Measurements and Uncertainties in Science or How Do We Know That!

Debra Blake Semmler

Introduction

Science is a human adventure into trying to understand and predict the Universe around us. Think of trying to understand the rules for a card game such as poker by watching others play the game. Over time you will be able to decipher some of the rules and predict who will win which hand. To reach the goal of understanding the Universe around us we make observations and develop theories to the rules and test our ideas by predicting the outcome of the next hand. This process is the scientific method. Everything scientists do in their search for understanding is based on observations and measurements. How much you know about something is related to how well you can measure it. The famous physicist Lord Kelvin stated that "when you can measure something and express it in a number, you know something about it." [1] Measurement in science is not a new concept but found its beginnings in ancient times when the sizes of the Earth, Moon, Sun and the distances between them were determined by indirect measurements. I want to use the measurements made by ancient astronomers to demonstrate that measurements have been made since the beginning of societies and many of the measurements were made to a high degree of precision. But even though science has grown in its understanding of the rules of the Universe there are always uncertainties in any measured quantity and some measurements even today have large uncertainties. Scientists must understand what uncertainties in measurements are and how they are propagated to correctly discuss the results for any experiment. Experimentation is the backbone of any scientific theory and is always based in measurements and measurements always have uncertainty.

Classroom Environment

I teach at an urban, partial magnet high school of about 1800 students. Approximately 600 students are part of the International Baccalaureate (IB) magnet. The school is approximately 49 % African Americans, 28 % white, 15% Hispanics and 6 % Asian. More than 50% of the student population is on free and reduced lunch. I teach introductory algebra-based physics to tenth, eleventh and twelfth grade students on a semester block program. The tenth grade students are part of the International Baccalaureate (IB) magnet program at my school. I also teach advanced placement (AP) algebra and calculus-based physics courses and the upper level IB Physics courses. The IB Physics 2 and 3 are taught on an A-day/B-day schedule over two years with standard level(SL) and higher level(HL) students mixed within the same classroom. I will use most of the curriculum unit materials in the upper-level IB physics classes. Knowledge of uncertainty analysis and error propagation is an integral part of the IB science curriculum. The ideas presented and the uncertainties analysis will be used by all IB science students and added to lower level physics courses.

History of Measurements

> *"Weights and measures may be ranked among the necessaries of life to every individual of human society. They enter into the economical arrangements and daily concerns of every family. They are necessary to every occupation of human industry; to the distribution and security of every species of property; to every transaction of trade and commerce; to the labors of the husbandman; to the ingenuity of the artificer; to the studies of the philosopher; to the researches of the antiquarian; to the navigation of the mariner, and the marches of the soldier; to all the exchanges of peace, and all the operations of war. The knowledge of them, as in established use, is among the first elements of education, and is often learned by those who learn nothing else, not even to read and write. This knowledge is riveted in the memory by the habitual application of it to the employments of men throughout life."*
> *JOHN QUINCY ADAMS - Report to the Congress, 1821*

The earliest societies needed tools to measure material to build shelter and clothing and for bartering for food. The first measurement tools were parts of our bodies such as forearm, hand or finger. Time was measured by the periods of the Sun, Moon or stars. When volume was needed to be measured gourds or clay vessels were filled with seeds and the seeds counted to estimate the volume. The "carat" that is still used as a mass unit for gems, is derived from the carob seed. In the third century BC measurements of the size of the Earth, Moon and Sun and the distance between them were made and the accuracy of some of these measurement are surprisingly close to the modern accepted values.[2]

How Big is the Earth?

In 235 BC, an Egyptian geographer and mathematician, Eratosthenes, calculated the circumference of the Earth based on the measurement of the shadow of a stick. Based on Eratosthene's observations he knew the Sun was at its highest point in the sky at the summer solstice around June 22. At the summer solstice in any city on the equator, the Sun is directly overhead and a vertical stick will not cast a shadow. At this time in Alexandria, 800 km north of the equator the shadow of a stick was measured to be one-eighth the size of the stick. From similar triangles, we see that this means that the ratio of the shadow length to the stick height is proportional to the ratio of the distance from Alexandra to the equator to the radius of the Earth. (See Figure 1 and 2)

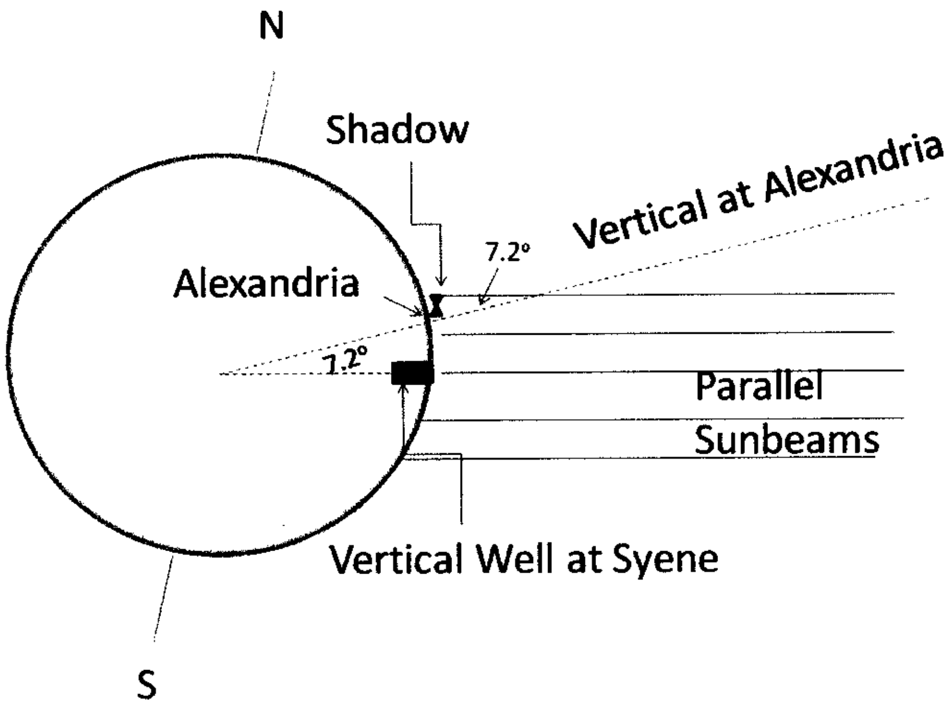


Figure 1

$$\frac{\textit{shadow length}}{\textit{stick height}} = \frac{\textit{distance between cities}}{\textit{radius of Earth}}$$

$$\frac{1}{8} = \frac{800\ km}{\textit{radius of Earth}} \therefore \textit{radius of Earth} = 8 \times 800\ km$$

Therefore, the radius of the Earth is 8 times the distance from the equator to Alexandria or 8 times 800km or 6400 km. We have since refined our measurements and the radius of the Earth is found to be 6370 km, which is within 1% of the value that Eratosthenes found. I wonder why we are taught that Columbus and the Europeans of the fifteenth century believed the Earth was flat, yet we knew the radius of the Earth as early as 235 BC.[3]

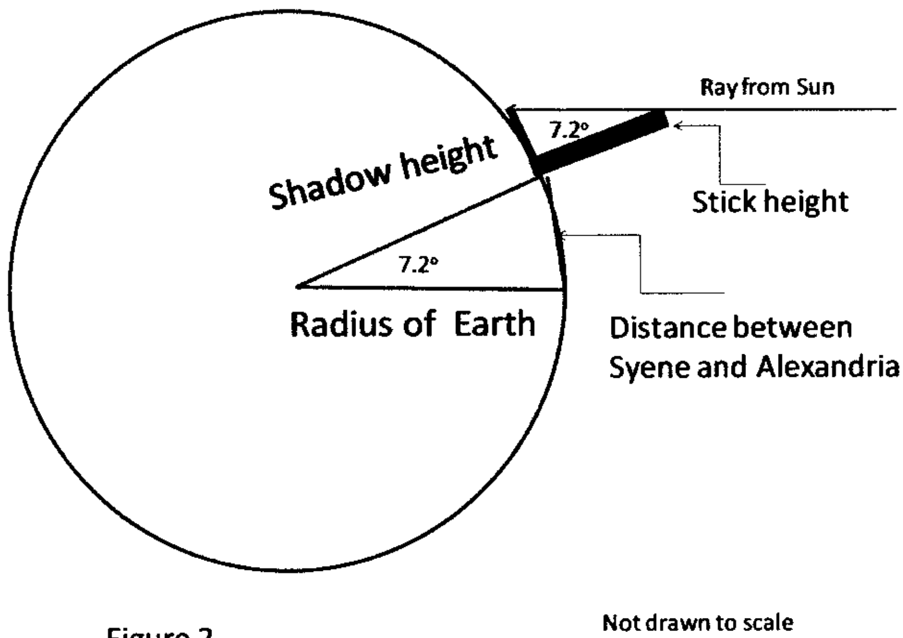


Figure 2

How big is the Moon?

In 240 BC, Aristarchus was perhaps the first to suggest that the Earth spins on an axis and this accounts for day and night and that the Earth revolves around the Sun once a year. Aristarchus precisely measured the diameter of the Moon by using the shadow on the Moon created during an eclipse of the Moon. An eclipse of the Moon is created when the Moon passes into the shadow of the Earth. By carefully studying the shadow of the Earth cast on the Moon, Aristarchus found that the width of the Earth's shadow was 2.5 Moon diameters. This would mean that the Moon's diameter is 2.5 times smaller than the Earth's diameter. However, shadows taper over large distances (such as the distance between the Earth and the Moon). Aristarchus determined that the taper of the Moon's shadow was equal to one Moon diameter by measuring the Moon's shadow during a solar eclipse, which was reduced to a point (see Figure 3). Using the data from the solar and lunar eclipses, Aristarchus concluded that the diameter of the Earth was 3.5 times the diameter of the Moon. So if the diameter of the Earth is 12,800 km the diameter of the Moon would be1/3.5 times 12,800 km or 3,660 km. Today the accepted value is 3,640 km and his value is within 5%.[4]

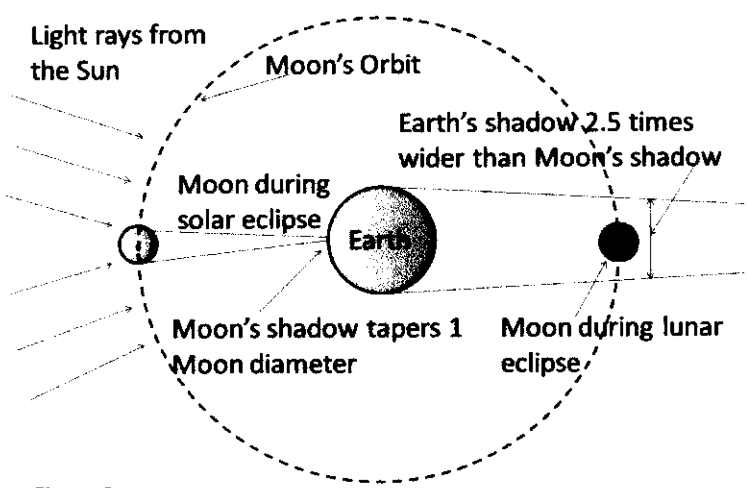


Figure 3

How Far is it to the Moon?

The early Greeks knew how to measure large distances by using ratios of sizes. If you tape a dime or other coin to a window and view it with one eye so that it just blocks out the full Moon, the ratio of the coin diameter to the distance from the coin is about 1 to 110. This is equal to the ratio of the Moon's diameter to the Moon's distance. (See Figure 4.) Therefore, the distance to the Moon is 110 times the diameter of the Moon and was determined to be 403,000 km. Today after visiting the Moon, the distance is known to be 384,000 km. These values are within 5 % of each other.

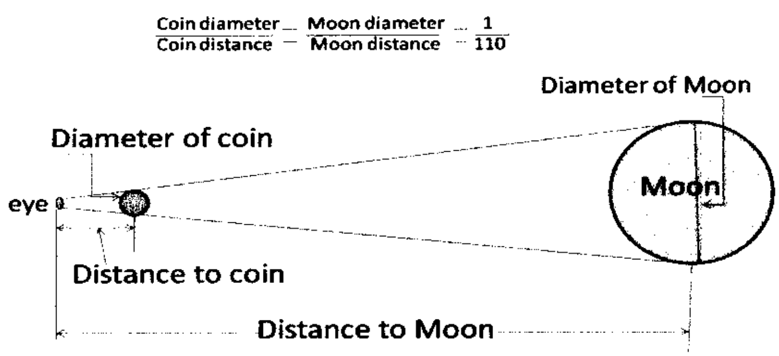


Figure 4

How far is it to the Sun?

Knowing the diameter of the Moon allowed Aristarchus to measure the distance to the Moon. But to use this same method to measure the distance to the Sun, Aristarchus needed to know the diameter of the Sun. The early Greeks repeated the coin on the window exercise for the Sun and the ratio of the Sun's diameter to the Sun distance is also 1 to 110. This is because the Sun and Moon both have the same size to the eye from the Earth and both taper at the same angle of about 0.5°. To determine the distance to the Sun, the Sun's diameter had to be known. Aristarchus made observations of the phases of the Moon. He determined that when the Moon was exactly half full, the Sun must be at a right angle to the observer's line of sight (see Figure 5). The lines formed between the Earth and the Moon, and the Earth and the Sun, and between Moon and the Sun form a right triangle. If Aristarchus could measure a second angle between his line of sight to the Moon and his line of sight to the Sun, the distance to the Sun and the diameter of the Sun could be determined. This angle was very difficult to measure to any degree of precision. One source of uncertainty is that the Moon and Sun are not points in the sky but rather large objects compared to other sky objects. The measurements had to be made to their centers (or either edge) and the angle was almost at ninety degrees. Aristarchus measured the angle to be 87° and the modern day angle has been measured to be 89.8°. Using right-angle trigonometry Aristarchus figured the Sun to be about 20 times more distant than the Moon, when in fact it is about 400 times more distant. Aristarchus's measurement was not close to today's accepted value of 150,000 ,000 km. Is this because his measurements were inexact or because he did not believe the Sun could be so far away? [5]

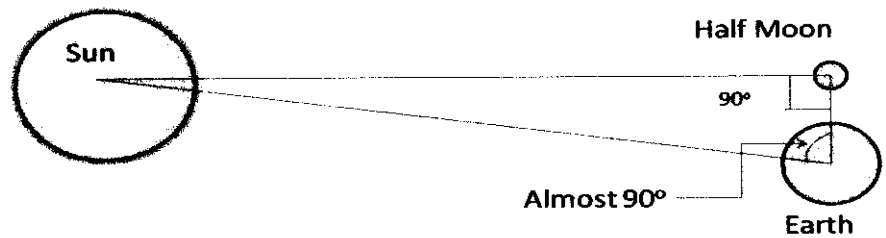
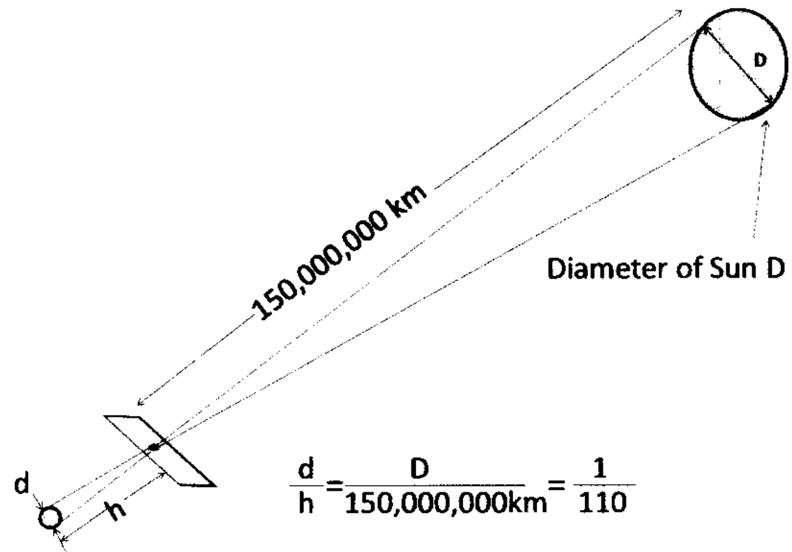


Figure 5

Once the distance to the Sun is known, using the ratio of the diameter of the Sun to the distance to the Sun of 1 to 110 allows us to calculate the Sun's diameter. Another way to measure the 1 to 110 ratio is to measure the diameter of the Sun's image that is made through a pinhole opening. (Figure 6)



$$\frac{d}{h} = \frac{D}{150{,}000{,}000\text{km}} = \frac{1}{110}$$

Figure 6

The ratio of the Sun's image size compared to the distance from the pin hole is 1 to 110, the same ratio of the Sun's diameter to the Sun's distance.[6]

Experimental Uncertainty

The early astronomers most likely knew their measurements were not exact. Even scientists from the early twentieth century had to round any calculated values because all calculations were completed without the aid of a calculator. Why would a scientist take the time or energy to make a calculation to five significant figures if the measurement were made only to two significant figures? In addition, any trigonometric function values were found in tables and only listed to four significant figures within one degree of angle measure. With the invention of the calculator, computation to twelve figures is processed in the blink of an eye. Trigimetric functions are stored to 12 decimal places for any angle, even angles that are humanly impossible to measure. How can we physically measure an angle to the 1/100,000 degree? I believe the use of calculators has lead to the misunderstanding of uncertainty of measurements by students. I know calculators are here to stay and I am glad to have my calculator, but it means that more effort must be placed on teaching students about significant figures, uncertainty and error.

Any measured quantity has some degree of uncertainty that can come from a variety of sources. Any complete statement of a measured quantity must include an estimate of the level of confidence associated with how the value was measured and a unit. Without an estimate of the uncertainty in a measurement it is impossible to answer the question of how well my result agrees with a theoretical prediction or other experiment.

There is a certain inherent inaccuracy or variation in any measurement we make. This inherent inaccuracy or variation is called experimental error. The word error is not meant to imply incorrect or incompetent, it merely reflects the condition that our measuring instruments are imperfect. We can control lack of perfection in our procedures and mistakes in calculation or writing data incorrectly. These mistakes have nothing to do with experimental error and everything to do with the ability of the experimenter and can be controlled.

Systematic Error

Systematic errors are reproducible inaccuracies that are consistently in the same direction. These errors are difficult to detect and cannot be analyzed statistically. If a systematic error is identified when calibrating against a standard, applying a correction factor to compensate for the effect can reduce the bias. Systematic errors cannot be detected or reduced by increasing the number of measurements. For example you would like to measure the mass of your apple you are eating for lunch and you find an electronic balance in the classroom. When you place the apple on the balance it reads 187.45 g. You know about repeating your measurement, so you repeat placing your apple on the balance three times and get 186.98 g, and 187.33 g. You take a look at the apple and wonder are these values correct? How do we know? So you decide to try and measure the apple on another electronic balance and you find the mass to be 115.75 g. You repeat the measurement three times with the same approximate value. All these measurement are very precise because they have a specific value that is repeatable, but the measurements are very different. This is because one or both of the balances is not accurate, that is it is not close to the true value of the quantity. In order to measure the accuracy of a value a true value must be known. In the case of the apple, the only way to find the true value for the mass of the apple is to check both balances to a known standard mass.[7]

Random Error

Random errors are statistical fluctuation in both directions in measured data due to the precision limitations of the measurement device. Random errors can be evaluated through statistical analysis and can be reduced by averaging over a large number of measurements. So how do you handle estimating experimental uncertainty for a single measurement? For example, you measure the diameter of tennis ball and you use a meter stick. The uncertainty might be ± 5 mm, because of parallax in using a ruler to measure the diameter of a round object. If you use a Vernier caliper the uncertainty could be reduced to ± 2 mm because the tennis ball is fuzzy. Do you measure to the ball or the outside of the fuzz? In both of these cases the uncertainty is larger than the smallest division on the scale because of the difficulty in using the instrument to it fullest precision. The person making the measurement reports the uncertainty in a way that clearly explains the quality of the measurement. All scientific measurements are reported in the following format

Measurement = (measured value ± standard uncertainty) unit of measurement

For example, the diameter of the tennis ball is 6.7 ± 0.5 cm using a ruler and 6.7 ± 0.2 cm when using a Vernier calipers.

Multiple measurements of the same quantity will reduce random error. For example if 25 people were to measure the width of a piece of paper using a typical metric ruler there would be 25 different measurements because each person would view the ruler from slightly different angles and make different judgments about the exact reading. All of the measurements would be distributed around a mean value of 21.6 cm. Most of the values would be within a few hundredths of this value. A graph of the frequency of the measurement versus the width of the paper would result in a histogram as shown in Figure 7. The smooth curve superimposed on the histogram is the normal distribution about a mean value of 21.6 cm.

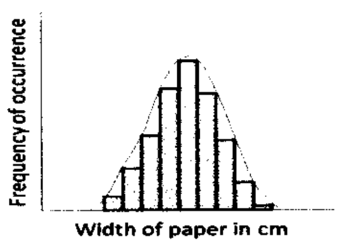


Figure 7

Random error almost always results in a bell shaped curve and the approximation of the uncertainties can be determined by calculating the average deviation or the standard deviation.

The average is the best available estimate of the measured quantity but still not exact. So how do we express the uncertainty for this average value? One method is to use the average deviation. This statistic tells us on average (50% confidence) how much the individual measurement varies from the mean. The average deviation is calculated by using the following formula:

$$\bar{d} = \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \ldots + |x_i - \bar{x}|}{N}$$

The standard deviation is the most common way to characterize the uncertainties in the spread of the data. The standard deviation is always slightly greater than the average deviation and is used because of its association with the normal distribution. To calculate the standard deviation for a sample of N measurements:

1. Sum all the measurements and divide by N to get the average, or mean.
2. Now, subtract this average from each of the N measurements to obtain N deviations.
3. Square each of these N deviations and sum.
4. Divide this result by (N-1) and take the square root.

The formula for the standard deviation is as follows. Let the N measurements be $x_1, x_2, \ldots\ldots x_N$. Let the average of the N values be $\bar{x}$. then the deviation is given by the expression:

$$\delta x_i = x_i - \bar{x}, \qquad for \;\; i = 1,2,3 \ldots. N$$

The standard deviation is:

$$\sigma = \sqrt{\frac{(\delta x_1^2 + \delta x_2^2 .. + \delta x_N^2)}{(N-1)}} = \sqrt{\frac{\sum \delta x_i^2}{(N-1)}}$$

The significance of the standard deviation is that within one standard deviation approximately 68% of the measurements will be measured compared to the mean value, 95 % of the readings will be within two standard deviations and nearly all, 99.7%, will lie within three standard deviations of the mean.

Propagation of Uncertainty

What if you use measurements to calculate a result that is dependent on the two or more measurements? How is the uncertainty in each measurement propagated in the result? For example, if you need to measure the area of a rectangle, Area = length × width. Let the area be the function f and for the area, $f = xy$. The error in x is σ_x and the error in y is σ_y. for a single-variable function the deviation in f can be related to the deviation in x using calculus: $\delta f = \left(\frac{df}{dx}\right) \delta x$, Then take the square and the average results in $\sigma_f = \left|\frac{df}{dx}\right| \sigma_x$ so for the function $f = xy$

$$\frac{df}{dx} = y, \qquad \frac{df}{dy} = x$$

$$\therefore\ \sigma_f = \sqrt{y^2\,\sigma_x^2} + \sqrt{x^2\sigma_y^2}$$

Dividing this equation by $f=xy$, will result in

$$\frac{\sigma_f}{f} = \sqrt{\left(\frac{\sigma_x}{x}\right)^2} + \sqrt{\left(\frac{\sigma_y}{y}\right)^2}$$

For velocity = displacement /time or resistance = voltage/current where the function is in the $f = x/y$ format, the result of the partial derivatives will give the same result. In the appendix is a table of common formulae and uncertainty formulae found using the partial derivative.

If one of the uncertainty terms is more than three times greater than the other terms, the root-squares formula can be skipped and the combined uncertainty is simply the largest uncertainty.[8] In the appendix are student handouts that include summary notes, the propagation of uncertainty table and student practice problems.

Modern examples of Uncertainties

I want students to know that even though we now have the ability to measure incredibly small masses such as the mass of an electron, 9.11 E -31 kg (notice only three significant figures) and incredible distances such as the distances to stars and galaxies, there are still limits to all measurements and in some cases large uncertainties. Even the best techniques and instruments have limits to their measurements.

To determine the distance to stars, astronomers use geometric techniques that are based on parallax. Parallax is the apparent displacement of an object because of a change in the point of view. Parallax can be observed when nearby objects appear to shift their positions against a distant background as you move from place to place. For stars, as the Earth orbits the Sun, stars appear to move relative to the background of the more distant stars.

The distance to a star can be determined by measuring the parallax (p). The parallax is half the angle through which the star's apparent position shifts as the Earth moves from one side of its orbit to the other. (Figure 8) If the angle p is measured in seconds of arc, then the distance d to the star in parsecs is given by the equation $d = \frac{1}{p}$. For example, a star whose parallax is $\frac{1}{2}$ arc sec is 2 parsecs from the Earth.

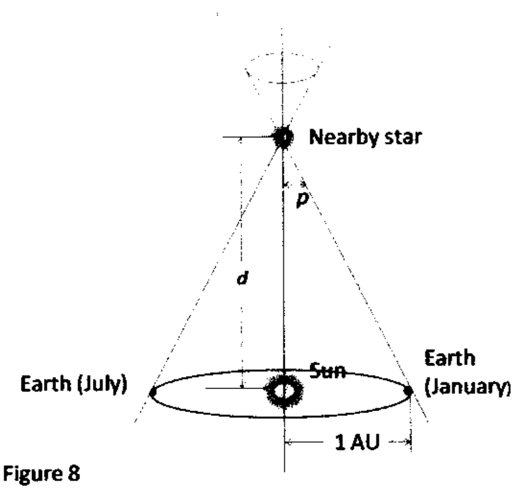


Figure 8

The parallax of Proxima Centauri, our nearest star, is comparable to the angular diameter of a dime seen from a distance of two miles (I could not see a dime two miles away!). Its parallax is 0.77 arc sec, and thus its distance is 1.3 pc. Because parallaxes smaller than about $\frac{1}{20}$ arc sec are difficult to measure precisely for the Earth observatories, the parallax method gives reliable distances only for stars nearer than about 20 pc.[9]

The Hubble Constant

The Hubble constant is one of the most important numbers in astronomy. It determines the rate at which the universe is expanding and thus tells us the age of the universe. Because of its importance astronomers are very interested in an accurate determination of the Hubble constant H_o.

In order to determine the Hubble constant, an astronomer must measure the Doppler red shifts and distances to many galaxies. Using the parallax method, from the Earth's surface, is impossible for distant galaxies. Astronomers use the apparent luminosity(brightness) and periods of pulsating stars.

Hubble's initial value for the expansion rate of the Universe, the Hubble Constant, determined the expansion age of the Universe was only 2Gyr, but by 1930 radioactive dating of rocks had shown that the age of the Earth was 3 Gyr (one Gyr is 10^{12}years). Hubble's initial value for the constant was greater than 600 km/s/Mpc, but by 1960 the value of H_o was consistently less than 100 km/s/Mpc but with uncertainties of ± 50 km/s/Mpc. Over the past few decades, leading astronomers reported values for the Hubble constant that varied between 50 (km/sec)/Mpc and 100 (km/sec)/Mpc. The difference of 50 km/s/Mpc is a difference in the age of the Universe of 10 Giga(10^{12})years (9.7Gyr for H_o = 100 and 19.4 for H_o = 50). In recent years data from the Hubble Space Telescope (HST) has greatly reduced the variation. [10]

Conclusion

Even with all the technological advancement of the last century, computers, calculators and lasers, scientific discovery continues to be based in observation and measurements. To measure the Hubble constant, astronomers must measure the distance to a galaxy using the apparent brightness and the red shift of the light from the galaxy. These measurements are as difficult as the ones made by Aristarchus in measuring the distance to the Sun. I am going to use these ancient and modern scientific measurements to teach students the importance of reporting uncertainty in experimental data and analysis. The drawings and descriptions in this document will be used as a presentation on measurements and uncertainties to my students. The handouts will be used as reference and practice of the techniques of finding averages and standard deviation and error propagation. The real test of this curriculum unit will be how students approach reporting and analysis of data in the experiments they perform.

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[1] Paul Hewitt, *Conceptual Physics* (New York: Harper Collins,1993), 2.
[2] NASA, "A Brief History of Measurements", (https://standards.nasa.gov/history_metric.pdf, 23 Oct.2010)
[3] Tom Apostol et al., *Mechanical Universe:Trigonometry Primer and Student Study Notes.* (Dubuque, Kendall/Hunt, 1986),7
[4] Hewitt, 3.
[5] Hewitt, 4.
[6] Hewitt, 6.
[7] UNCCH Lab Staff, *Physics Laboratory Manual*, (Chapel Hill, Course Pack Publishing, 2007), 13.
[8] UNCCH Lab Staff, 15.
[9] Roger A. Freedman And William L. Kaufmann, *Universe*,(New York, W.H. Freeman, 2005)586.
[10] Freedman, 588.

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Appendix 1 Student handouts

Review-Significant Figures Rules

Significant figures are critical when reporting scientific data.
The rules for significant figures are:

1. All non-zero numbers are always significant
2. All zeros between non-zero numbers are always significant
3. All zeros which are to the right of the decimal point and at the end of the number are significant
4. All zeros which are to the left of a decimal point and are in a number greater than or equal to 10 are significant. If you need the zero when writing the number in scientific notation then it is significant.

When adding and subtracting numbers using significant figures round to the least measured decimal point. When multiplying and dividing numbers using significant, figures round to the least count in the measurements.

Guidelines for using uncertainties and significant figures.

The uncertainty, σ, in the final result should have, at most, two digits, and generally only one digit. All uncertainty calculations are estimates, and there is no such thing as an "exact uncertainty." The rule is if the first digit is 1, use 2 digits for sigma, e.g. $\sigma = 0.14$ g or $\sigma = 0.3$ not $\sigma = 0.34$g. If σ is especially large, you will lose significant digits. For example, suppose that multiple measurements are made with an instrument that is precise to three sig, digs, and the mean value of 9.52 s is found, but for other reasons the data points varied so that the standard deviation of the mean was 2 s. The result would have to be reported as 9 ± 2 s.

If the measurement is so inexact that σ is larger than the value itself, you will not have significant digits, but only know the order of magnitude. This case is most common when the quantity in question is expected to be close to zero.

If σ is calculated to be much smaller than the smallest digit of your measurement, then assume that σ is equal to one of the smallest digits. For example, if a measurement of a mass gives exactly 8.45 g ten times, the result should be stated as m = 8.45 ± 0.01 g. Thus you may need to round your uncertainty up to the least significant digit in your measurement.

Do not confuse round-off errors with uncertainty. With calculators and computers, there is no reason to round an intermediary result, just because it is found to be uncertain. If properly used, the formulas for propagation uncertainty will take care of the uncertainty in the final result. So keep your extra digits as you go but make sure to adjust the final result when you present your measurements for comparison.

Table 1:
Common formulas for propagating uncertainty. These equations can be combined to form more complicated formulas.

Functional Form	Formula	Uncertainty formula
Product or Quotient	$f = xy \text{ or } f = x/y$	$\sigma_f = \sqrt{\sigma_x^2 + \sigma_y^2}$
Sum or difference	$f = x + y \text{ or } f = x - y$	$\sigma_f = \sqrt{\sigma_x^2 + \sigma_y^2}$
Product raised to powers	$f = x^m y^n$	$\sigma_f = \sqrt{m^2\sigma_x^2 + n^2\sigma_y^2}$
Constant multipliers	$f = Kx$ (K is constant)	$\sigma_f = K\sigma_x$
Logarithmic functions	$f = \log_e x$	$\sigma_f = \sigma_x$
	$f = \log_{10} x$	$\sigma_f = 0.4343\sigma_x$
Exponential functions	$f = e^x$	$\sigma_f = \sigma_x$
	$f = 10^x$	$\sigma_f = 2.303\sigma_x$

Practice Questions:

1. A standard household thermometer has one mark for every two °F. What is the minimum uncertainty that you should assign to the temperature that you read for such a thermometer? What do you think is the best uncertainty to assign to this reading? Do you think larger than this? Explain your reasoning.

Standard deviation problem

Trial	Group 1	Group 2
1	9.7	16.4
2	9.9	4.6
3	10.5	2.6
4	10.1	20.3

5	9.6	5.2
6	9.7	
7	9.4	
8	9.8	
9	10.0	
10	9.6	

2. Find the mean and standard deviation for the two lab groups and report the values for the measurement including the uncertainty. Do both measurements have the same precision?

3. What would the value of measurements be if you found the product of group 1 times group 2 include the correct uncertainty. What would be the result for the quotient and sum of the two groups of data?

Annotated Bibliography

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Appendix 3

NC state standards met in this Curriculum Unit

Competency Goal 1: The learner will develop abilities necessary to do and understand scientific inquiry.

1.01 Identify questions and problems that can be answered through scientific investigations
1.02 Design and conduct scientific investigation to answer questions about the physical world
1.03 Formulate and revise scientific explanations and models using logic and evidence

1.05 Analyze reports of scientific investigations of physical phenomena from an informed scientifically literate viewpoint including consideration of

- Adequacy of experimental controls
- Replication of findings
- Alternative interpretations of the data