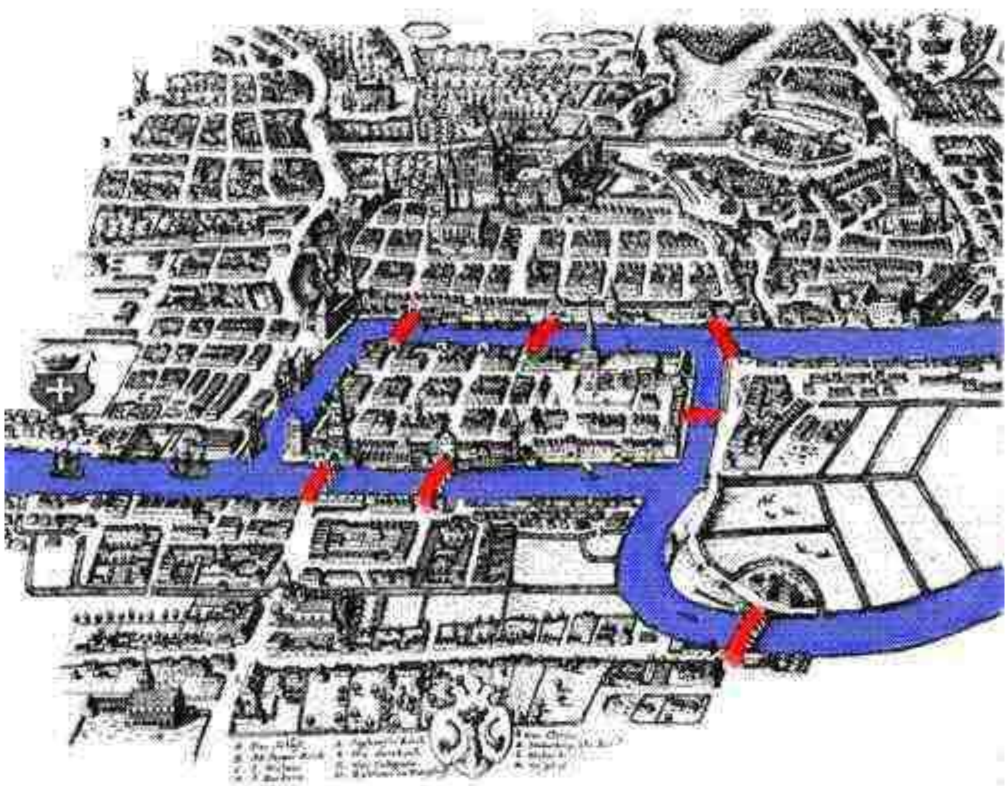


Figure 7. Euler's Konigberg Bridge Problem

"In 1736 Euler solved the problem of whether, given the map below of the city of Konigsberg in Germany, someone could make a complete tour, crossing over all 7 bridges over the river Pregel, and return to their starting point without crossing any bridge more than once." [7]In looking at the problem Euler found that it was impossible to cross all bridges in one trip without crossing the same bridge twice. To prove this, Euler drew a diagram of the river and bridges using points and lines. The insect world activity that follows in the activity section introduces these diagrams to students.

Strategies

There are seven lessons in this unit that will enhance the geometry curriculum in my class. Students will keep drawings or glue into their sketchbooks all of the art from these lessons that is not three dimensional.

Lesson 1 – Lines and Intersections

At the beginning of this lesson I will ask the students to look at several pieces of art and then to discuss how the artist used lines in the pieces. I have shown some examples of art that I might share with the students in Figures 8 through 11, however, any art using lines is acceptable. I expect students to identify curved and straight lines and possibly vertical, horizontal, diagonal, and zigzag lines. I will ask questions such as: What is a line? What is a line made of if it is broken into pieces? Why did you come to these conclusions? I will introduce a dot and ask students what that is and what happens if you put a bunch of dots in a line very close together and then go back to the definition of a line in hopes of getting them to understand that a line is successive points in alignment.

Figure 8. **Jackson Pollock**, *Guardians of The, 1943*

Figure 9. **Jackson Pollock**, *Number 7*, 1951

Figure 10. **John McCraken**, *23,* oil on canvas, 1964

Figure 11. **Kenneth Noland**, *Beginning*, magna on canvas, 1958

Once students have an understanding of what a line is, I will move to a discussion of straight lines and how to describe them. Here we will have a brief discussion of infinity and develop terms and drawings for lines, line segments, and rays. Students will look for examples of these in the artwork. One of the pieces will include a vanishing point so that the concept of an infinite line may be further discussed.

Next, I will ask students how the different lines in the artwork make them feel. I will ask students to explore other ways to describe the lines and what might be an artist's purpose for using a particular line in a piece of art. I will ask students to walk as they want to around the room to a piece of music. I will ask them to look at the pathway they are making, which I predict will probably be curved. Next, I will ask them to exaggerate the path. Afterwards, I will have students walk in straight lines, turning sharply every four counts or three counts depending on the measure of the music, i.e. four/four time, three/four time. Then I will ask them to walk in a spiral pattern. I will ask them to change the tempos of their walks. Afterwards, we will discuss the energy and feelings they felt with each path they walked, how those paths might look on paper, and how can we change the look of one path if it stays basically the same path. Here I expect students to change the width/thickness of their lines and the frequency. I will also have students discuss the effect of changing the color of the lines in an artwork.

The last part of our discussion will be on intersecting lines. I will ask students to describe the different ways lines cross in the artworks. We will identify and define intersecting, perpendicular, and parallel lines, as well as obtuse, acute and right angles. Students will continue to draw and label these lines in their sketchbooks.

In culmination, students will draw three pieces of art in their sketchbook. One will be using only straight lines, another only curved lines, and one using both. These drawings will be in black and white because I want the focus to be on the lines and their paths, instead of the colors. They will add to the pieces a description of what and why they did what they did and/or what the painting is supposed to represent or how it makes them feel. Next we will discuss how their drawings would change if they added color, what colors they would add, and why.

Extension

For an extension of this lesson students will make a picture that is just from random doodling and then describe any forms they see within the doodle.

Lesson 2 Hatching Lines

For this lesson students will each need a piece of lumber, some nails, and different color embroidery thread to make string art. Ideally, the boards should be painted, so that the artwork has a background color. Poster board and thumbtacks may also be used for this project. See Appendix A for more detailed directions.

First, I will show students a board that has nails hammered in 1 inch increments along the perimeter of a piece of wood. I imagine most pieces of wood will be rectangular in shape, but circular or irregular shapes are fine as well. I will ask students to predict what the wood would look like if we connected all of the nails together with string. After taking their predictions I will ask students to use rulers and their sketchbooks to develop a plan to connect each nail with every nail that is not on the same straight side that it is on. After they develop their plan I will ask them to test it out. Plans will be shared with the class. We will discuss the effectiveness of each plan and then I will have students design their own artwork first in their sketchbooks, using appropriate colored pencils to represent the embroidery thread, and then to recreate it on their board. Figure 12 shows a completed project.

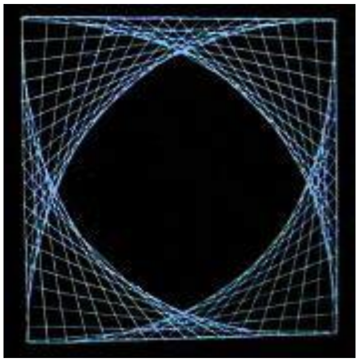


Figure 12. An Example of Curve Stitching or String Art

Following this we will move to hatching lines in artwork. Some examples of cross hatching are shown in Figure 13.

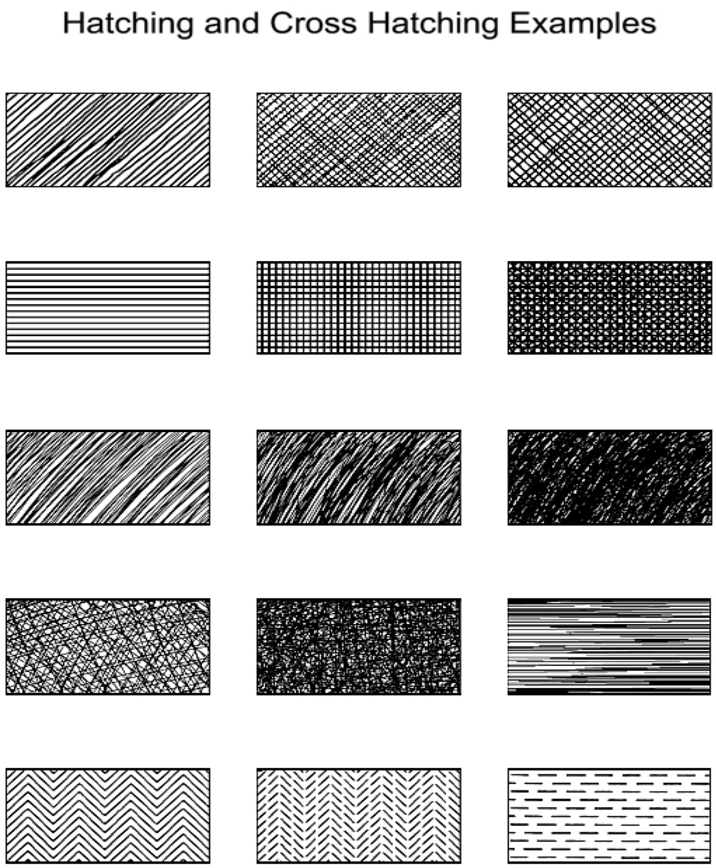


Figure 13. Examples of Different Types of Cross Hatching

I will display different works of art and explain that the artists used hatching lines to create the appearance of light and dark in their artwork. One example of cross hatching used in artwork is shown in Figure 14. I will ask students to point out the areas of light and dark in the picture and describe how the artist created this effect. I will ask them to describe the different hatching lines using the terminology we have developed about lines so far, such as perpendicular, intersecting, frequency, parallel, curved, and thickness. I will ask them to compare the hatching lines in the artwork to their own string art.

Figure 14: **Vincent Van Gogh**, *Fountain in the Garden of the Hospital, St Remy,* ink on paper, 1889

Next, students will practice using hatching lines to add shadow and depth to drawings of three dimensional figures. Students will be given circles to be shaded in to look like a sphere without using hatching lines and with using hatching lines, so they may compare some of the differences in using hatching lines and different types of hatching lines, see Figure 15. Afterwards, they will draw a shadow underneath a sphere by using hatching lines to give the appearance of the sphere being suspended in air.

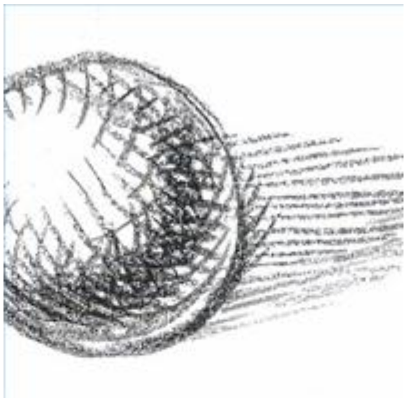


Figure 15. Shading in a Sphere Using Hatching Lines

Lesson 3 Perspective

In preparation for this lesson students need to bring a photograph from home that has a landscape in the background.

For this lesson I will show the students a simple landscape of train tracks vanishing into the distance. I will ask students to describe the tracks. I will show them additional pictures of streets and fences vanishing into the distance. I will ask students to draw the tracks that they see in the picture on a scrap sheet of paper. Though this discussion and exercise I want students to notice that the tracks are parallel in reality, but on paper they are drawn as lines that eventually intersect. Next they will identify the same characteristics of vanishing lines in the other pictures.

Figure 16. Photo of Vanishing Train Tracks

I next want to see if they can notice the horizon line and the vanishing point. After I have made sure that students understand these terms, I will have students observe me draw the vanishing points from a couple of photographs that I bring from home. Then they will try to draw their own photographs by making the objects in the foreground larger than the objects in the background, using a horizon line, and using a vanishing point as appropriate for their photograph. I also want them to use only lines in their drawing and make an outline of everything they see, such as trees, clouds, the tops of the blades of grass, and people. Students will either draw this on paper to attach inside their sketchbooks or they will draw it directly in their sketchbooks.

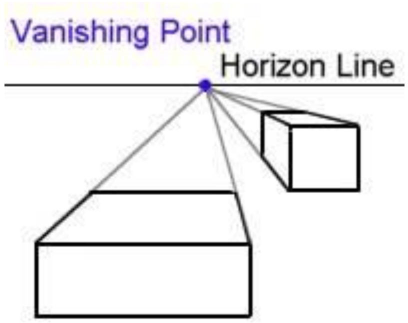


Figure 17. Example of One Point Perspective

Following the photograph activity I will teach students how to draw a cube in one point perspective (see Figure 17) and two point perspective (see Figure 18). To begin I will draw a square parallel to the viewing plane with a horizon line that is slightly above the cube and a vanishing point. Using rulers I will connect the corners of the square to the vanishing point and then draw the other edges of the cube based on the diagonal lines. We will try moving the square around as well as the horizon line, so that the perspective changes, and then examine where the viewer is standing.

After examining one point perspective I will introduce two point perspective to the students by examining a cube with edges parallel to the viewing plane and none of the faces parallel to the viewing plane. First, I will put two vanishing points on the horizon line. I will then connect the bottom and the top of the edge to the vanishing points (VP). Using these diagonals I will draw in the other two horizontal edges. After drawing the next diagonals to connect the new edges to the vanishing points I will show students how to draw in the next sets of edges to complete the cube. I will then have students draw a rectangular prism in the same way.

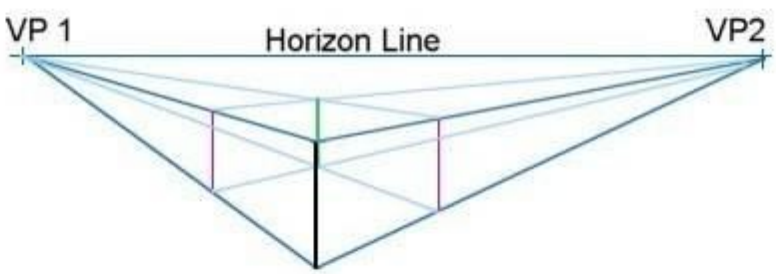


Figure 18. Example of Two Point Perspective

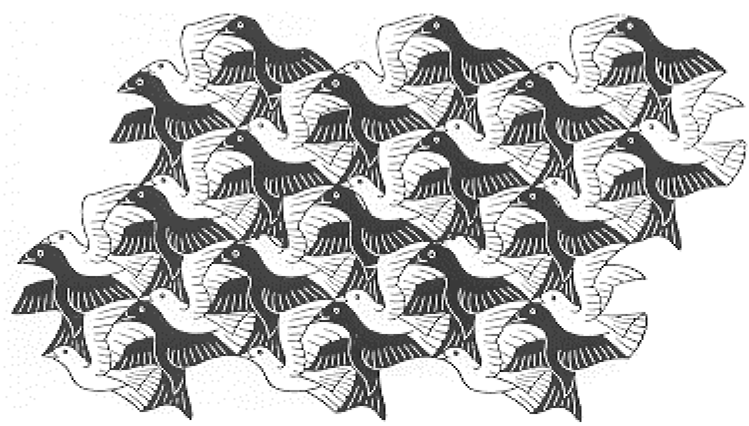
For our final activity in this lesson I will ask students to draw a picture of their choice using the perspective techniques we have learned: 1) objects in the foreground appearing bigger than those in the background (and that they change progressively in size), 2) one point perspective drawn with a ruler, 3) two point perspective drawn with a ruler.

Lesson 4 Symmetric Lines

At this point in the year some students are more familiar with symmetry than others. Before this lesson I will review the terms rotation, translation, symmetric, congruent, and reflection.

This lesson will begin with students using pattern blocks (the square, rectangle, equilateral triangle, scalene triangle, circle, pentagon, hexagon, and parallelogram) to see which ones will completely tile a plane by only using translations (slides). No rotational symmetry should be used. Students will use one shape at a time to ascertain whether or not it will cover the paper without overlapping the shapes and without any of the paper showing between the tiles. Students should find that the equilateral triangle, parallelogram, and hexagon are the shapes that will tile the plane.

I will show students some examples of Escher's drawings that are made using symmetric lines with plane tilings. I will then ask paired students to highlight the symmetric lines in copies of three different Escher drawings made with different plane tilings – triangle, square, and hexagon. All lines that are symmetric should be highlighted in the same color. Then I will put a transparency of the drawings on the board and we will highlight the drawings together. Next, we will make a key for the drawing by labeling the lines that are in the same color "A" and the other color "B". Students will put this key on the side of their drawing with a picture of the line and its corresponding letter. We will then discuss any patterns they see.



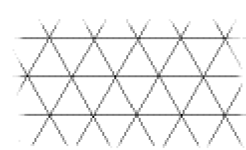


Figure 19. Triangle Tiling **M. C. Escher**, *Regular Division of the Plain with Birds*, woodcut, 1949

Figure 20. Hexagon Tiling **M. C. Escher**, *Reptiles*, lithograph, 1943

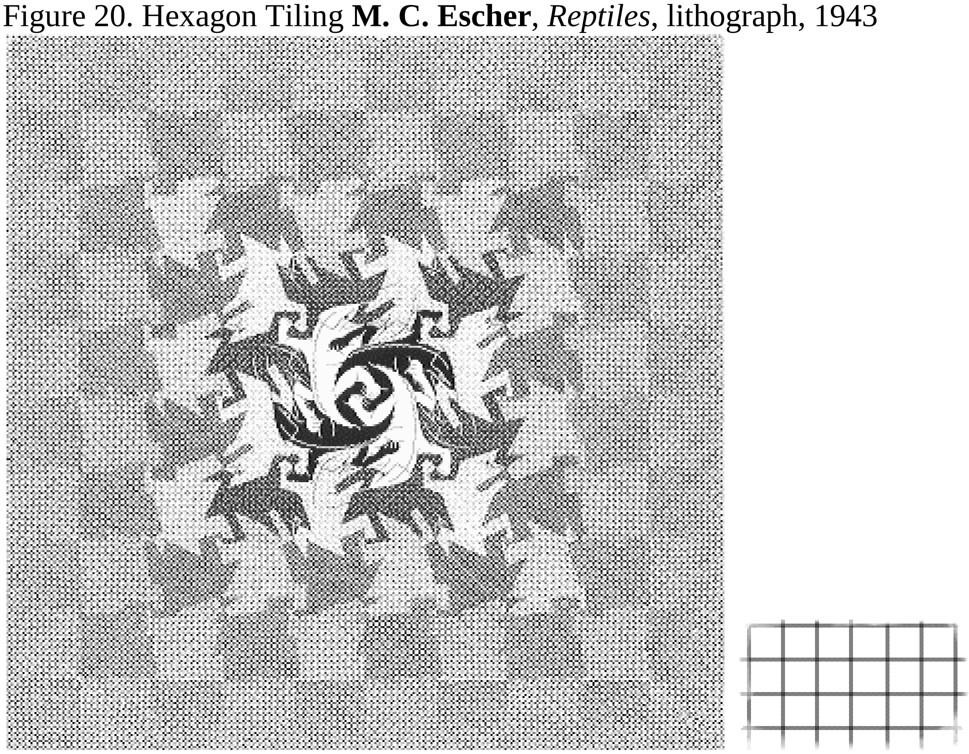


Figure 21. Square Tiling **M. C. Escher**, *Development 1*, woodcut, 1937

In the next activity we will create a drawing together on a grid using a Smartboard. This may also be done with an overhead projector and transparency. Using a background of a plane tiled with squares, we will draw symmetric lines in the appropriate places and then pull the shape out of the grid and redraw it on the side of the grid. Next students will contribute ideas on what the shape should be until we reach an agreement. We will then reproduce the shape across the plane. After this is completed I will ask the students to consider how color and hatching would enhance the picture.

Students will next create a drawing of their own and I will let them use their choice of grid paper. Students will add this drawing to their sketchbooks.

Continuing with symmetry, students will use websites, such as Tessellations.org, to explore and produce different designs. As they work with this website I will ask students about the relationships of the lines and shapes to one another and see if they can tell if the lines that they see are made using translations, rotations, and/or reflections. At school, I have access to a poster maker that produces wonderful color illustrations from computerized images. Using the website, I would like to have the students each produce a final design of their choice and print them as posters to decorate the school. This will depend upon the principal approving the cost of using the poster maker for this purpose. Alternatively, I may be able to have them printed at a local business or university. Using the terminology we have learned so far regarding lines and symmetry, students will describe their designs on 4x6 index cards to post next to their artwork. I will send the artwork home with students in electronic format by emailing it their parents or by writing it to a CD for them to keep. The posters will stay in the school.

Lesson 5 Celtic Knots and Knot Theory

To introduce knot theory, I will have students use Push and Pull Twizzlers to create a knot by making a few crossings or twists in their long piece of candy and then attaching the ends by pressing them together. Next, they will lay their knot flat on their desk and I will ask them to discuss the question “What is a knot?” After the discussion I will make sure that students understand that for knot theory a knot is the same as another knot if you can twist it around or manipulate them so that they look the same. I will ask them to see if they can tell if their knot is the same as their neighbor’s knot. At some point in this activity I expect someone to find an “unknot”, a knot with no crossings, and we will stop to define the unknot or I will introduce the unknot.

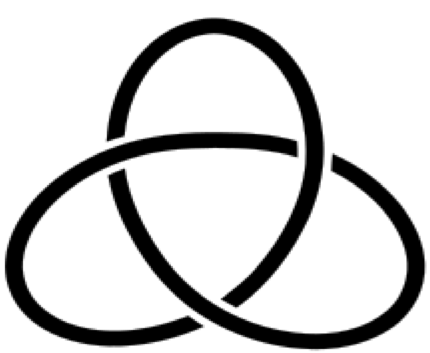


Figure 22. Knot Diagram of the Trefoil Knot

Next I will teach the students to draw a knot diagram by leaving a gap in the line that crosses under another line (see Figure 22). Students will then practice creating and drawing different knots using their Twizzlers. Then using the drawings we will examine the knots for tri-colorability. Students will use three colors to color a knot diagram. They will color each strand and see if they can apply the colors so that the strands are all the same color at a crossing or that they are all different colors at a crossing. Two strands out of a crossing of three strands may not be the same color. After students explore this concept, I will ask them if the unknot is tri-colorable. Then we will determine that a knot that is not tri-colorable is not the unknot. I will then ask if this means that a knot that is not tri-colorable is the unknot, and then give them the diagram of a Figure 8 knot to tri-color and then reproduce with their Twizzlers, so they may see that this knot is not an unknot and not tri-colorable. I will then ask students if what we are doing with knots is math and why or why not?

Extension

Have students color an outline map of the United States using as few colors as they can so that a state of one color does not touch another state of the same color anywhere on the map (see Appendix B for a U.S. map). This is the Four Colored Theorm. Suppose regions which share a border of some length must have different colors. Then any map of regions on a plane or sphere can be colored in such a way that only four colors are needed. Conjectured by Gutrie in 1852; proved by Haken and Appel in 1976.[8]

To bring in the art for this activity we will move to Celtic knots. Celtic knots are interesting because they are produced by a pattern of lines. I will ask students to look at some examples of Celtic knots and explain that we are going to learn the pattern for making these knots and then design and create our own knots in a different medium.

First, students will learn to draw the knots on paper by placing an X over each side in a plane figure. I will give students a variety of plane figures, so they can practice drawing the X's and connecting the lines correctly. I will tell students to place the broken X so that when the X is rotated counter-clockwise the solid line of the X crosses the line on which the X was drawn. I will define the X's so that an X with a solid line originating on the bottom right of the X is a right crossing and a solid line originating on the bottom left of the X is a left crossing. If the broken line crosses this line before the solid line does, they need to switch the direction of the X in regards to its position on top of the side of the plane figure. Students will then connect the ends of the X's by connecting the lines that are closest to one another and when connected, pass a corner on the original plane figure. Once finished, students will check to see if the lines alternate in an over under pattern by tracing each line, one at a time, around the knot.

After students become proficient at this procedure for making Celtic knots, I will ask them to design a knot to produce in three-dimensional form. Students may use the medium of their choice, although I may only be able to provide some of this material in the classroom. If they want something particularly expensive or difficult to get that I don't have access to, they will need to bring it in from home. Some ideas for mediums that come to mind are pipe cleaners, floral or craft wire, bendable straws, clay that hardens, beads strung upon stiff materials such as wire or pipe cleaners, strips of metal if there is no potential for being cut by the metal, braided rope or paper or other material that will braid, fabric or cheesecloth that is dipped in a solution of two parts glue to one part water so that it will harden when dry, or hinges to include hinged Legos. Knots will then be mounted for display.

Extension

Students will make a truncated icosahedron using CD's and railroad ties.

Lesson 6 Graph Theory and Insect Worlds

One of the most interesting ideas I have come across in my study of lines for this unit is graph theory and I would like to introduce the idea to my students using the concept of insect worlds as presented in Knots and Surfaces.[9] I enjoyed working with these problems and spent hours trying to answer some of the first questions in the book, so needless to say I will not be doing too much with this in the interest of time.

I will begin by telling the students that there are worlds where insects live and each world has cities and tunnels that connect those cities. These insects cannot determine the length of the tunnels or the direction that the tunnel is heading, so they do not determine a map of their world by distance or direction. If fact, these words/concepts don't even exist in their worlds! Insects, however, can communicate by cell phone with other insects and so some insects are friends with insects in other worlds. (I have no idea how they established contact with each other originally, but somehow it happened- maybe a wrong number?!) These insects are very curious to know if the maps of their worlds are the same, so we need to find out how they may figure this out. If I give you the insect worlds below (Figure 23), how can you tell me if they are the same or not?

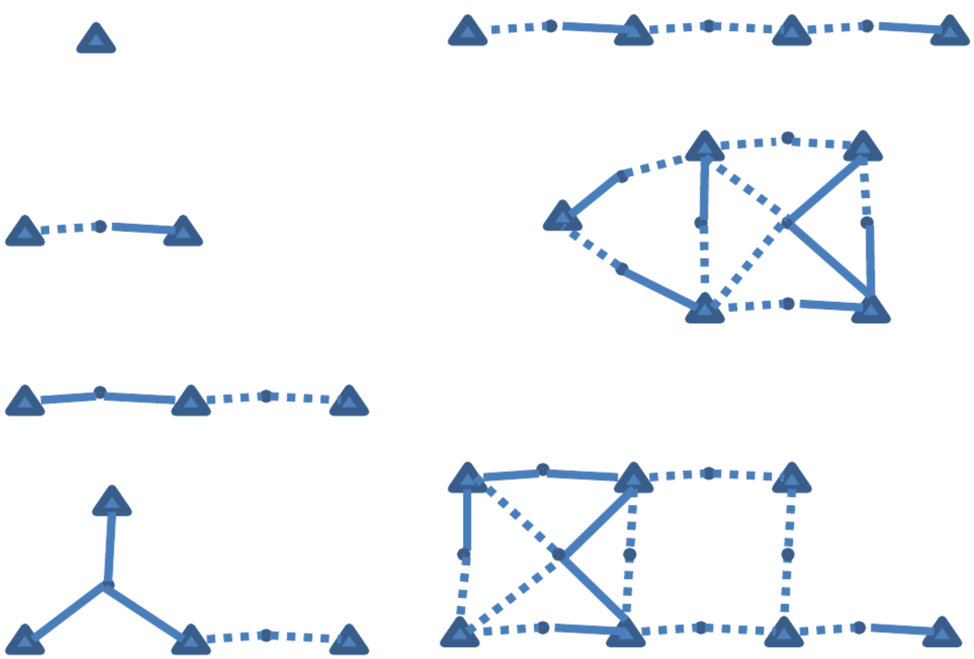


Figure 23. Insect World Diagrams (There is no difference between the solid and dotted paths)

I will present a variety of problems here on determining if the graphs are the same or not and we will discuss how they know. Ultimately, we will establish rules for determining if graphs are the same: 1) they have the same number of cities, 2) they have the same number of tunnels for each city.

After students have become proficient at identifying equivalent worlds I will explain that this is one way that mathematicians look at graphs. Anything in our world can be graphed using the same type of drawings that we have been making of our insect worlds. I will ask students what words they know from their study of mathematics that might represent insect worlds, insect cities, and insect tunnels. Worlds are graphs. Cities are vertices. Tunnels are edges. These graphs represent three dimensional objects. We have just flattened the object onto a plane. If we blew it back up, like we blow up balloons, the number and arrangement of vertices and edges will still be the same. I will ask students how they think this is different than the ways that have learned to graph objects, hoping they can generate the answer of measurement and direction being insignificant for these graphs. I will explain that this is called “graph theory” to mathematicians and ask how they think this may be used in the real world and how is it connected to our study of lines.

As a culminating exercise I will ask students to find all of the graph diagrams with 5 vertices, after we find all of the graph diagrams of 4 vertices as a class. To bring art into this area I will show some examples of sculpture and have students discuss how the sculpture would be represented using graph theory.

Annotated Bibliography

ArtyFactory, "Pen and Ink Drawing Examples of Cross Hatching." http://www.artyfactory.com/pen_and_ink_drawing/ink_drawing/cross-hatching_examples.htm (accessed 11/28/2010). This website had excellent resources for hatching. There is a slide show under the pen and ink drawing section that is fantastic for showing the process of how an artist goes through the process of adding cross hatching to a drawing.

Beasley, J E. "OR- Notes." http://people.brunel.ac.uk/~mastjjb/jeb/or/graph.html (accessed 11/30/10). This short paper had information on the Euler's Konigsberg bridge problem and its solution.

"Curve Stitching." http://www.montessoriworld.org/Handwork/stitch/stitch1.html (accessed 11/28/2010). The string art directions in Appendix A were adapted from this website. It has some very good lessons on string art and other types of art projects using symmetry and lines.

Edutopia, "http://www.edutopia.org/." edutopia.org (accessed 11/16/2010). Fanastic site for teachers and students on a variety of topics to include geometry resources.

Escher, M.C.. "Development 1." 1937.http://www.mathacademy.com/pr/minitext/escher/ (accessed 11/28/2010). This Escher artwork illustrates the way a plane tiled with squares may be distorted to create art.

Escher, M.C.. "Regular Division of the Plane with Birds." 1949.http://www.mathacademy.com/pr/minitext/escher/ (accessed 11/28/2010). This Escher artwork illustrates the way a plane tiled with equilateral triangles may be distorted to create art.

Escher, M.C.. "Reptiles." 1943.http://www.mathacademy.com/pr/minitext/escher/ (accessed 11/28/2010). This Escher artwork illustrates the way a plane tiled with hexagons may be distorted to create art.

"Jackson Pollock ." jacksonpollock.com (accessed November 28, 2010). This website is where I found the paintings *Guardian of The* and *Number 7* by Jackson Pollock to illustrate the use of lines in artwork.

Kalajdzievski,Sasho. *Math and Art An Introduction To Visual Mathematics.* Boca Raton: Taylor & Francis Group, LLC, 2008. This book was used for the entire unit and has in-depth information on most of the topics mentioned in this unit. It explains much more of the math involved in the various areas of art.

Malak, Mike. "Railroad Tracks Perspective." 05/23/2006.http://en.wikipedia.org/wiki/File:Railroad-Tracks-Perspective.jpg (accessed 11/28/2010). This is a photo to illustrate a vanishing point and a horizon line, which are key terms in studying perspective.

McCraken, John. "23.” 12/9/2010. http://en.wikipedia.org/wiki/File:%2723%27,_oil_on_canvas_painting_by John_McCracken,_1964,_Smithsonian_American_Art_Museum.jpg (accessed 11/28/2010). This is the website I accessed the painting by John McCraken, 23, to illustrate the use of lines in artwork.

Noland, Kenneth. "Beginning."9/15/2008.http://en.wikipedia.org/wiki/File:%27Beginning%27, Magna_on_canvas_painting_byKenneth_Noland,_Hirshhorn_Museum_and Sculpture Garden, 1958..jpg (accessed 11/28/2010). This is the website I used for the image of the painting, Beginning, by Kenneth Noland. This painting illustrates the use of lines and color.

"One Point Perspective Drawing." http://www.la-art-tutor.com/2007/10/17/one-point-perspective-drawing/ (accessed 11/30/2010). This is a website that teaches art technique for the Greater Los Angeles area. It had a comprehensive site of non-profit organizations connected with art education.

"Online-Instructions - Learn to Paint and Draw." http://www.art-class.net/art-site/learn-to-draw/drawing-perspective.php (accessed 11/30/2010). This website has instructions and illustrations for perspective drawing. I used this website for the Figure 3 on two point perspective.

"Perspective Drawing." http://www.explore-drawing-and-painting.com/perspective-drawing.html (accessed 11/30/2010). This website has good instructions on basic perspective drawings and I used it for the illustration in Figure 1.

"Review of Basic Geometry." http://www.art-class.net/art-site/learn-to-draw/drawing-perspective.php (accessed 11/30/2010). I used this to check my geometry term definitions.

"Tesselation Do-It-Yourself." http://www.tessellations.org/index.htm (accessed 11/30/10). Fantastic, award-winning site for kids. Excellent instructions and on-line interactive tools for making tilings/tessellations.

Van Gogh, Vincent. "Fountain in the Garden of the Hospital, St Remy." http://www.artyfactory.com/pen_and_ink_drawing/ink_drawing/cross-hatching_examples.htm (accessed 11/28/2010). I found this painting to use as an expressive example of cross hatching

Notes

[1] See David W. Farmer and Theodore B. Standford, *Knots and Surfaces* (Providence, RI: American Mathematical Society, 1996), back cover.

[2] See "http://www.edutopia.org/." edutopia.org (accessed 11/16/2010).

[3] See Rosalind Ragans, Willis Davis, Tina Farrell, Cloria McCoy, Nan Yoshida, Bunyan Morris, Jane Hudak, and Jackie Ellett, *SRA Art Connections* (Columbus, OH: SRA/McGraw-Hill, 2005), 22.

[4] See Sasho Kalajdzievski. *Math and Art an Introduction to Visual Mathematics* (Boca Raton, FL: Taylor and Francis Group, LLC, 2008), 170-172.

[5] See David W. Farmer, *Groups and Symmetry A Guide to Discovering Mathematics* (Providence, RI: American Mathematical Society, 1996).

[6] Farmer, *Knots and Surfaces*.

[7] Beasley, J E. "OR- Notes." http://people.brunel.ac.uk/~mastjjb/jeb/or/graph.html (accessed 11/30/10).

[8]Farmer, *Knots and Surfaces*.

[9] Farmer, *Knots and Surfaces*.