

Dilations and Linear Equations – A Project Based Unit

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School and Classroom Background

Bailey Road Middle School is an urban public middle school located in Cornelius, North Carolina. There are approximately 1,400 students at Bailey Road Middle School, which is part of the Charlotte-Mecklenburg School District. The school is made up of mostly Caucasian students, with the following breakdown of non-Caucasians; 13% African American, 7% Hispanic, and 2% Asian. The surrounding communities are mostly middle and upper class families, which makes Bailey Road Middle School different from the average middle school in Charlotte-Mecklenburg Schools. We do, however, serve communities that are impoverished, and approximately 25% of our students are on free or reduced lunch. 7% of our students have Limited English Proficiency, and another 7% of our students are on Individualized Education Plans for a specific learning or behavioral disability^[1].

At Bailey, we have two levels of math classes. There is Honors math, which in the 8th grade is Algebra I, and there is Standard Plus math. The honors classes have a little more consistency as far as the achievement level of the students. In the Standard Plus classes, there are a variety of different levels of learners, ranging from far below grade level to slightly above grade level. Every year I have a couple students that make me wonder why they aren't in Algebra as an eighth grader. Each year that I have been at Bailey Road Middle School, I have taught both Algebra and Standard Plus classes. This year in the 8th grade, there are 7 Algebra classes (honors) and 5 Standard Plus classes, which speaks to the extraordinarily high number of high achieving students at my school. At some schools in CMS with the same number of students as Bailey, there is only one 8th grade Algebra class.

I teach from the objectives stated in the North Carolina Standard Course of Study (NCSCOS), and next year, CMS will be adopting the Common Core Standards, which are a national set of learning standards. At present, the NCSCOS for 8th grade math states two objectives that relate to dilations in the coordinate plane and the slope-intercept form of a line. They are Objective 3.03, "*Identify, predict, and describe dilations in the*

coordinate plane,” and Objective 5.01C, “*Find, identify, and interpret the slope (rate of change) and intercepts of a linear relation.*”^[2]

This unit is designed for the implementation in 7th or 8th grade, as students begin to explore the concepts of graphing, slope, intercepts, the slope-intercept form of a line ($y = mx + b$), and dilations in the coordinate plane. It can also be used with Algebra students, as most of these topics translate to both the 8th Grade Standard Course of Study and the standards that represent Algebra I. CMS has recently adopted the “Math Investigations” textbooks and online resources, and it is important to note that this unit *does not* draw from these textbooks or resources.

Rationale and Objective

In my opinion, one of the biggest hindrances for the middle school student when it comes to learning math is motivation. Most 8th grade students see learning math as a chore, much like washing the dishes or cleaning their room. In my experience, very few students come into my classroom proclaiming that math is their favorite subject. One objective of this unit is to allow students to be creative, while at the same time taking ownership of the unit. Studies have shown that students working on open-ended projects can outperform their counterparts that learn in a more traditional, direct instruction driven classroom. In a study by Jo Boaler, associate professor of education at Stanford University, students that learned through an open-ended project were shown not only to score better on math questions requiring analytic and or conceptual thought, but also math questions that involved rote learning^[3]. I feel that giving my students a quasi open-ended project will give them more motivation to want to learn about the mathematics behind their project.

In my experience, when students jump into the concepts of slope, intercepts, dilations, and transformations, many of them don’t see the point. They are constantly asking when they are going to use this in real life. Through this unit, I hope to illuminate some real world examples as to why these concepts are important in some aspects of everyday life, while giving the students some chance to be creative and flexible in their learning. Teaching these concepts through direct instruction can make the material abstract and difficult to understand, and this is why this unit is focused on a project based method of teaching. As students progress through the stages of the project, they will begin to take ownership of these concepts, which will help them to better retain their understanding of them in the future, as well as provide a deeper understanding of the stated topics.

In addition, the concepts of dilation and slope-intercept form are typically taught in isolation. By using a project that incorporates both skills at different phases of the unit, I hope to make the learning of these two topics even more authentic.

History of $y = mx + b$

To provide students with background information on how this puzzling equation came to appear in Algebra textbooks across the world, I will provide a brief background on the slope-intercept form of a line. The first notation of the slope-intercept form of a line appears to have come from an Italian mathematician named Vincenzo Riccati, who lived from 1707-1775. Although most of his work focused on mathematical analysis and differentiated equations, in his book *Vincentii Riccati Opusculorum ad res Physica, & Mathematicas pertinentium*, he refers to the equation $y = mx + n$. In his book, he credits a Swiss mathematician named Jacob Herman with coming up with this equation^[4]. As to *why* the letter m is used to represent the slope of a line, the consensus is that nobody really knows. Students ask all the time why m is used to represent the slope of a line, and they are usually a little let down when, as a teacher, you have to admit that you don't know and that nobody really knows.

Before Riccati and Herman, however, the *concept* of a linear equation can be traced back to ancient Egyptians and Babylonians. In the ancient Islam world, in the 9th century, an Arabic mathematician named Muhammad al-Khwarizmi wrote one of the first comprehensive documents about Algebra and equations. In Arabic, the word *al-jabru* means “restoration”, and it is the root word of “Algebra”. The first person to start using symbols to represent the unknown, such as “ x ” and “ y ”, was a French philosopher and mathematician named Rene Descartes. He proposed using symbols to represent the unknown in his book *La geometrie*, which was published in 1637^[5].

Background Knowledge of Students

In order for students to successfully complete this unit, they must be proficient in several areas of math. First off, they must be able to do basic operations (multiplying, dividing, adding, subtracting). They must also be proficient multiplying with fractions, although it is not essential that they be able to do this without the use of a calculator. Students must be able to graph coordinate points, and in lesson 1, it is essential that the teacher monitor the students and ensure that they can graph coordinate points correctly. In addition, students need to be familiar with the concept of using variables to represent the unknown,

as well as substituting numbers in for variables, and following a given formula, in this case, $y = mx + b$. Students will have to eventually plug numbers in for x in order to find y . Since a portion of this project will be done in groups, students should have experience working with groups, and the teacher should be familiar enough with their students to ensure the groups are constructed properly (more on this later).

UNIT

Lesson 1

The unit will begin by ensuring that all students review the properties of graphing coordinate points on a coordinate plane, and will continue on the first day with students making shapes by connecting coordinate points, which will be essential for the first part of this project. The teacher will first put a series of coordinate points on the board that, when connected will form a right triangle. The students should graph the points $(-6, 7)$, $(-6, -5)$, $(4, -5)$, and connect them in order. The teacher should look to ensure that all students remember how to graph these points correctly to make identical right triangles (**Figure 1**). The reason that it is important to ensure that all students can graph coordinate points correctly is that many times, when students have not worked with coordinate points for a while, they will have a tendency to make careless errors. Frequently they will confuse the x and y coordinate, and they also can put points in the incorrect quadrants by not paying close enough attention to the positive or negative value of each point.

Once the students have shown that they can graph basic coordinate points, the teacher will give them each a picture to draw on graph paper. This will be done by the teacher giving them a list of coordinate points, and the students will have to create the picture by connecting one point to the next. The teacher should give “Image A” to the half of the class that he/she feels are among the upper half of the class in terms of mathematical ability, and the teacher should give “Image B” to the lower half (**Figures 2, 3, and 4**). The teacher should assist students that need further clarification, and, of course have something extra for students to work on that finish early.

Once all students have completed their drawings based on the coordinate points they have been given, the teacher should pair up the students in carefully selected, pre-made pairings. One strategy that the teacher could use to ensure the quick transition into partner work is to write the name of each student’s partner on each worksheet before they

are passed out. This way, the teacher is not spending time trying to put each student with their partner. All that the teacher has to say is, "Please pair up with the student whose name is on your worksheet."

There are many things that should be considered when the teacher makes these pairings. The dyads should have one student that drew Image A, and one student that drew Image B. In addition, the teacher may want to consider pairing up students of the opposite gender, pairing up strong egos with students that are shy, as well as considering which students are the highest achieving with which students are the lowest achieving. Once the students have been paired up, they should describe the image they have drawn by reading the coordinate points, one at a time, and having their partner draw the image as they go. Making students have a mathematical discourse about the coordinate points, even in something as basic as this, will help the students retain information about graphing a picture using coordinate points, particularly the lower achieving students.

Although this lesson seems basic, it is *critical* that students can correctly work with ordered pairs in the coordinate plane, as well as explain their drawing to another student. During this lesson, the teacher should assist as much as necessary, as it is important that all students feel successful during this first phase of the unit. Research has shown that when students feel successful, it increases their motivation and reduces their anxiety^[6].

Figure 1

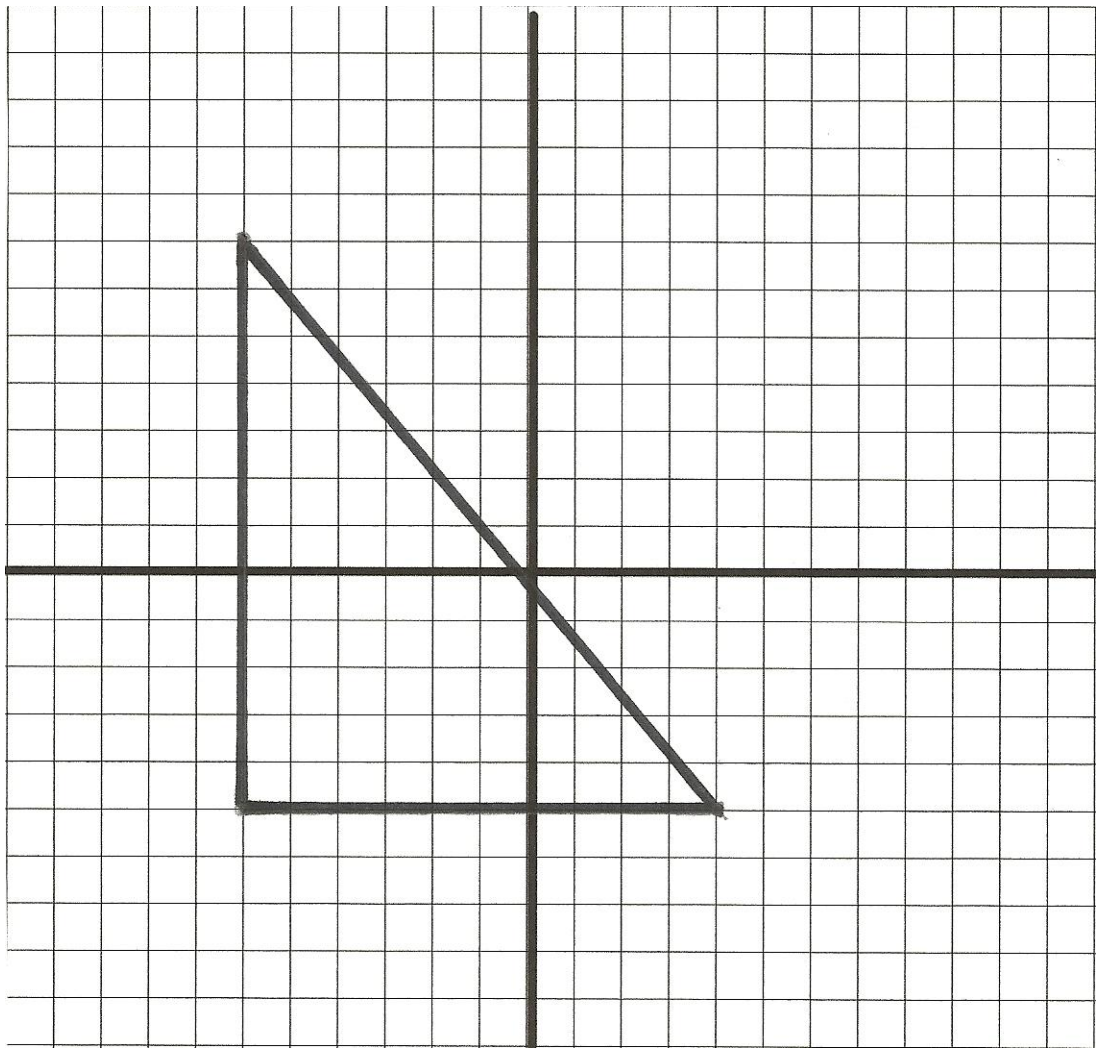


FIGURE 2

Coordinates for Image A – Connect each pair of points in order. When you see the word ***STOP*** start a new line.

$(8, 5) \rightarrow (6, 3) \rightarrow (-13, 3) \rightarrow (-13, 9) \rightarrow (-11, 7) \rightarrow (6, 7) \rightarrow (8, 5) \rightarrow (5, 5) \rightarrow (5, 7)$ ***STOP***

$(2, 5) \rightarrow (-3, 1) \rightarrow (-8, 5) \rightarrow (2, 5)$
STOP

$(-12, 2) \rightarrow (-17, 2)$ ***STOP***

$(-14, 5) \rightarrow (-17, 5)$ ***STOP***

$(-14, 7) \rightarrow (-17, 7)$ ***STOP***

$(-11, 10) \rightarrow (-16, 10)$ ***STOP***

$(7, 9) \rightarrow (9, 9) \rightarrow (9, 11) \rightarrow (7, 11) \rightarrow (7, 9)$ ***STOP***

$(7, 9) \rightarrow (6, 8)$ ***STOP***

$(8, 9) \rightarrow (8, 8)$ ***STOP***

$(9, 9) \rightarrow (10, 8)$ ***STOP***

$(9, 10) \rightarrow (10, 10)$ ***STOP***

$(9, 11) \rightarrow (10, 12)$ ***STOP***

$(8, 11) \rightarrow (8, 12)$ ***STOP***

$(7, 11) \rightarrow (6, 12)$ ***STOP***

$(7, 10) \rightarrow (6, 10)$ ***STOP***

Coordinates for Image B – Connect each pair of points in order.

$(3, 10) \rightarrow (5, 8) \rightarrow (4, 8) \rightarrow (6, 6) \rightarrow (5, 6) \rightarrow (7, 4) \rightarrow (6, 4) \rightarrow (8, 2) \rightarrow (4, 2) \rightarrow (4, -1) \rightarrow (2, -1) \rightarrow (2, 2) \rightarrow (-2, 2) \rightarrow (0, 4) \rightarrow (-1, 4) \rightarrow (1, 6) \rightarrow (0, 6) \rightarrow (2, 8) \rightarrow (1, 8) \rightarrow (3, 10)$

FIGURE 3

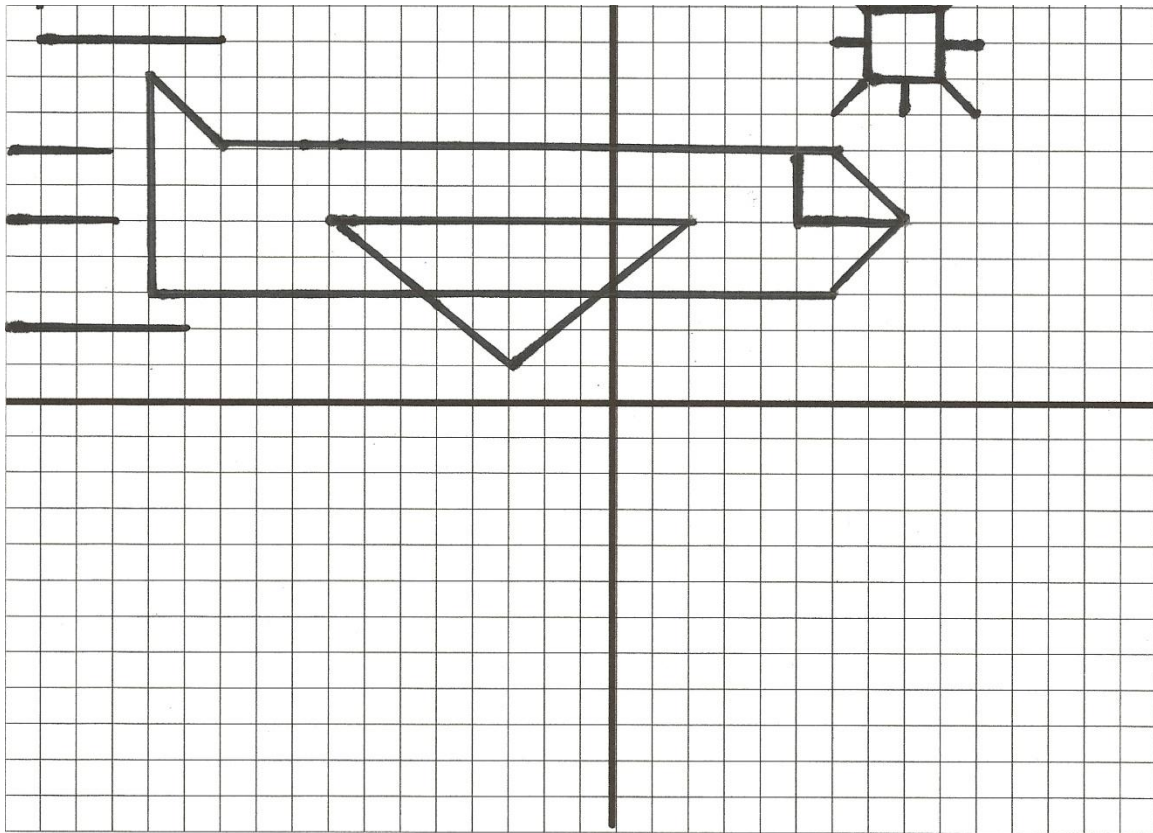
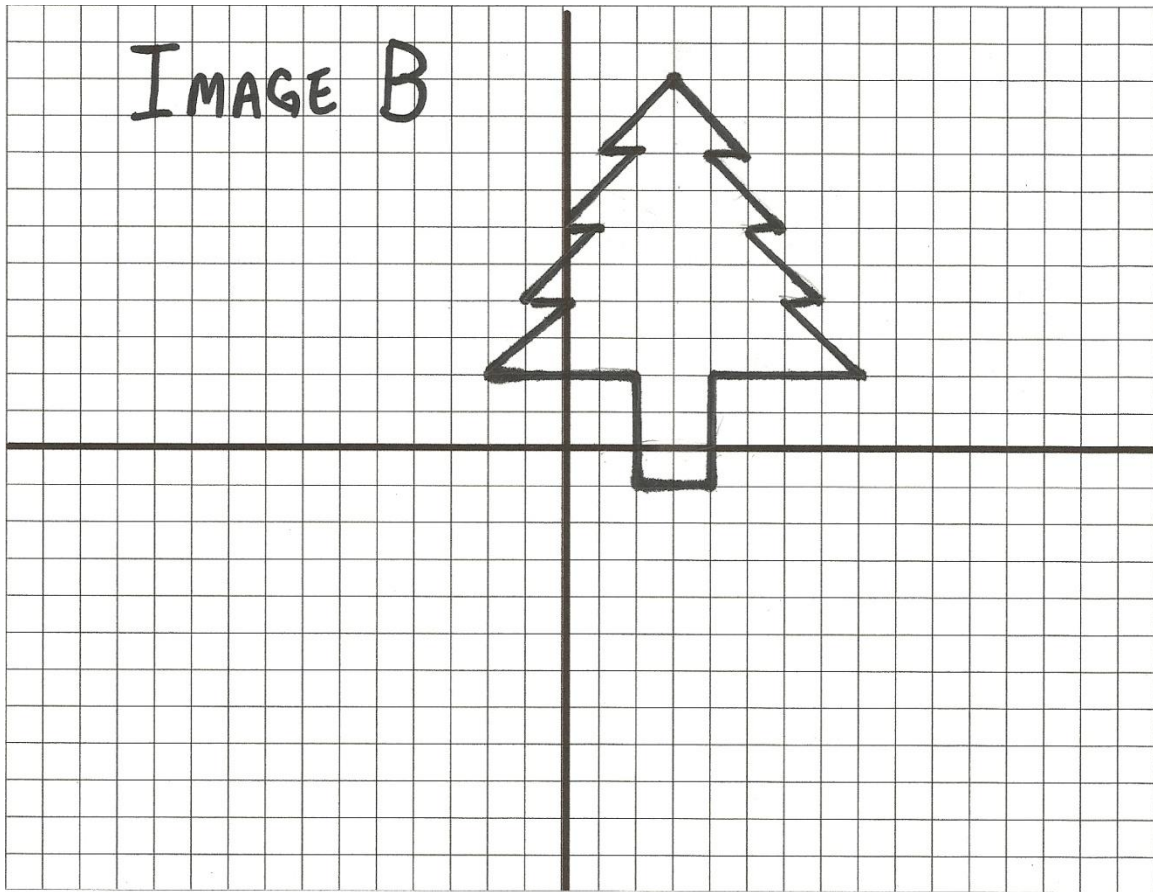


FIGURE 4



Lesson 2

In this lesson, the teacher will inform the students that they are going to design the homepage of a brand new website. The teacher can tell the story of Mark Zuckerberg and Facebook if they are familiar with it. A good website that sums up his life and rise to fame can be found at <http://www.biography.com/people/mark-zuckerberg-507402>^[7]. The teacher can read this article with the students, or read it him/herself and summarize how, by creating a website, he rose to fame and fortune.

Or, as a motivator, the teacher can show this clip:

<http://www.youtube.com/watch?v=RyhI8NuxNyY&feature=related>, which is a 5-minute video discussing the career of web design^[8]. The reason for showing this video is to introduce to the students the concept of web design. As mentioned previously, getting students motivated is one of the key challenges facing any middle school math teacher, and this video should help motivate students to want to stay focused throughout the course of this unit. Once the video has ended, instruct students that they are going to come up with an idea to design their own website. The students will need to choose from the following choices as the theme for their website; *Sports, Fashion, Gaming, or Music*. Once the students have chosen which theme they are going to go with, they should brainstorm ideas of what specifically their website is going to do. Is it going to provide a service? Is it going to be informative in nature? Are they going to try and connect with people that share their love of a common theme? The teacher may want to do a starter activity prior to this lesson to help generate some ideas as a class, since many students may feel overwhelmed by the amount creativity that this part of the unit needs. For example, at the beginning of class, the teacher may want to write on the board, “What makes a website good?” As a whole class, the students can brainstorm reasons that some websites are considered good, while other websites are not so good. This starter should help students wrap their heads around some of the things that go into starting a website.

As a Language Arts connection, the teacher should tell the students that they are going to need investors to fund their website, and they must write a proposal to investors describing, in detail, what the purpose of their website is, and, just as importantly, how they plan to make money from their website. The teacher should remind all students that advertising through their website is one way to make money, as long as they get enough traffic on their site. Other ways to make money through their website may include a charging membership fee, selling items on their website, collecting donations, etc.

Lesson 3

In this lesson, students will have to recall the activity from lesson 1. Their job is to create the cover page of their website on a piece of 20-unit by 20-unit graph paper with the following guidelines:

- 1) They must use one image from the choices that follow (**Figures 5 and 6**) – As they re-draw their image of choice, they must correctly label each “dot” with the coordinate points that they have placed it on.
- 2) The title of their website must appear on their cover page.

Aside from that, the students can be as creative and colorful as they wish. This part of the unit may seem a little slow moving, and perhaps a bit “artsy” for some teachers, but it is an essential part of the students claiming ownership over their website, and thus, the learning of the mathematical concepts at hand. Once students finish designing their cover page, the teacher should carefully pair them up, making sure not to pair up two students that have chosen the same image to put on their website. Tell the students that, one at a time, they are to pretend that one of them is describing their image to a web designer, and that the web designer can only create the image by using coordinate points. Since the students have done a similar activity already, this should run pretty smoothly with little need for teacher support.

Lesson 4

In this lesson, students will explore the concept of *dilation*. The teacher should instruct all of the students that their web designer has made a mistake. The web designer was using out-dated software, where everything had to be described on a 20 by 20 grid. If the web designer wants to use that old software, he will have to charge the students double. The newer software, which is half the price as the old software, uses a 10 by 10 grid. The teacher should ask the students, *How much are we going to have to scale down our designs?* The students should respond that their designs need to be *half as large* or that their designs need to be *cut in half*. The teacher should then ask students to partner up and try and develop a way to make their chosen image half as large. If the students are having trouble, tell them to focus their attention on the coordinate points. The students should come up with the idea (with the teacher’s help) that if all of the coordinate points are cut in half, then the image will be cut in half. The teacher should eventually elicit the response out of the students that they can *multiply* each point by one-half to cut all of the coordinate points in half. The teacher should then introduce the concept of a *scale factor*.

FIGURE 5

Directions: Based on the theme of your website, create an image using the coordinate points below. Plot the points and connect them as indicated. **DO NOT** connect the last point to the first point, unless indicated. Whenever you see the word *STOP*, start a new line.

MUSIC

$(-5, 5) \rightarrow (-7, 5) \rightarrow (-7, 3) \rightarrow (-5, 3) \rightarrow (-5, 9) \rightarrow (-11, 9) \rightarrow (-11, 3) \rightarrow (-13, 3) \rightarrow (-13, 5) \rightarrow (-11, 5)$

FASHION

$(5, 10) \rightarrow (6, 9) \rightarrow (7, 10) \rightarrow (7, 7) \rightarrow (9, 4) \rightarrow (3, 4) \rightarrow (5, 7) \rightarrow (5, 10)$

VIDEO GAMES

$(-3, -2) \rightarrow (0, -6) \rightarrow (-3, -6) \rightarrow (-4, -5) \rightarrow (-10, -5) \rightarrow (-11, -6) \rightarrow (-14, -6) \rightarrow (-11, -2) \rightarrow (-3, -2)$

STOP

$(-3, -3) \rightarrow (-4, -3) \rightarrow (-4, -4) \rightarrow (-3, -4) \rightarrow (-3, -3)$ STOP

$(-5, -3) \rightarrow (-6, -3) \rightarrow (-6, -4) \rightarrow (-5, -4) \rightarrow (-5, -3)$ STOP

$(-7, -5) \rightarrow (-7, -6) \rightarrow (-9, -8)$

SPORTS

Plot the following points and then make a CIRCLE around them, using the coordinate points as a guide. Do not use straight lines for these first four points:

$(2, -5) - (6, -1) - (10, -5) - (6, -9)$

Connect the following points with STRAIGHT LINES

$(4, -2) \rightarrow (5, -3) \rightarrow (5, -7) \rightarrow (4, -8)$ STOP

$(8, -2) \rightarrow (7, -3) \rightarrow (7, -7) \rightarrow (8, -8)$

Figure 6

