The Journey to Discover Right Triangles and Trigonometry in the Real World

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Objectives

Trigonometry is a division of mathematics that carries great value with its application to the real world. Students get their first exposure to trigonometry in geometry class where it is used to explore the relationships between sides and angles in right triangles. Through my observation, trigonometry and the use of the three functions: sine, cosine, and tangent, proves to be one of the most difficult mathematical concepts for students to grasp in the geometry classroom. This unit is designed to take an approach to the three trigonometric functions that allows students to understand where they come from and why they are used. In a typical geometry class, the teacher presents the basic trigonometric rules without giving students the opportunity to explore the origins of the trigonometric functions or the history behind the uses of trigonometry. The implementation of this unit will prove that there is merit in introducing these aspects of trigonometry to students. Students will be able to master the operation and identification of the three trigonometric functions, as well as apply the functions to real world situations more successfully.

This unit involves several discovery activities in which students will derive formulas and rules to be used throughout each lesson. Once the students have mastered the basic uses of the functions, the teacher will provide instruction to students on how to apply the functions to real world application problems. This unit will also encourage students to look into practical uses of trigonometry in different careers, and to explore the foundations of trigonometry. Students will be able to compose their own answer to the common question of “Why?”

This unit was designed to be used with honors level students, but can be easily adapted for standard level students. There are several discovery activities throughout the unit for students to encounter that require more abstract and independent thinking from the students. For standard level students, these discovery activities could be more teacher directed, while at the same time pushing students to think more for on their own.

This unit is not intended to go in depth with the laws of the trigonometric functions or the reciprocal functions of secant, cosecant, and cotangent. This unit will also strictly be used for trigonometry involving right triangles and angle measures in degrees. The concept of radians and the unit circle will not be incorporated into the unit. Per the North Carolina Standard Course of Study[1], these listed concepts are taught in higher level mathematics where students are expected to have previous knowledge of the basic operations of sine, cosine, and tangent. So, although this unit will simply touch on the surface of trigonometry as a whole, the rules and formulas to be applied are essential to students understanding of all other concepts to be covered in geometry and will be essential to student success in all levels of math higher than geometry.

Time Period

This unit is designed for a block schedule geometry class to be completed in a semester. The class periods are 90 minutes long. This entire unit will take up to two weeks, including the final unit assessment to cover all aspects of the topic. Each part of the unit is easily adaptable to accommodate a year long schedule by splitting up each lesson into two or three days.

Each lesson in this unit contains details for how to present/instruct/facilitate for the information that represents the focus of the lesson. Teachers may want to incorporate several skill based practice activities into the lessons to make sure that students master the concrete concepts presented throughout the unit.

Prior Knowledge

Based on the NC Standard Course of Study, students should have mastered these objectives before entering geometry class:

- Apply geometric properties and relationships to solve problems.
- Use formulas to solve problems.
- Define and use linear expressions to solve problems.
- Operate with matrices to model and solve problems.

For the Charlotte-Mecklenburg School System Geometry Pacing Guide[2], trigonometry is the fourth unit on triangles that students cover in the school year. Students should already have prior knowledge of the parts of a triangle, angle measures in a triangle, triangle congruence and similarity, and bisectors and medians in a triangle.

Outcome

From this unit, students should be able to obtain the following information:

1. The main focus of the unit is for students to be able to answer the Essential Question: WHY and HOW do we use trigonometry in our world today? The

teacher may have the students complete an open ended assessment to go along with the objectives mastery assessment at the end of the unit.

2. Students will understand the uses of trigonometry. Through exploration and teacher driven lessons, students will discover why and how we use trigonometry in the world today.

3. Students will master the application of the trigonometric functions of sine, cosine, and tangent. Students will use these functions in solving and creating real world application word problems.

Strategies

Lesson 1: Parts of a Right Triangle

Objectives:

1. Identify the angles and sides of a right triangle.

2. Use the geometric mean relationship within right triangles to find side lengths and altitudes of triangles.

Opening:

The focus of the first part of this lesson should involve the teacher reinforcing the concept of similar triangles. The teacher should begin the lesson with a review of the similarity in triangles. The review may consist of a few warm-up problems involving proofs of similar triangles, or identification of similar triangles and their corresponding parts in different pictures. This is material that is covered in the previous unit where students explore proportions and ratios in similar triangles. Key ideas that students should recognize from similar triangles are as follows:

- Similar triangles have congruent angles and proportional corresponding sides
- Scale factor is the ratio of corresponding sides in similar triangles.
- Three theorems prove that triangles are similar:
    - Angle-Angle: two angles in two triangles are congruent
    - Side-Side-Side: all three sides in two triangles are proportional
    - Side-Angle-Side: Two corresponding sides in two triangles are proportional with one included angle congruent in the two triangles.

Discovery Activity 1: Similarity in Right Triangles (see appendix A)

To further drive the idea of ratios within similar triangles, students will complete an activity in which they explore the relationships among similar right triangles created by an altitude with its origin at the vertex of the right angle in a right triangle. The purpose of this activity is an informal assessment of student understanding of similar triangles with an introduction to more detailed properties of right triangles. Students will explore the side and angle relationships within the triangles without measuring tools; instead students will use properties of similar triangles and triangle classifications (sides: scalene, isosceles, equilateral, and angles: acute, right, obtuse). Recognizing similar right triangles will lead to the discussion of geometric mean within triangles.

During the activity, students will create a model to represent the similar triangles out of building tools. Students will label their own model and record the properties that they identify for the right triangles based on the segment lengths that they used to create the triangles.

The teacher may have students build the model on their own, or the teacher may want to have some models already constructed for each group. If the teacher is having students build models on their own, it may be easiest to build the model with students so that they will not be confused as to what the model should look like. The relationships will be easier to manipulate if the original triangle is an isosceles right triangle. This activity can be completed with the following building tools:

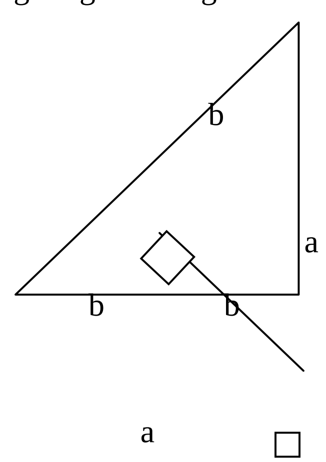
*Zometools: a combination of segments and connectors that can be connected to form various geometric shapes

*Pipe cleaners: connect the pipe cleaners by twisting joint ends together

*Popsicle Sticks: connect the sticks by gluing them together.

Figure 1

The finished structure should look as follows.



Students will need two different size pieces to build the figure. These two lengths will create an isosceles right triangle with two similar isosceles right triangles inside created by an altitude from the right angle. The side length labeled “a” should have a ratio of the length of b multiplied by the square root of two.

Teacher Notes

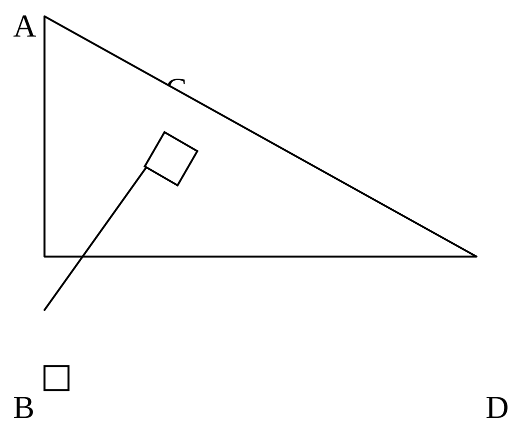
The teacher should share the following information with students to record in their notes for future reference. The teacher should also complete example problems representing geometric mean in triangles with the students:

1. The parts of a right triangle include two legs, and a hypotenuse. The hypotenuse is always the longest side of the right triangle because it is the side across from the largest angle in the right triangle, the 90 degree angle. Since one angle of a right triangle will always be 90 degrees, we can make a conjecture that the acute angles in a right triangle will always equal 90 degrees also to complete the total of 180 degrees for the triangle.

2. Altitudes in right triangles: The altitude drawn from the right angle in a right triangle creates three similar triangles. All three triangles can be proven similar by observation of the shared angles. The large triangle, ΔABD, is similar to each of the two smaller triangles by the Angle-Angle Theorem because it shares an angle with each triangle, and all three triangles include a right angle. Therefore, ΔABD is similar to ΔBCD because they both share <D and a right angle. ΔABD is also similar to ΔACB because they both share <A and a right angle. By the transitive property, all three triangles will be similar.

Figure 2

The similar triangles are ΔDBA, ΔDCB, and ΔBCA



3. Geometric Mean:

   a. The geometric mean of two numbers is the positive square root of the product of the numbers. For example, the geometric mean of 5 and 4 is the square root of 20.

b. The altitude from the right angle in the triangle represents the geometric mean of the two select segments of the hypotenuse created by the intersection of the altitude. Therefore, to find the length of BC in figure 2, we would compute the geometric mean of the lengths of AC and CD.

c. Either leg of the right triangle represents the geometric mean of the length of the hypotenuse and the select segment adjacent to the chosen leg of the triangle. Therefore, to find the length of leg AB, we would compute the geometric mean of AC and AD. Similarly, to find the length of leg BC, we would compute the geometric mean of CD and AD.

Lesson 2: Pythagorean Theorem

Objectives

1. Derive the Pythagorean Theorem formula by separating a square into four congruent right triangles and a smaller square.
2. Use the Pythagorean Theorem to solve problems involving right triangles.

Opening

The teacher should have students again review similarity in triangles, while also incorporating a few examples of geometric mean into a warm up activity. The teacher should stress the two types of geometric mean problems: using geometric mean to identify lengths in similar right triangles with an altitude created from the right angle, and finding the geometric mean of two arbitrary numbers.

Discovery Activity 2: WHY does the Pythagorean Theorem Work?

To understand trigonometry, students must first understand what a right triangle is. The second lesson will involve students exploring the parts of a right triangle using the Pythagorean Theorem. Students will complete a discovery activity to explore how the Pythagorean Theorem works. Most students remember the Pythagorean Theorem from middle school mathematics; however, they may not understand why it works or where the formula comes from. The teacher should not discuss the formula with students before the activity; allow the students a chance to derive the formula on their own from their findings through the discovery lesson.

This activity takes the Pythagorean Theorem to its simplest form and allows students to create an equation that will result in the Pythagorean Theorem when simplified. Students will be given a square with side lengths c. Students will then construct four right triangles within the square, all adjacent to each other. Each of the four right triangles will have a side length of a (shortest leg of the triangle) and b (longer leg of the triangle). The completed figure should look as follows:

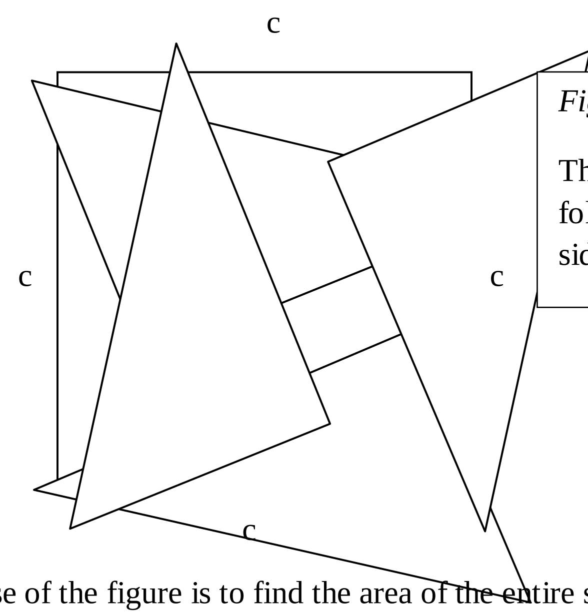


Figure 2

The finished structure should look as follows. Students should also label sides A and B on the figure.

The purpose of the figure is to find the area of the entire square in relationship to the area of the four right triangles and the smaller square in the middle. Before allowing students to work on solving the problem, the teacher may want to briefly go over the formulas for area of a square and triangle so that each student starts their proof using the correct formulas. The area of the entire square will be represented by c^2. Each triangle has an area of ½(ab) and the area of the square in the middle is $(b-a)^2$ (the length of each side in the smaller square is created by taking the longer leg (b) and subtracting the shorter leg (a) of each right triangle in the figure. After setting the two expressions equal, students should notice that the Pythagorean Theorem is the result of simplifying:

$$c^2 = (b - a)^2 + 4(1/2)(ab)$$

$$c^2 = b^2 - 2ab + a^2 + 2ab$$

$$c^2 = b^2 + a^2$$

PYTHAGOREAN THEOREM: $a^2 + b^2 = c^2$

After the activity, the teacher should lead a discussion on the equation that was derived, and the result of simplification. The teacher should ask discussion questions such as:

1. What relationship do we find when comparing the areas of triangles and the areas of squares?
2. What relationship do we find between sides c, a, and b?
3. How do we know if this relationship will always work?

Teacher Notes

The teacher should share the following information with students to record in their notes for future reference. The teacher should also complete example problems with students using the Pythagorean Theorem to find missing legs in right triangles:

1. The Pythagorean Theorem: the relationship between the legs and hypotenuse of a right triangle. The sum of the squares along the legs is equal to the square along the hypotenuse. $a^2 + b^2 = c^2$

    d. Pythagorean Triple: three whole numbers that represent the side lengths of triangles that satisfy the Pythagorean Theorem.

        i. Examples: 3,4,5 and 5,12,13 and 8,15,17

Lesson 3: Ratios in Triangles

Objectives

1. Discover and identify special ratios in 30-60-90 and 45-45-90 angle measure right triangles.

Opening

1. MathTV.com[3]: to give students an introduction to problem solving involving the Pythagorean Theorem, the teacher may want to use the video under the following headings found in the menu on the left side of the website: *Geometry: Pythagorean Theorem: #2*. This video solves an application problem involving the formula to find the missing length in a right triangle created by a chairlift.
2. Students should be comfortable with completing practice skill based problems using the Pythagorean Theorem. The teacher should walk through a few application problems involving Pythagorean Theorem with the students. A helpful problem solving format is shown below in figure 3. This format allows students to explore each part of the problem in order to understand what the problem is asking, the important information and vocabulary needed to solve the problem,

and the steps for developing a plan to solve the problem, with a result that ensures a correct answer.

Example application problems may include:

1. The foot of a ladder is placed 6 feet from a wall. If the top of the ladder rests 8 feet up on the wall, how long is the ladder?

2. Donna's TV screen is 20 inches long. If the diagonal measures 25 inches, how long is the width of Donna's TV?

3. Town A is 9 miles from town B, and 12 miles from town C. A road connects towns B and C directly. Find the length of this road.

Problem:	Important Facts and numbers:	
Important Vocabulary with definitions:	Picture:	Rewrite the Question:
	Explain your plan of action:	Solve the problem:
Explain Why the Answer Makes Sense:		

Figure 3

This template is called the Breaking It Down word problem format. This format allows students to analyze every part of a word problem so that they can be successful in solving the problem. After completing the problem, students should analyze why their answer make sense.

Discovery Activity 3: Cutting Corners using the Pythagorean Theorem

Students may better understand the application of Pythagorean to real life situations by completing this hands-on activity. Discuss with students the following question: What is the shortest way to get from any destination at point A to a destination at point B? The teacher may want to use the classroom as an example. Have two students start at a corner of the room. Walking at the same pace, have one student walk at a diagonal and have the other student walk along the walls to reach the opposite corner of the room. Students should notice that the diagonal is a faster walk. Discuss why this happens, how much shorter of a walk it is, and why. Have students work in groups to practice measuring the distance between two points by measuring the legs that create the right triangle for the situation. This gives students a chance to see kinesthetically how the formula works (see appendix B for more examples).

Discovery Activity 4: Introduction to Special Right Triangles

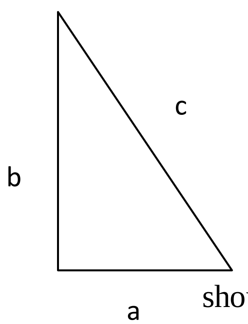


Figure 4

This activity will explore the relationships between side lengths and angles in a right triangle. Students will be given three triangles of different sizes, but with all the same angle measures. Along with discovering the continuity of the ratios for sine and cosine, students will also discover the special relationship for the ratios in a 30-60-90 angle measure right triangle. Each triangle will be labeled with sides a, b, and c. This is the idea for the side lengths that had been formulated in the discovery activity for lesson two. Each triangle will look as figure four shown to the left. The letter a will represent the shorter leg of the triangle, b will represent the longer leg of the triangle, and c will represent the hypotenuse of the triangle. Students are to measure each side length, and find the following ratios: a/c, b/c, a/b and b/a. After completion of the ratios for each triangle, students will recognize that the corresponding ratios are the same amongst the triangles.

After the discovery activity, the teacher should lead the students with the following questions:

1. How do our findings correspond to what we already know about similar triangles?
2. How is the information different?
3. What other relationship did you notice between the side lengths a, b, and c in these particular triangles?

Teacher Notes:

The teacher should share the following information with students to record in their notes for future reference. The teacher should also complete example problems with students in order to find missing side lengths in the special right triangles:

1. Theorem: the shortest distance between any two points is the straight line connecting the two points. In a right triangle, the shortest distance between the two acute angles is the hypotenuse. The hypotenuse alone is longer than either leg, but because of the triangle inequality theorem, the hypotenuse must always be less than the sum of the two legs.
2. 30-60-90 triangle: Scalene right triangle with the hypotenuse twice as long as the shortest leg
    a. The 30 degree angle is across from the shortest leg
    b. The longer leg = short leg x square root 3
    c. Origin: equilateral triangle split in half by an altitude
3. 45-45-90: Isosceles triangle with the hypotenuse = leg x square root two
    a. Legs are congruent
    b. Origin: Square split in half along the diagonal

Lesson 4 (2-Day lesson): Uses of Trigonometry

Objectives:

1. Discover the origin of trigonometry and use sine, cosine, and tangent to solve right triangles.

Opening:

The teacher may provide a warm up for students to complete involving practice skill based and application problems with Pythagorean Theorem. Students should be more comfortable with working on application problems on their own.

Discovery Activity 5: The Pythagorean Theorem in a New Light

This activity will build upon the formula for the Pythagorean Theorem. Students will explore the Pythagorean Theorem for use in obtuse and acute triangles. Students will discover that the Pythagorean Theorem allows us to classify triangles by their angles based on the inequality formed between the shorter sides and the longest side in any given triangle. Students will be asked to use a ruler and protractor to draw the following types of triangles:

*Two acute triangles

*Two obtuse triangles

*One right triangle

Specific classification of the triangles by the side lengths is not necessary when considering how to draw each triangle. Students will label all side lengths and angle measures for each triangle. Students will then plug each set of side lengths into the Pythagorean Theorem formula. In each case, students should pick out the two smallest sides to represent a and b in the formula, and use the longest side to represent c. Students will observe the relationships between the results for each set of triangles. The teacher may want to discuss results with students as they work. Some questions that the teacher may ask are as follows:

1. What relationship do we find when using the Pythagorean Theorem with the sides in an obtuse triangle? In an acute triangle?
2. Why do these relationships make sense (based on the types of angles formed in each triangle)
3. How do we know if this relationship will always happen?

Teacher Notes:

The teacher should share the following information with students to record in their notes for future reference. The teacher should also complete example problems with students representing each of the three types of problems encountered when using sine, cosine, and tangent (listed in #5 of *Teacher Notes*).

1. Application of the Pythagorean Theorem: The Pythagorean Theorem can be used to determine if three triangle side lengths create acute or obtuse triangles by comparing the sum of the squares of the two smaller sides to the square of the longest side:
    a. In an acute triangle, the sum of the squares is greater than the square of the third side
    b. In an obtuse triangle, the sum of the squares is less than the square of the third side.

2. Trigonometry: the study of the relationships in a right triangle between the sides and angles.

3. Trigonometry was created between 190 and 120 BC by Hipparchus, a Greek astronomer and mathematician. Hipparchus created trigonometry using the 360 degree measure that was evolved by the Babylonians. The original purpose of trigonometry was to be able to calculate the position of a planet in space[4].

4. Focus angle: the focus angle for use with trigonometry is one of the acute angles in a right triangle. The focus angle tells us how to identify each of the following sides in a right triangle to prepare for use with the three trigonometric functions:

    a. Hypotenuse: the longest side in a right triangle. This is the side directly across from the right angle.

    b. Opposite: the side directly across from the focus angle in the triangle.

    c. Adjacent: the side joined to the vertex of the focus angle in the triangle.

5. Trig Functions:

    a. Sine: opposite / hypotenuse

    b. Cosine: adjacent / hypotenuse

    c. Tangent: opposite / adjacent

    d. Every trig problem should be set up in the following format

    Function (angle) = ratio

    *Function: sine, cosine, or tangent

    *Angle: the acute angle that you are using for your ratio

    *Ratio: the rule for the function chosen in fraction form

6. The trigonometric functions cannot be used with the right angle in a triangle. Each side has to uniquely represent the opposite, adjacent, and hypotenuse based on the position of the focus angle. If we were to use trigonometry for the right angle in a triangle, the hypotenuse and the opposite side would be the same, which is not possible in right triangle trigonometry.

7. Types of trigonometric problems:

    a. Finding a ratio: Set up the ratio using the rule for the desired function and simplify the fraction

b. Finding a missing side length: set up an equation and solve for the missing side, which represents either the numerator or denominator in the ratio.

c. Finding a missing angle: set up an equation and use the inverse function to solve for the missing angle.

Lesson 5: Angles of Elevation and Depression

Objective:

1. Identify angles of elevation and depression and discover the relationships between the angles and the trigonometric functions.

2. In this lesson, students will further their knowledge of how to apply the rules of sine, cosine, and tangent to right triangles. This lesson will be the introduction to the real life application of the functions. One important fact that students will take away from this lesson is that the angles of elevation and depression form alternate interior angles within one uniform setting.

Discovery Activity 6: Navigating through Trigonometry

The teacher may want to complete this activity during day two of this lesson. It will be essential for students to be more familiar with the rules for sine, cosine, and tangent before completing the WebQuest (see appendix A). Students will explore the uses and history of trigonometry and will become more familiar with the three functions through the completion of this activity. Students will have a set of instructions for navigating two websites to research essential facts that will aid in the understanding of this lesson. The first website that students will explore is Math Careers[5]. Students will learn about three professions that specifically use trigonometry. Students will then brainstorm other professions that may use trigonometry and explain their conjectures. The second website in the WebQuest will be MathTV.com. This website features several teaching videos for trigonometry. Students will be introduced to two application problems in the video involving angles of elevation and depression. Based on the instructor's presentation of each problem, students will derive their own definition for angles of elevation and depression. The application problems will give students more insight to how trigonometry is used in everyday life.

Teacher Notes:

The teacher should share the following information with students to record in their notes for future reference:

1. Angle of elevation: the angle between the horizontal and the line from the object to the observer's eye. The angle formed when looking up.

2. Angle of depression: the angle between the horizontal and the observer's line of sight. The angle formed when looking down.
3. The angles of elevation and depression form a right triangle with the distance between the observer and the object as the hypotenuse, and the horizontal and vertical lengths that connect the observer and the object as the legs of the triangle.
4. Angles of elevation and depression are always acute angles.
5. An angle of elevation and an angle of depression formed in the same picture are alternate interior angles, therefore they are congruent.

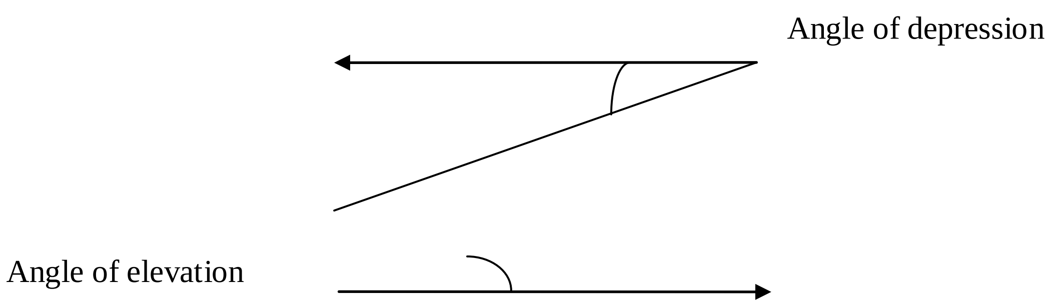


Figure 5

Lesson 6: Real World Application

Objective:

1. Apply trigonometry to word problems involving angles of elevation and depression.

2. This lesson will focus solely on real world application using the three trigonometric functions to solve problems.

Opening Activity:

The teacher should walk through the following problems with students using the word problem outline Breaking It Down (figure 3):

1. A boat is observed by a parasailer 75 meters above a lake. The angle of depression is 12 degrees. How far away is the parasailer from the boat?
2. A person is looking up at the top of a tree. The angle of elevation from the person's line of sight to the top of the tree is 56 degrees. The person is 6 feet tall. If the person is 48 feet from the tree, how tall is the tree?

3. From the top of a tower that is 150 feet tall, an air traffic controller observes an airplane on the runway. The angle of depression is 12 degrees. How far away is the airplane from the base of the tower?

Discovery Activity 7: Applying Trigonometry to My World

In this activity, students will get a chance to create their own application problems involving angles of elevation. Students will create their own sextant to measure angles using a protractor, note card, straw, paper clip, and string (see appendix B). The teacher should have students develop a plan for the objects that they would like to focus on for measurement. Some examples may include measuring the height of the school building, a tree, flagpole, etc. Students should work in groups of two to complete the activity, creating at least three application problems that they will solve on their own after they have created their sextants. One partner will need to hold the sextant to focus the angle while the other partner measures the angle by reading the intersection of the string and the protractor. If time permits, the teacher may also have students exchange problems with other groups to see if they can successfully solve other problems created by their classmates.

Conclusion

After the close of Lesson 6, and time for review at the teacher's discretion, students should complete an assessment based on skill and application problems to show mastery of the content covered in each lesson. An example assessment can be found in appendix A. The teacher may also want to include an open ended question to add to the assessment in order for students to show their mastery of answering the essential question throughout the unit of WHY and HOW do we use trigonometry in our world today. The teacher should also continue to review these concepts as they become important to student success with several other topics that students will encounter in geometry including area, volume, and polygon angles and sides. This unit spends twice as much time as normally allotted to teach the concepts of right triangles and trigonometry, but will prove valuable to teachers when introducing the difficult topics of areas and volumes in polygons and solids.

Notes

1. NC Department of Public Instruction, "North Carolina Standard Course of Study: Geometry," http://www.dpi.state.nc.us/curriculum/mathematics/scos/2003/9-12/47geometry.

2. Charlotte Mecklenburg School System, “Geometry Pacing 2009-2010,” http://cms.k12.nc.us.

3. Math TV, “Math TV: Geometry and Trigonometry,” http://www.mathtv.com/#.

    a. ***NOTE: You must register a free account in order to view the videos on the website.***

4. Think Quest, “Early Trigonometry,” http://library.thinkquest.org/C0110248/trigonometry/history1.htm.

5. Council For Mathematical Sciences, “What’s the Use of Trigonometry,” www.mathscareers.org.uk/14_-_16/maths_in_everyday_life/trigonometry.cfm.

Appendix A

Unit Discovery Activity 1: Similarity in Right Triangles Review

Use your Zometools to create a right triangle with two different side lengths. The legs should be congruent, and the hypotenuse should be formed from congruent smaller pieces. Then, connect the right angle to the intersection of the two pieces of the hypotenuse to create two more right triangles inside of the large one. Before moving on with the activity, present your model to the teacher for approval.

Answer the following questions based on your figure created.

1. Define a vertex for each connection in the figure. Redraw your model below and label each vertex with a different letter:

2. List at least three properties that the right triangles share without using any measuring tools. Justify each property.

    a. ____

    b. ____

    c. ____

3. Without using a protractor, determine all of the angle measures for each of the similar triangles. How do you know that these angle measures are correct?

4. Determine how the triangles are similar, and name the similar triangles.

Δ________________ ~ Δ________________ ~ Δ________________

5. Identify two congruent triangles in the figure:

Δ________________ = Δ________________

6. Determine the relationship for the hypotenuses of the three similar triangles. Explain this relationship.

Appendix A

Unit Discovery Activity 6: Navigating Through Trigonometry

In this WebQuest, you will explore two websites to increase your knowledge of the uses of trigonometry. Be sure that you ONLY visit the websites listed and answer all questions based on the information that you find.

I. Math Careers

WEBSITE: www.mathscareers.org.uk/14_-_16/maths_in_everyday_life/trigonometry.cfm

1. Give a detailed definition of trigonometry:
2. List two ways that trigonometry is used in navigation:
   a. ____
   b. ____
3. Under the topic of navigation, list two careers that may find trigonometry useful based on what you have read. Why do you think that trigonometry directly affects these careers?
4. How is sound travel related to trigonometry?
5. What other careers study the production of sound, light, or movement waves? How would trigonometry be important to these careers? List two.
   a. ____
   b. ____

II. MathTV.com:

WEBSITE: www.mathtv.com

Follow this path on the menu on the left: Trigonometry: Right Triangle Trigonometry: Applications: #1. View the video with the instructor and show your work for each of the two problems below:

1. Based on your observations in the video, give a description to each of the following terms:

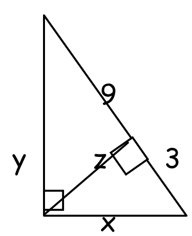
    a. Angle of elevation: ____

    b. Angle of depression: ____

Appendix A: Unit Test

Apply geometric mean and Pythagorean Theorem formulas to solve problems

1. Find x, y, and z.

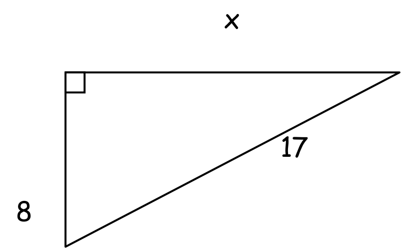


x = __________

y = __________

z = __________

2. Using Pythagorean Theorem, find x.



3. Find the geometric mean of 8 and 15.

4. Find the geometric mean of $6\sqrt{3}$ and $8\sqrt{3}$.

5. Is the set of numbers 6, 13, 15 a Pythagorean triple? Justify your answer.

6. Do the following three sides form right triangles? If not, what kind of triangles are they? (acute or obtuse)

    a. 16, 34, 30

    b. 5, 10, 12

Apply rules for sine, cosine, and tangent to solve right triangles

7. Find the following for the triangle to the right: (leave in fraction form and simplify)

a) sin A ____________ d) sin B ______________

c) cos A ____________ e) cos B ______________

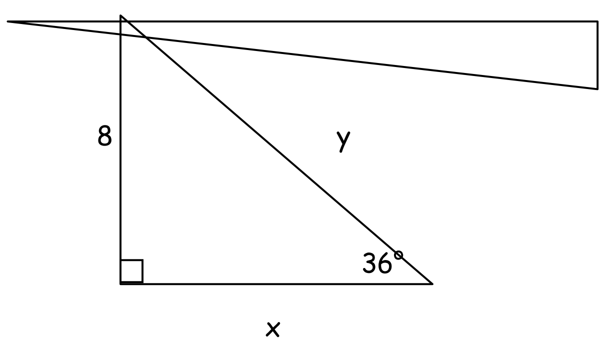
b) tan A ____________ f) tan B ______________

A 12 C

9

15 B

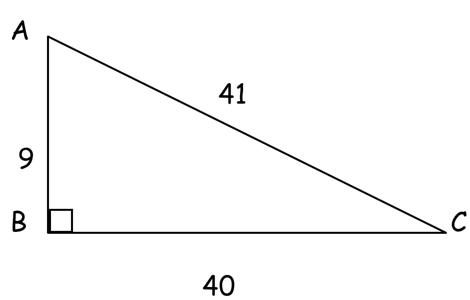
8. Find the MISSING SIDES in the right triangle round to TWO DECIMAL PLACES:



x=

y=

9. Find TO THE NEAREST HUNDREDTH: m∠A =_____ and m∠C=_____



Solve word problems involving trigonometry and angles of elevation and depression

MULTIPLE CHOICE: For questions 10-, draw an illustration to represent and then find the requested value, choose the correct answer. NO WORK SHOWN = NO CREDIT

10. A ski slope is 1,000 yards long with a vertical drop of 172.6 yards.

Find the angle of depression of the slope. _______

Picture: Answer Choices:

a. 9.9°

b. 0.003°

c. 8.4°

d. 0.5°

11. A kindergartener is laying on the ground looking up at the top of the swing set. The kid is 27 feet from the base of the swing set. If the angle of elevation from the kids eyes to the top of the swings is 21°, how tall is the swing set?

____

Picture: Answer Choices:

a. 28.9 feet

b. 10.4 feet

c. 75.3 feet

d. 70.3 feet

12. A large balloon for a parade is anchored to the ground by a string that is 80 feet long. If the angle of elevation from the ground to the balloon is 68°, how high is the balloon?

____

Picture: Answer Choices:

a. 198 feet

b. 86.3 feet

c. 30.0 feet

d. 74.2 feet

13. The length of an altitude of an equilateral triangle is 6 yards. ______

Find the length of a side of the triangle.

Picture: Answer Choices:

a. 6 yards

b. 3.46 yards

c. 6.9 yards

d. 10.4 yards

Appendix B

Discovery Activity 3: Cutting Corners Using the Pythagorean Theorem

Additional Examples

1. Have students measure the shortest distance between two points in a hallway on opposite sides of the hall by counting the tiles along the horizontal and vertical distances between the points. Students are not able to "count the tiles" along a diagonal piece. Have students check their conjecture by using a yard stick to measure the actual distance.

2. Have students measure the shortest distance between two corners of a wall connected by a diagonal. Use a yard stick or measuring tape to measure the vertical height of the wall and the horizontal stretch. These will represent the legs of the right triangle. The corners when connected will create the length of the hypotenuse in the triangle.

3. Have students work in groups to estimate the shortest distance between two points on a field or track. Have some students walk the vertical and horizontal distances that connect the two points, and have some students walk the diagonal that connects the two points. Students should count their paces as they walk, and compare the square of the horizontal and vertical distances to the square of the distance along the diagonal.

Appendix B

Discovery Activity 7: Applying Trigonometry to My World

Steps for Creating a Sextant

1. Materials: 6” protractor, straw, 4x6 note card, paper clip, string
2. Tape the protractor to the note card.
3. Tape the string to the note card along the straight edge of the protractor. The straw should be long enough so that both ends hang off the note card.
4. Attach the paper clip to the string so that it hangs on the end of the string.
5. Attach the other end of the string to the center point of the protractor where the protractor lines up with the angles.
6. TO MEASURE: with the protractor held upside down, look through the straw at any specific object. The paper clip will move at the angle of your observation. Use the intersection of the string and the protractor to determine the angle of elevation. This angle should always be an acute measure.

Resources

Bibliography for Teachers

Charlotte Mecklenburg School System, “Geometry Pacing 2009-2010,” http://cms.k12.nc.us (Accessed September 2009).

The Charlotte Mecklenburg pacing guide for geometry takes the North Carolina Standard Course of Study and adapts it to the block schedule for the high school classroom. The pacing segments each objective into a chronological timeline that follows the Glencoe Geometry textbook that students use in the class. The timeline outlines the objectives that should be covered in the class for each quarter of the semester. The pacing guide also suggests a specific amount of days for teaching each objective.

NC Department of Public Instruction, “North Carolina Standard Course of Study: Geometry,” http://www.dpi.state.nc.us/curriculum/mathematics/scos/2003/9-12/47geometry (Accessed September 2009).

The North Carolina Standard Course of Study is implemented for each specific content area in mathematics. In the geometry standards, trigonometry constitutes for one objective, but also has implementation in the objectives involving areas and volumes of two and three dimensional figures. The standards also give an outline of student prior knowledge to help teachers prepare their direction for instruction.

Student Reading List

Council For Mathematical Sciences, “What’s the Use of Trigonometry,” www.mathscareers.org.uk/14_-_16/maths_in_everyday_life/trigonometry.cfm (Accessed September 2009).

This website describes how trigonometry is used in three career areas and also gives examples of specific careers that use the functions and graphs of the trigonometric functions. Students will gain insight to how trigonometry is applicable to practical real life situations in our everyday lives.

Math TV, “Math TV: Geometry and Trigonometry,” http://www.mathtv.com/# (Accessed September 2009)

Math TV provides free videos demonstrating different mathematical topics (MUST SIGN UP FOR A FREE ACCOUNT). There are several videos relevant to the unit in the sections notated as Geometry and Trigonometry in the menu on the homepage of the website. This is a resource that can be accessed by students at school or at home. The trigonometry videos can be used to introduce word problems to students or review skill based problems.

Think Quest, “Early Trigonometry,” http://library.thinkquest.org/C0110248/trigonometry/history1.htm (Accessed September 2009).

This website is student friendly in navigation and easy comprehension. Students can learn about the history of trigonometry as well as its development in different parts of the world over the centuries. Students can also get detailed descriptions of the trigonometric functions and Pythagorean Theorem to reinforce what they have learned about these topics.

Materials for Classroom Use

Lesson 1: Zometools, Pipe Cleaners, or Popsicle Sticks

More students are learning when they can create a tangible product that allows them to see the content to be mastered. The discovery activity in this lesson calls for students to build a model of similar triangles. Although there are several ways that this could be accomplished, Zometools, pipe cleaners, and popsicle sticks prove to be the most sufficient for building the model. Students will be able to identify each of the separate segments that combine to make up each triangle. Students will also be able to manipulate the placement of the segments to compare lengths for classifying each of the angles and sides formed in the similar triangles.

Lesson 3: Tape Measure and Yard Sticks

The discovery activity in this lesson allows students to physically be a part of the problem solving. These measuring tools will be used to help students measure long distances throughout the classroom or halls for the problems that the teacher presents to the students. The tape measure will allow students to measure vertical distances that they may not be able to reach with a yard stick.

Lesson 4: Rulers and Protractors

Students will be able to practice angle measurement using a protractor in the discovery activity for this lesson. Students will use the angle sum theorem for a triangle to make sure that their measurements are precise. The use of measuring tools allows students to recognize the properties of angles and sides and related inequalities as they measure the six lengths for each of their triangles.

Lesson 6: Sextant

This unique tool will allow students to measure angles of elevation to different objects at different heights. Students will be able to calculate great distances using this tool. Students will also obtain experience with creating their own application problems using the tool.