More than Just the Basic Facts, Please

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Introduction

I remember enjoying mathematics as a child. We sorted all kinds of objects any way we wanted and gave each pile a name. We counted shells by putting them in piles of 5 or 10. We made patterns by stamping geometric shapes in a row. It was simple and fun and I felt powerful because I was in control. As the years went by, math became more of a book and a paper and pencil thing. We learned how to manipulate larger and larger numbers by learning the basic operations and their properties. There was a "way to do it". Math was okay, but mathematical thinking was getting less fun because there was so much to remember. In fourth grade, I wanted to get on the Batman board for knowing my basic multiplication facts. The teacher made a big deal about getting your math quiz on the board. It was cool. Mom bought flash cards. I memorized most of them so I could be on the Batman board. Then, you had to memorize ways to do area, perimeter, long division and multi-digit multiplication. Where you put in zeros became steps to follow. There were times when fractions had to be flipped and cross multiplied or you found common numbers inside bigger numbers before you could even add or subtract them. My powerful feelings regarding math were waning. My control over manipulating numbers was fading, but my determination to "get it" was still strong. In 7th grade, rumors around school said, "Algebra is hard." As we sat isolated in rows that first day, my teacher confirmed that talk in the hallways. My peers and I felt like mathematical thinking was a NASA rocket blasting off and we were left on the platform amid the smoke and flames. My teacher was on board that flight, with a starry look in his eyes like, "Soon I will see the glories of the universe." That sparkly look meant he knew the language of math. I was very much left behind wondering how I would get through it.

Now, I am the teacher of the fourth grade classroom. I see the same glimpses of doubt in the eyes of those nine year olds as we begin our study of mathematics. As I watch them perform some preliminary assessments, many use their fingers to add combinations to 20. They count out the first addend, then count out the second addend, and then count them all again. I ask a few if they have ever tried "counting on" as a strategy. "Put the bigger number in your head and then count on the second number to get your sum." They look nervously at me as if they are thinking, "...but my way works for me". When we review subtraction with regrouping, some moan. Within one group of 27 students, there is everything from mastery to "I don't remember how to do that." When I look at a child thinking that 434 – 148 = 314, then, I ask if that answer makes sense. The child shrugs. "Can you subtract 8 from 4?" I ask. The student replies, "Oh yeah, I need to borrow." One third grade objective is mastering the multiplication combinations to 5 X 10. They are adept at using fingers to repeat add when we solve 3 X 6. I am amazed as I walk around the groups seeing fingers flicking quickly under the desks. When I ask if there is another way besides repeat addition, a few students report, "You just know it!" The others hang on to the technique they learned in grades 2-3 because those are familiar and give them a sense of power. Conversations where I assess place value result in similar pockets of either confident understanding of the one way to "get it" or lack of remembering the process. I can compare their thinking of mathematics to Swiss cheese. There are random holes in their comprehension of math concepts which shake their foundation of mathematics.

Rationale

I want this unit to help students think about math at a deeper level of understanding than memorization of facts and algorithms. In elementary school, learning basic facts and how the basic operations work is critical to building a strong foundation for future mathematical thinking. Two major difficulties with problem solving are figuring out what operation to use and then computing the data correctly. Multiplication facts are used in solving problems with all strands of mathematics. Some examples follow: Fractions: 1/8 + 2 1/3 Decimals: What is the cost of 24 sodas at $2.89 per six-pack? Geometry: What is the area of a playground that is 48 yards by 25 yards? Measurement: How many cups are in 5 gallons of milk? Probability: Joe has a probability of 1/4 chances of winning the game. What are his chances of winning if he plays 36 times? When asked, fifth grade teachers' number one desire at my school is for incoming students to know their **multiplication facts**. I also want that for my students. That is my first objective. They need to understand why numbers like 4, 6, and 24 have a special relationship. I want them to explain why 6 X 4 equals 24 by using models or showing different ways to prove the product is 24. When any two of those numbers, 6, 4, and 24 are spoken or seen, their brains should automatically think of the missing number because of the special fact family relationship. They should be empowered by knowing there is more than one way to find the product of a multiplication fact.

Students should have the tools and strategies that promote confidence in **solving word problems** involving multiplication and its inverse, division. Empowerment comes from sufficient experience manipulating arrays or drawing groups of objects to visualize the problem. I will introduce the Singapore Model as another strategic tool in their problem solving repertoire. Then students need a variety of interesting problems to solve where they work together with peers to discuss their thinking. They cannot rely on specific words in the problem to tell them the operation to use. Instruction in the different structures of multiplication and division problems is critical. Recent research shows students need to develop deep conceptual knowledge of multiplication and division in order to apply and use these operations to solve problems. The National Council of Teachers of Mathematics recommends students work with a wide range of representations and problem situations to learn the properties of these operations and fluency in problem solving. (1) Discussing mathematical thinking with your peers promotes more positive energy about mathematics as students are not working in isolation. They find multiple ideas for a solution, increase development of expressive math vocabulary, and self-worth as a mathematician.

I want them to understand how and why the algorithm for **multi-digit multiplication** works. If they can see the models, then they can make a connection between the steps in the algorithm and the portions in the model. Understanding why an algorithm works is important because our brains like making sense of things. The more synapse connections we make with a concept, the better retention of the process or idea. Even if we use a calculator to problem solve involving multi-digit multiplication, we will have developed the mathematical sense through understanding the model to know if the product on the calculator screen appears accurate. I will do this by applying strategies from our school district's new series, Investigations and including interesting problems from my understandings of the seminar with Harold Reiter called "Understanding Fundamental Ideas in Mathematics at a Deeper Level".

Context

Our Charlotte, North Carolina suburban school of grades K-5 is one of 99 elementary schools in a county wide school system. About 950 students make up a diverse group economically and racially with 48% on free or reduced lunch, 49% African American, 25% Caucation, 11% Hispanic, and 15% other.

Our school district adopts a math curriculum and encourages all elementary teachers to use it as designed. This year they adopted Investigations by Scott Foresman. We attended a one-day workshop where the sequence and lay-out of the units, plus the available resources were introduced to us. Our school has a math/science facilitator who assists us with supplies, lessons, and in-service needs. The district designs a scope and sequence calendar that follows the state math guidelines using the units in the new adoption. This calendar assists teachers in pacing units to cover all the math objectives over the course of the school year and before state End-of Grade testing.

Our school's philosophy empowers teachers to develop and use curriculum that meets the needs of each teacher's particular class. The principal places the professional responsibility on us to provide the best teaching for the students so that each child makes Annual Year's Growth. I am not tied to the scope and sequence calendar by my administration, but it is a helpful guide, as finding enough quality time to fit in state objectives is challenging. The grade level teams follow the Standard Course of Study for the state, design pre and post assessment, and seek out supplemental activities in addition to those in the adopted series. Our math facilitator attends our planning sessions and assists us in finding materials to suit our instruction.

My class composition of 27 students is about one third each of Talent Development students, average ability students, and students below grade level who do not qualify for special services. Two of these students are from third grade and accelerated in math. Talent Development is defined as students who perform in the 90th percentile on state testing in math and reading. The talent development teacher comes into my classroom twice a week during math block to team teach with me for one hour. The implication of this for my class is that I have a wide range of: student abilities, student attitudes towards math, and math instructional techniques from the ten third grade teachers from which these students came.

Timeline

This unit will be taught in chunks and practiced throughout the school year. Learning the multiplication combinations and properties will occur for 3 weeks right at the beginning of the year. More complex multiplication problems will be taught for 2 weeks at the beginning of the second quarter. Problem solving will be laced throughout the year to provide cooperative experiences and practice with tools and strategies.

Background Information

Teachers are vitally important to improving mathematics education. "If we teach our children merely to compute while we teach our computers to think ever more intelligently, who is going to rule the world in the future?" (2) I was taught how to compute in my grade 1-12 education. My college education in mathematics for elementary teachers opened a window for me to help children think mathematically. But continued education for teachers is critical as well. I've met too many teachers in my career who dislike math, admit they don't spend a lot of time and effort teaching math to children, and are comfortable "just using the textbook". We must take the time in the classroom to use effective teaching strategies with students. Yes, the math scope and sequence timeline for your grade level is unrealistic for a 180 day school year. Your classroom is not homogeneous in skill mastery or concept development. The timeline does not account for the time needed to explore mathematics deeply enough to promote deeper math thinking. Let's look at recognized teachers of mathematics and research in math education for making the best of math instruction.

Children need to learn mathematical concepts and to see relationships among these concepts. Because mathematical concepts and relationships are constructed by people and exist only in their minds, to learn mathematics, children must construct these concepts and relationships in their own minds. (3)

Investigations math curriculum is developed by many groups involved in math research and teacher education. TERC, Technical Education Research Center, is dedicated to improving math, science, and technology learning and teaching. NSF, National Science Foundation, supported the development of Investigations through a grant. Pearson/Scott Foresman is the publishing company that wrote the text format for the math curriculum to be used by teachers and students. The Investigation units use a constructionist model. Students build and draw arrays to represent the basic multiplication facts. They manipulate the models to discover properties of multiplication like commutative, associative, and distributive. This math series will be used to develop concepts about multiplication facts.

Our seminar leader, Harold Reiter, professor of mathematics at the University of North Carolina at Charlotte, takes the position that a way to develop deeper math understanding and increase motivation in students is to use interesting problems. My peers, teachers from various grade levels, worked in small groups for 100 of the 120 minutes each Thursday evening, using our experiences and math vocabulary to solve various puzzles and problems. But, it wasn't the solution that interested our leader. It was the reasoning we used that took up the lion's share of each seminar. That is what put the sparkly look in his eyes and gave us "ah ha" moments at different points in our discussion. At times we were asked not to speculate but reason out everything before writing numerals into a Ken Ken puzzle. These many strategies are exactly what we should be doing with our students: teachers delivering short, exact direct instruction, students spending most of the math work time solving interesting problems, students working together in teams and speaking the language of math, students using what they know to figure out an approach to the solution, teachers acting as the coach who cheers on the investigations, students creating appropriate models using tools of their choice, and then explaining what they discovered to other teams.

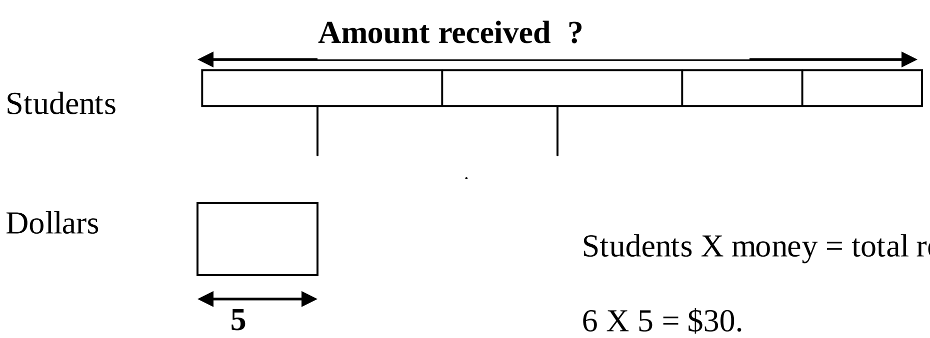
So, what makes a suitable problem? Marilyn Burns has set up criteria for good math problems. (4)

1. There is a perplexing situation that the student understands.
2. The student is interested in finding the solution.
3. The student is unable to proceed directly toward a solution
4. The solution requires use of mathematical ideas.

Harold Reiter might sum up the question by saying irresistible problems are mathematical play. (5) Using interesting problems and best teaching practices will guide me in teaching more complex multiplication.

The Singapore Model Method teaches students, in every year of math instruction, to create a visual. First the word problem is read and discussed so there is an understanding of how it could be solved. A diagram of bars is drawn on paper to represent known data about the problem. These bars are lined up horizontally and/or parallel to one another to show the comparison of known data to unknown data. Bars are then studied to figure out what piece or pieces of bars are missing to solve the problem. In the operations of addition and its compliment subtraction, this comparison model has the students seeing whether one of the addends is missing or maybe the sum is missing. In the case of multiplication or its compliment division, students first solve for the value of the unit piece, or factor, before they calculate the value of the unknown bar.

Example: 6 of the 25 students brought in five dollars for the weekly news magazine. How much money has the teacher received so far?



Students X money = total received

6 X 5 = $30.

The Singapore Model uses Polya's four-step problem-solving sequence. 1. Understand the problem 2. Devise a plan 3. Carry out the plan 4. Look back to check for reasonableness. Singapore Model involves many and varied learning experiences, where students use the same model construct from kindergarten through the teen years, with increasingly difficult problems. (6) This Singapore Model idea will be used as another technique to help students know what operation to use in word problems.

Major concepts/skills and North Carolina objectives in this unit

Goal 1: The learner will read, write, model, and compute with non-negative rational numbers.

1.01 Develop number sense

- Build understanding of place value
- Connect model and number using a variety of representations

1.02 Develop fluency with multiplication and division

- Strategies for multiplying and dividing numbers
- Relationships between operations
- Two-digit by two-digit multiplication (larger numbers with calculators)

1.05 Develop flexibility in solving problems by selecting strategies and tools (7)

Strategies

Place value is a major concept for the foundation of understanding operations in mathematics. Nine year olds can name the places in decimal notation and know their value. When representing multiplication, they can draw four groups of nine stars and skip count to 36. But when multi-digit operations present themselves, students can lose sight of the importance of lining up digits within place value columns while working on a problem. In his essay, Roger Howe comments on the importance of getting students to appreciate the effect of lining up digits of the same magnitude. (8) It's difficult to see the magnitude of 14 bags of 48 candies when you're learning the algorithm and not seeing the model for what is happening in all those steps.

Creating a model that accurately represents multiplication is a skill and concept students need to understand. Models using snap cubes, manipulating arrays cut from centimeter paper, and diagrams of rectangles where the dimensions are sub-divided into tens and ones can show the value of a number and the value of the position of digits within a number. During the process of multi-digit multiplication, lining up the place value columns makes sense when viewing the parts of the area multiplication model.

Models for Multiplication Combinations

Students will create models to show the products of multiplication combinations. Snap cubes will be used when learning about arrays. The first factor represents the number of rows of cubes in the array. This column is snapped together and recounted to insure the number of rows for the first factor is correct. It can be confusing to students that they are making a column to represent rows, but continued dialogue with the teacher and peers will reinforce this idea. The second factor is the total number of identical columns needed for the array. When the columns are lined up in an array, the child can see the rectangular shape that is formed by the combination of two factors. It is important that the rows and columns are recounted and adjusted to make the array fit the combination. The teacher can check for understanding by asking, "Which part of your array shows the first factor?" Students are encouraged to use the words: rows, columns, dimensions, array, and product in their discussions about the model. "How do you know you have the correct number of cubes to match the product?" A student can respond by counting by one of the factors from the array. If the child counts by ones or twos, touching the blocks as s/he counts, s/he is encouraged to try counting by one of the factors.

Once children have several experiences making the arrays with cubes, they can use centimeter graph paper to construct the arrays. It's encouraged that all students count the rows first and then the columns to promote an organized way of thinking about math. Later on, they will see arrays can be rotated and the commutative property allows them to reverse the factors resulting in the same product. The paper arrays are bordered with a colored pencil and then cut out to be pasted in a math journal with the rows and columns labeled with the corresponding factor. The multiplication combination is recorded by the array with its product.

Then, students can learn to draw arrays on paper. These are free-hand drawings on notebook paper or blank paper. Attention to the relative length of the sides of the arrays is important so that the dimensions correspond to the number or factor. Students draw in the vertical and horizontal lines to make the squares of the array. Again, this is done in combination with labeling the sides of the array and recording the equation for the array. Students also explore writing word problems to match the array made. Here they learn the wording for showing that multiplication is the necessary operation.

After assessing mastered multiplication facts 0-5, students learn how to "break apart" a difficult combination, like 7X8. In the beginning we use the first factor as the rows and the second factor as the columns so that we are all using the same procedure. Later on, they learn the order does not matter. We discuss how 5 times any number is easy to recall. They use the paper model to draw a line between the fifth column and the sixth column of the array. This is called "break apart". Children discuss how we haven't changed the array and 5 + 3 is still 8. Students discover that they can multiply the 7X5 array to get 35 and multiply the 7X3 array to get 21, then we add the products to get 56. Later we show the algebraic expression for "break apart" or the Distributive Property.

Now that students have several ways to "figure out" the basic combinations they do not have memorized, we use the "Easy Does It Chart" (9) The worksheet shows the children that they do know most of the facts. The doubles, nines, and last ten remain to be mastered. Personal paper flash cards are made for "The Doubles". This is a set of 12 cards from 0X0 to 12X12. Each paper slip has "Start with __________" on the bottom. The owner of the cards decides what math fact s/he can start with to figure out the unknown double. This is home practice for learning the facts and mentally practicing the Distributive Property. Make a set of these facts and staple them to the cork strip in your school hallway. Cover the products with a small piece of cardboard that can be flipped up to check the answer. As children line up in the halls, they can practice the basic combinations at least twice a day! After the doubles are mastered, we move on to the 9's and finally "The Last Ten". This part is learned to memory but uses mathematically thinking to remember the products.

Do I Multiply or Divide?

Students should be exposed to various kinds of word problems involving multiplication and division over an extended period of time to develop conceptual knowledge of the relationship between multiplication and division. Here are explanations and examples of the types of word problems and a graphic organizer for easy reference. Remember, many experiences with these problem types involving peer discussion, the use of models, and math tools is necessary for children to build the multiplication/division relationship. (10)

Equal Grouping Problems

There are 8 peaches in each of 6 sacks. How many peaches are in the bags? This is an equal grouping problem where one factor tells the number of groups, the multiplier, and the other factor tells the number in each group. These problems are solved using multiplication.

With equal grouping division problems, either the number of groups is unknown, quotitive division, or the number in each group is unknown, partitive division. Quotitive-Joe puts 8 peaches in each bag. How many sacks does he need for 48 peaches? This is also called measurement division as the child is putting a measured amount of objects into each group. Partitive-Joe needs to bag 48 peaches into 6 sacks. How many peaches will go into each sack? In this action, students are partitioning or evenly dividing the total number of peaches into 6 groups.

Rate Problems

Two different quantities are being compared in rate problems. Examples: heart beats per minute, miles per gallon, cost per item. In these problems one factor tells the rate and the other acts as the multiplier or number of sets. Movie tickets cost $6.50. How much will it cost a family of 4 to attend one movie?

Rate problems can also be express as a division problem. Mom spends $26. for 4 movie tickets. How much was each ticket? This is partitive division as the size of the group or ticket price is unknown. Movie tickets are $6.50 and Mom spends $26. for the tickets. How many tickets did she buy? This is quotitive, the number of tickets is unknown.

Compare Problems

In these multiplication problems, one factor gives the amount in one set while the other number is a comparison factor. Ralph types 40 words per minute. Joe types twice as fast as Ralph. How fast does Joe type? Students would use division if one of the factors was missing. Ralph types 40 words per minute. Joe types 80 words per minute. How much faster does Joe type than Ralph? The language of these problems can be difficult. It is important for students to discuss and model the situations in these problems.

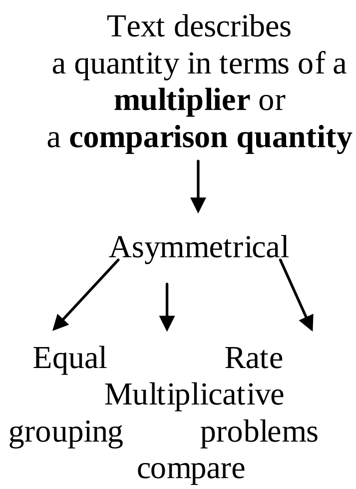
Rectangular Array Problems

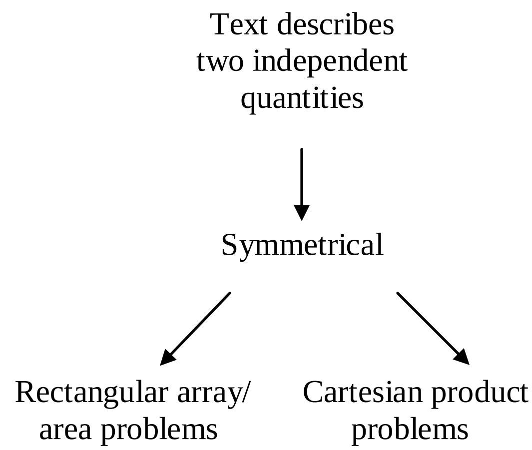
These may be familiar to the students from studying the multiplication facts. Two dimensions are given and students are asked to solve for the number of square units. An array is 12 units long and 8 units wide. What is the total number of square units? These problems are called symmetrical. Either factor can be the multiplier. Again, if one factor is missing, division would be used. The array uses 96 squares. One side is 8 units. What is the dimension of the longer side?

Cartesian Product Problems

These are logic problems where the pieces of two or more sets are paired together to make all the possible combinations. Joe has 3 different shirts, 2 different pairs of shorts and 2 styles of flip flops. How many different combinations of the three sets can Joe make? These problems are also symmetrical, as the role of the factors is interchangeable.

Here is an example of a missing factor division problem. There are 18 meat the cheese sandwich combinations from which to choose. If there are 3 kinds of meat, then how many kinds of cheese are offered?





Area Model for Multiplication

Now that students are familiar with using arrays and working on mastering their basic facts, we can move on to more complex multiplication. To solve problems like 243 X 7, students draw a rectangular array without the squares inside. They draw lines in relatively correct position to show the left side of the array is broken into 200 + 40 + 3. Now they record the product for each section of the array. 200 X 7 = 1400, 40 X 7 = 280, and 3 X 7 = 21. The partial products are added to get the answer of 1701.

Calculations like 83 X 44 are done in a similar way. Students add the partial products to get the answer of 3652.

The teacher observes when peer discussion about the area model has resulted in concept understanding between the model and the partial products. The algorithm for multi-digit multiplication can then be shown and compared to the area model. Students would have two ways to solve a similar problem. With further activities, students may learn how to solve some problems mentally or solve from left to right and use compensation to arrive at the answer.

Classroom Activities

<u>Lesson 1</u>

Objective: Students use area model of multiplication to discover the rectangle with the greatest area

Strategy: Students are given two numbers to construct a rectangular garden. They can manipulate the place values of these numbers to arrive at the largest garden area possible. They must prove their answer by using the area model of multiplication.

Skills Practice: NC objective 1.01 understanding of place value- expanded form; NC objective 1.03 creating a model to solve a problem; NC objective 1.02 basic multiplication combinations and two-digit by two-digit multiplication; NC objective 5.03 distributive property, using mathematical vocabulary to communicate with others

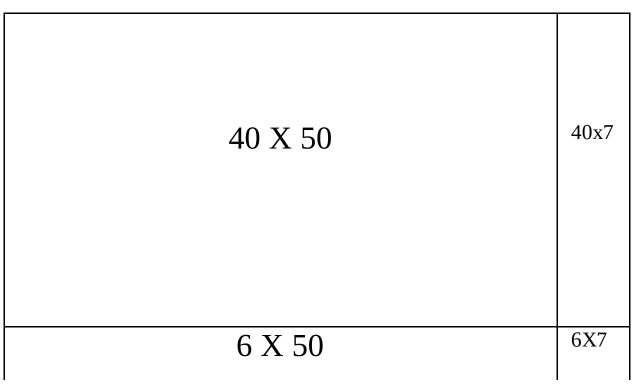
Procedure: "Today, you and your partners will receive a ball of kite string and a tape measure. Your problem is to design a model of the largest garden area using the values of these two numbers: 46 and 57. You can break down each number into tens and ones, using any combination of tens and ones for the garden's dimensions. In doing so, you will discover and show most of the possible garden sizes and areas. Use the string to help visualize how each garden size will appear. Use the area model of multiplication to prove which garden would result in the largest area. Explain, on paper, what you are learning as you investigate this problem. As I observe your work session, I would like to hear wonderful mathematical vocabulary as you discuss ideas with your partners and other peers. Now, get into groups of 3 or 4, find your math journal, pencil, and a place in the room to work.

Students might precede in a variety of ways, but below shows the specific results.

Step one: **46** = 40 + 6

57 = 50 + 7

Step two:



40 X 50 = 2000
6 X 50 = 300
40 X 7 = 280 +
6 X 7 = 42
2,622 feet squared

T

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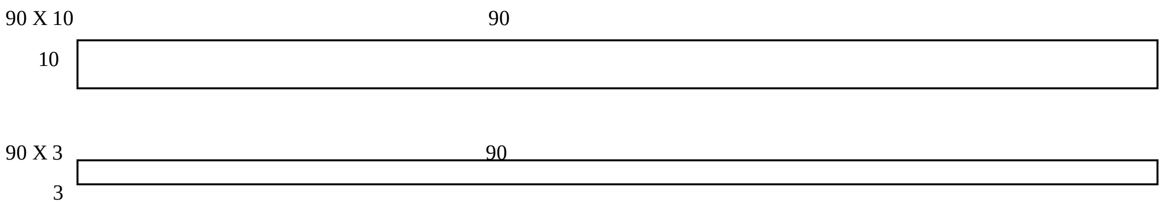
L for one dimension and adding the ones for the second dimension. Maybe adding the larger place values together will give us a bigger garden area.

40 + 50 = **90** for one dimension

6 + 7 = **13** for the second dimension



90 X 10 = 900

90 X 3 = <u>270</u>

1170 squared feet

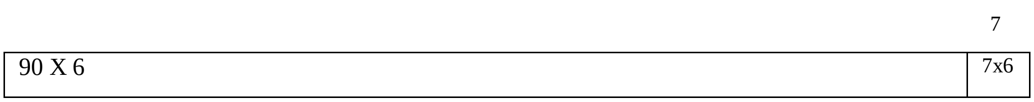
That made the garden even smaller in area and a very odd shape.

Step four:

So, if we combine the tens plus one of the ones and multiply that by the other ones, then we should get an even smaller area garden, that is longer and thinner. Let's try it.

40 + 50 + 7 = **97** or 90 + 7

6



90 X 6 = 540

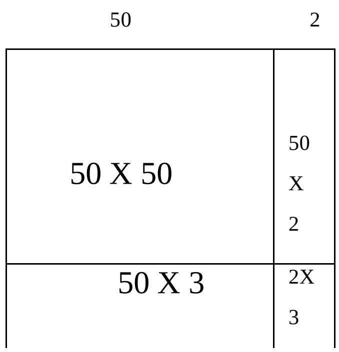
7 X 6 = <u>= 42</u>

582 squared feet

We are correct; it is a much smaller area. So far the first garden's dimensions give us the largest area and the garden's shape is more like a square compared to the longer, narrower gardens of trial 2 and 3.

Step five:

Let's change the given numbers completely to get the dimensions of the rectangle more equivalent. It should give us a garden with an even larger area. 46 + 57 = 103   103 = 52 + 53



50 X 50 = 2500

50 X 2 = 100

50 X 3 = 150

2 X 3 = 6

2,756 squared feet

Wow, that is the largest area yet! The closer we get to a square the larger the area of the rectangle.

Lesson Conclusion and Assessment: Students share with the class what they learned and how they went about finding their answer to the problem. Teacher uses checklist to monitor skill mastery and ability to share mathematical thinking.

<u>Lesson 2</u>

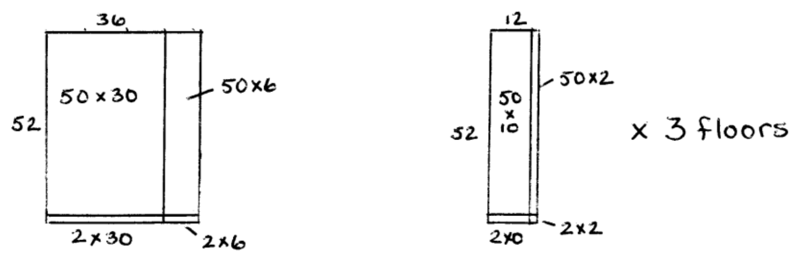
Objective: Students use area multiplication model to problem solve a comparison.

Strategy: Students are given two structures in which they compare the areas. A one-story building is compared to a multi-story building. Students must use area multiplication model and written explanation to explain their results.

Skills Practice: 1.03 creating a model to solve a problem; NC objective 1.02 basic multiplication combinations, factors, multiples, and multi-digit multiplication; NC objective 5.03 distributive property, using mathematical vocabulary to communicate with others

Procedure: "Today's exploration is this question: Is a one-story building that is 52 meters by 36 meters larger in area than a three story building with floors 52 meters by 12 meters? Create a model using the math tools, if you like. In your written work, use the area multiplication model and your mathematical thinking for your explanation.

Students might use a variety of ways but here is one specific example.



50 x 30 = 1500
50 x 6 = 300
2 x 30 = 60 +
2 x 6 = 12
1,872 m^2

50 x 10 = 500
50 x 2 = 100
2 x 10 = 20 +
2 x 2 = 4
624
x 3 floors
1,872 m^2

We noticed that the area is the same in both building designs. That makes sense since

3 floors X 12 meters = 36, which is the width of the one story building. The length, 52 meters, did not change.

So, 36 X 52 = 3 X 12 X 52

Extension: "If you wanted a total area of 360 X 540 square meters, how many stories would the building need to be if each floor was 60 X 90 meters?"

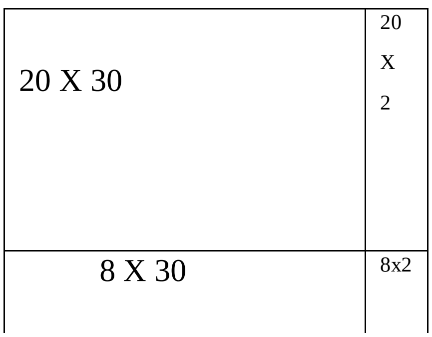
Lesson 3

Objective: Students use area multiplication model to come up with the dimensions for a fraction of a given area.

Strategy: Students are asked to explain how to solve for ¼ of the area of a classroom and give the dimensions for the new area.

Procedure: "Wouldn't it be great if we could build a reading loft that is ¼ the area of our classroom! How would you go about figuring the area of the loft? What would be the dimensions of the loft? Use the area model of multiplication in your solution. Be sure to write about your discoveries throughout the investigation.

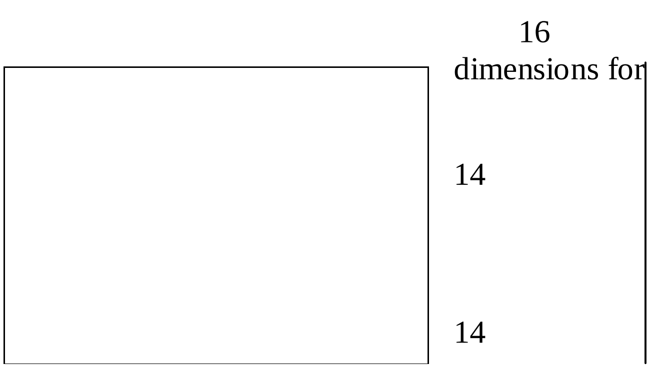
Students measure the classroom to be 28 feet by 32 feet.



20 X 30 = 600
8 X 30 = 240
20 X 2 = 40
8 X 2 = 16
896 ft squared

The area of the classroom is 896 feet squared. If we divide that into fourths, we get 224.

Now, we need to figure out the dimensions of the rectangle with an area of 224 feet squared.



The dimensions for ¼ of the classroom that is 32 X 28 must be 16 X 14.

16 X 14 = 224 feet squared and that is ¼ of 896.

Extension: "Why can't we just take ¼ of each dimension, 32 X 28, and then multiply the two new dimensions to get the new area? It seems like that would be much easier. Would this strategy work or not?"

¼ of 32 is 8

¼ of 28 is 7

8 X 7 = 56 squared feet

When I draw the model it is evident that this idea doesn't work. 56 is 1/16 of the total area.

Appendix A: Terminology

Multiplication Combination is a synonym for multiplication fact.

Example: 4 X 6 = 24

Fact Family is a set of equations showing all the possible multiplication and division combinations for 3 numbers. It shows the Commutative Property of Multiplication, where the order of the factors does not affect the product. It shows the inverse operations of multiplication and division, which means the two operations, undo each other.

4 X 6 = 24 6 X 4 = 24 24 / 4 = 6 24 / 6 = 4

Array is an arrangement of dots or squares to represent a multiplication combination. The rows represent one factor and the columns the second factor.

* * * * * *

* * * * * * 4 X 6 = 24

* * * * * * 4 and 6 are the factors. 24 is the product.

* * * * * *

Factors are the numbers multiplied together to find the product.

Product is the answer to a multiplication problem.

Multiplier is the factor that tells how many groups are in the problem.

Distributive Property or "break apart" is a characteristic of multiplication where you can break apart one factor to create two simpler problems, and then add the partial products to get a correct solution.

* * * * */ * * * 3 X 8 = 3 X (5 + 3)

* * * * */ * * * = (3 X 5) + (3 X 3)

* * * * */ * * * = 15 + 9

= 24

Expanded form is a number written as the sum of the value of its digits.

Example: 46 = 40 + 6 2081 = 2000 + 0 + 80 + 1

Area model for multiplication is a way of showing the process of multiplication by using a rectangular model. The model's width and length shows the value of each factor in expanded form. It can be as simple as modeling a multiplication combination. If used in multi-digit problems, it can show where the partial products originate. Using the "break apart" idea, the model is subdivided to show the product of each section inside the rectangle. This model aids in understanding the step in the algorithm for multi-digit multiplication problems.

Appendix B: Word Problems for sorting by structure type and solving

1. Damaris earned $48. babysitting. She earned 3 times more than Grace. How much did Grace earn?
2. Mom has 3 wigs and 9 different pairs of earrings. How many different combinations can she make?
3. Each soap dispenser holds 8 ounces of liquid. The refill bottle has 72 ounces of liquid soap. How many dispensers can be filled with one refill bottle?
4. Six boys are planning to share a bag of 64 pieces of Laffy Taffy. How many pieces will each boy receive?
5. The dimensions of a rug are 17 feet by 10 feet. What is the area of the rug in square feet?
6. The turkey cost $8.80. It weighs 8 pounds. How much did the turkey cost per pound?
7. Josh rides 6 miles an hour on his bicycle. How far can he travel in 3 hours?
8. Jordan walks 12 miles at a rate of 4 miles per hour. How long did it take him to walk the 12 miles?
9. Madison read 3 times as many books as she read last year. Madison read 42 books this year. How many did she read last year?
10. Chick-fil-A has 4 sandwiches, 5 drink choices, and 3 sides. How many different meals can be made?
11. Gasoline is $1.55 per gallon. How much would 20 gallons cost?
12. A rectangular storage room is 100 square feet. It is 20 feet long. How wide is it?
13. The room has 8 rows with 7 chairs in each row. What is the total number of chairs?
14. Dad is 3 times older than your 15 year old brother. How old is dad?

15. The arrangement of checkers is 8 rows by 4 columns. How many checkers are there?
16. You must clean the bathroom, make your bed, and do the dishes. How many different sequences can you make for the jobs to be done?
17. The packer puts 5 apples in each bag. There are 75 apples. How many bags are needed?
18. The moped goes 25 miles/hour. The cheetah goes 75 miles/hour. How much faster is the cheetah than a moped?

Teacher Resources

Burns, Marilyn. *About Teaching Mathematics Second Edition*. Sausalito: Math Solutions Publications, 2000

Her books are easily read, great resources for methodology and pedagogy. You can find lesson ideas for the next day right at your finger tips.

Chapin, Suzanne H. and Art Johnson. *Math Matters Second Edition*. Sausalito: Math Solutions Publications, 2006

This grade K-8 resource book covering all strands of math was helpful in understanding the various problem solving structures for multiplication and division.

Goldish, Meish. *Making Multiplication Easy*. New York: Scholastic Inc. 1991

This paperback is rich with activities for memorizing and seeing patterns in the multiplication tables.

Howe, Roger. "Taking Place Value Seriously: Arithmetic, Estimation and Algebra." January 2008, http://www.maa.org/pmet/resources/PlaceValue_RV1.pdf.

This paper was interesting to read and a great review of the foundation for understanding decimal notation and arithmetic. Of special interest to me were strategies for understanding more deeply, the four operations and their properties.

Kenschaft, Patricia Clark. *Math Power How to Help Your Child Love Math, Even If You Don't*. New York: Pi Press, 2006

This professor of mathematics encourages parents to take a close look at the math education your child is getting in school and be more proactive with concrete strategies, discussions and math games at home.

Mokros, Jan, Russell, Susan Jo, Economopoulos, Karen. *Beyond Arithmetic Changing Mathematics in the Elementary Classroom*. Cambridge: Dale Seymour Publications, 1995.

This is written by the TERC people who wrote Investigations. It explains the value of the constructivist approach, the job of the math curriculum, and the teacher as a "guide at the side".

Reiter, Harold B. http://math.uncc.edu/~hbreiter/Yale/

His webpage has a wealth of irresistible problems used in our seminar sessions.

Schwarz, Valerie. "Dr. Word Problem-Solving Word Problems with the Four Operations Using Singapore Bar Models." Yale National Initiative, Unit 07.06.05, July 2005, http://teachers.yale.edu/curriculum/search/viewer.php?id=initiative_07.06.05_u&skin=h

She has a page of word problems all ready for you!

TERC, *Investigations*. Cambridge: Pearson Education, Inc. 2008

Van de Walle, John A. *Teaching Student-Centered Mathematics Grades K-3*. Boston: Pearson Education, Inc. 2006

Here's another great book of ideas and strategies for helping kids better understand math.

Internet Sites

http://www.aplusmath.com/Games/index.html

http://www.coolmath.com/0-math-practice-problems.html

http://www.mathplayground.com/games.html

http://multiplication.com

Notes

1. Suzanne H. Chapin and Art Johnson, *Math Matters second edition* (Sausalito: Math Solutions Publications, 2006), 76.
2. Shirley Hill, President of the National Council of Teachers of Mathematics, speech 1980.
3. Marilyn Burns, *About Teaching Mathematics* (Sausalito: Math Solutions Publications, 2000), 24.
4. Marilyn Burns, *About Teaching Mathematics* (Sausalito: Math Solutions Publications, 2000), 17.
5. Harold Reiter, Mathematics Professor at University of North Carolina, Charlotte, Seminar class 8/27/09 and website.

6. Kho Tek Hong, Yeo Shu Mei, and James Lim, *The Singapore Model Method for Learning Mathematics* (Singapore: EPB Pan Pacific, 2009), 5-67.
7. *North Carolina Teacher's Desk Reference and Critical Thinking*, (New York: Educational Tools, Inc. 2007-2008),17.
8. Roger Howe, "Taking Place Value Seriously: Arithmetic, Estimation, and Algebra", January 2008.
9. Meish Goldish, *Making Multiplication Easy*, (New York: Scholastic, 1991)
10. Suzanne H. Chapin and Art Johnson, *Math Matters second edition* (Sausalito: Math Solutions Publications, 2006), 76-83.