

This would be a good time to put the word *scale factor* on a word wall, if there is one present in the classroom. Tell the students to start with their chosen image, and re-draw it using coordinate points that are half of their original. Once they have systematically dilated their chosen image in half, (something that can easily be checked by the teacher), they can try and dilate the rest of their webpage. For 8th grade students, it is unrealistic that their *entire* web page will be dilated correctly, but if they can correctly dilate their image using its coordinate points, then they have been introduced to and learned the concept of dilation. Again, it is important for the teacher assist each student to make them feel successful, so that math anxiety and self doubt do not come in to the picture in the students' minds.

The teacher should then ask the students, *What could we have done if we wanted to make our designs twice as large, instead of cutting them in half?* The students should eventually respond that they could multiply all of the coordinate points by 2. *What would the number 2 be called in this instance?* (The scale factor). The teacher should ask multiple questions like this (*What would we do if we needed to make our designs a third of the size of the original?*) .

At this point, the students should be given a sheet of 10 by 10 graph paper. Tell the students to start with their chosen image, and re-draw it using coordinate points that are half of their original. Remind the students that they can multiply all of the coordinate points of their image by one-half, or they can simply divide all of the points by two. They will have to estimate points that lie between two integers, like 3.5 for instance. After they have re-drawn their original design, the teacher should give the students a ruler and have them measure several lengths of their line segments on both designs and compare them. Students should come to the conclusion that, not only are the coordinate points half of what they were in the original design, but the side lengths are also half as long.

Once this process is complete, the teacher should explain that they have all used a mathematical process known as *dilation*. This would be a good time to put the word *dilation* onto the word wall if one is present. To reinforce this skill, the teacher should give a homework assignment where the students have to dilate different objects in the coordinate plane using a variety of different scale factors. Students should come to the conclusion that, any time a scale factor is less than one, the object shrinks, or gets smaller. Likewise, any time a scale factor is greater than one, the object is enlarged.

Starting with this lesson, the students will transition from learning how to dilate an image to the concept of the slope intercept form of a line. The students will begin by choosing the company that will help launch their website. The teacher will give the students three choices that will provide creative advice, put their site online, and provide advertising for their website. Website group #1, which is called **Web-X Designs**, will cost the student \$2,000 in start-up fees, but they have an excellent reputation for promoting new websites. Website group #2, which is called **Net Surfer Plus**, costs \$500 in start-up fees, but they are an unknown group, and so the promotion of the students' websites is a bit of a risk with investment group #2. The third option is to pay the computer teacher at their school \$100 to launch and promote their website. Although he swears he will do a great job, he seems a little strange, but you can't beat \$100 to launch your website.

After each student chooses how they are going to start their website (Web-X Designs, Net Surfer Plus, or their computer teacher), they are now locked-in to a course of action that will take them through the rest of the project, learning about a linear equation and the slope-intercept form of a line along the way. The students that started with Web-X Designs should be told that their website will make a steady stream of revenue of \$250 per month. The teacher can be creative as to explain how this happened. They can even individualize the profit for each student, if they have time. For example, if a particular student wrote that their website would make money by charging a membership fee, the teacher can say that the membership fees are coming in at a rate of \$250 per month. If the student explained that their website would make money through advertising, then the teacher could tell them that Web-X Designs predicts that their advertising revenue will amount to an extra \$250 each month. The students that chose Net Surfer Plus will make a steady stream of income of \$150 per month. The students that chose to have their computer teacher launch their website will receive an income of \$50 per month. Pair up students that chose the same group (keeping in mind that factors that were discussed earlier), and have them project how much money they will make after 1 month, 2 months, 3 months, etc., up to six months. Remind students that they have to subtract the amount that they paid to launch their website. It might be helpful to provide a table that looks like this:

Month	Profit Made (Don't forget to subtract your start-up fee)
1	
2	

3	
4	
5	
6	

Although they won't know it at the time, the students are already using the concepts of functions and linear equations. They are using linear equations, like $y = 250x - 2,000$ if they chose Web-X Designs, without really even knowing it. This makes the learning authentic to each learner.

Lesson 6

In this lesson, students will start graphing their tables on a coordinate plane. Since the students have already plotted so many points in the earlier lessons of this project, the mistakes of the actual graphing of points should be minimal. The teacher should, as a whole class, display how to set up their graphs. (**See Figure 7**). The teacher should guide the students to make sure that they label the x-axis with Months, the independent variable, and the y-axis with Profit, the dependent variable. Also guide the students to understand *why* profit is the dependent variable, because it *depends on* how many months the website has been online. Working with a partner, they should individually graph the first 6 months of their profits. The graphs should show that, after 6 months, the students that chose Web-X Designs have not made a profit yet. In fact, they are still at negative \$500. Net Surfer Plus has made \$400 in profits and the third group (the computer teacher group) has made \$200. Next, have them graph the next 6 months of profits, so that they will have profit data for one full year.

After a year, Web-X Designs will have made a profit of \$1,000, Net Surfer Plus will have made a profit of \$1,300, and the computer teacher will have made the students a profit of \$500 (**Figure 7**). Once all students have graphed out their profits for one year, arrange students in groups of 3, so that each "company" is represented. The teacher should once again consider the factors of ability, gender, self-esteem, etc. Have each student copy the lines that represent the other groups on to his/her graph. The teacher may want to have each student color code each line. For example, the line that represents Web-X Designs could be blue, the line that represents Net Surfer Plus could be red, and the computer teacher line could be brown. Have them analyze each other's graphs, and have them answer the following questions: *Where does each graph touch the y-axis? What does this number represent? (The start up fees). Which line is the steepest, and*

which line is the least steep? Why is this? (Web-X Designs is the steepest because they make the most per month. The people who chose their computer teacher have a line that is not very steep at all, because they make the least per month). *If these websites were on the internet for 2 years, who would make the most money? Why?* (Web-X Designs would make the most money because their graph grows at a higher rate than the other two groups).

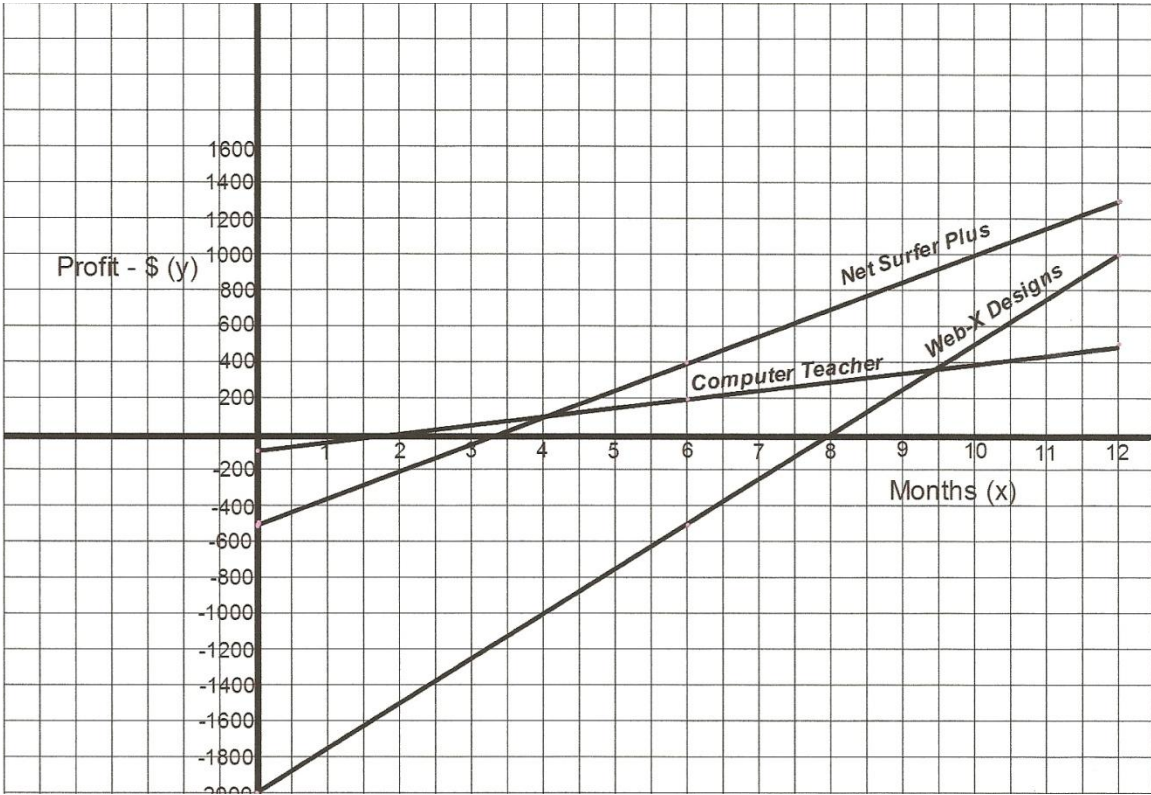
Lesson 7

In this lesson, students will make the connections between their graphs, the y-intercept of a line, the slope of a line, and the slope-intercept equation ($y = mx + b$). First, as a whole class, have students see if they can come up with an equation to match each of the 3 groups' profits. Tell them that, just like on their graphs, we will let x represent the amount of months the website has been online, and we will let y represent their profit after a given month. Since the equation is going to calculate the profit of each group, the teacher should guide the students to realize that the equation should start with " $y =$ ". Next, guide the students to come up with the 3 equations to represent the three groups. Web-X Designs will be represented by the equation $y = 250x - 2000$, Net Surfer Plus will be represented by the equation $y = 150x - 500$, and the computer teacher will be represented by the equation $y = 50x - 100$.

Ask the students *Which line was the steepest? How does that relate to the equation for that group? Which line was the least steep? How does that relate to the equation for that group?*

This is a perfect time to introduce the concept of *slope* to the students. By looking at how the steepness of the graph matches the amount made per month, students should see the connection between the slope of the actual line and the slope, or " m ", of the equation. The students should see that the slope is the same thing as the amount made per month. This is another opportunity to add the word *slope* to the word wall.

FIGURE 7



Next, ask the students *How does the point at which the line crosses the y-axis relate to each equation?* Students should notice that each line touches the y-axis at the same number that ends each equation, i.e. the “+ b ” part. This is a great opportunity to introduce the concept of “*y-intercept*.” Students should also see the connection between the *y-intercept* and the fact that, for each group, it is their start up fee. In many linear equations, the *y-intercept* represents some sort of starting point. At this point, the teacher should put *y-intercept* on the word wall.

And now, finally, the teacher can introduce the algebraic way to write the slope-intercept form of a line, the infamous $y = mx + b$. The students will almost certainly ask, “*Why does the letter m represent slope?*” And the teacher can give the honest, but frustrating answer of “*Nobody knows.*”

Once this part of the unit is complete, teacher can begin to extend on the concept of $y = mx + b$ with more real word situations. Although the amount of situations that a teacher can create and have the students represent both graphically and algebraically are limitless, some examples include:

- A taxi service charges a \$3.00 flat fee and then charges that passenger \$1.25 per mile
- A leaky bucket has 10 gallons of water and is losing water at a rate of a $\frac{1}{2}$ gallon every minute. (This situation is good to introduce the concept of a *negative* slope as opposed to a *positive* slope).
- Little Johnny and his mom arrange an allowance that pays Little Johnny \$5.00 a week, plus an additional \$2.00 that Johnny works around the house.
- A local gym charges an up front, \$50.00 fee, plus an additional \$30.00 per month for a membership.
- A 350 pound man starts a diet, which enables him to lose 6 pounds a week.

In summary, this unit is designed to teach students about the slope-intercept form of a line and the dilations of objects, without these objectives coming off as abstract and non-relatable. By trying to hook students in by telling them the story of Mark Zuckerberg and letting the students be creative with the design of their own website, the goal is to set a stage where each student will relate to these topics. In the end, the goal is for the students to understand that, in dilation, an object is enlarged or scaled down, and on a coordinate plane, this can be accomplished by using coordinate points, and multiplying them by a scale factor. Students should also understand the relationship between a real life situation, a table of values, an equation, and a graph. They should realize that the slope

of a line is the rate of change for that line, and that the y-intercept of a line is not only where a line crosses the y-axis, but it often relates some sort of “starting point” of a linear equation.

Works Cited

1. GreatSchools.org. "Learn About This School's Students." Greatschools.org
<http://www.greatschools.org/cgi-bin/nc/other/3383#toc> (Accessed October 3, 2011).
2. North Carolina Department of Instruction. "8th Grade Math Standard Course of Study."
<http://www.dpi.state.nc.us/curriculum/mathematics/scos/2003/k-8/38grade8> (Accessed October 3, 2011).
3. Edutopia. "PBL Research Summary: Studies Validate Project-Based Learning."
Edutopia.org
<http://www.edutopia.org/research-validates-project-based-learning> (Accessed September 12, 2011).
4. Askville. "In the slope intercept formula $y=mx+b$, why is the letter "m" chosen to represent slope?" Askville.com. <http://askville.amazon.com/slope-intercept-formula-y%3Dmx%2Bb-letter-chosen-represent/AnswerViewer.do?requestId=6061505> (Accessed September 12, 2011).
5. Derbyshire, John. *Unknown Quantity: A Real and Imaginary History of Algebra*. Washington, DC: Joseph Henry Press, 2006
6. Turner, J. C., et. al., "Motivating Mathematics Learning: Changes in Teachers' Practices and Beliefs During a Nine-Month Collaboration." *American Educational Research Journal* v. 48 no. 3 (June 2011): p. 718-62
7. Biography. "Mark Zuckerberg. Biography" Biography.com
<http://www.biography.com/people/mark-zuckerberg-507402> (Accessed November 15, 2011).
8. YouTube. "Cool Careers: Website Designer." Youtube.com
<http://www.youtube.com/watch?v=RyhI8NuxNyY&feature=related> (Accessed October 4, 2011).

Annotated Bibliography

Turner, J. C., et. al., "Motivating Mathematics Learning: Changes in Teachers' Practices and Beliefs During a Nine-Month Collaboration." *American Educational Research Journal* v. 48 no. 3 (June 2011): p. 718-62. This is a great research article about the research involved with motivating students to learn math. It talks specifically about how anxieties and fear of math can be a roadblock to student achievement, and it touches on how confidence is a very important factor in determining the success of a math student.

Derbyshire, John. *Unknown Quantity: A Real and Imaginary History of Algebra*. Washington, DC: Joseph Henry Press, 2006. This book provides a detailed history of algebra. It is full of interesting facts and stories about the mathematicians that invented what we know today as Algebra.

Edutopia. "PBL Research Summary: Studies Validate Project-Based Learning." Edutopia.org.
<http://www.edutopia.org/research-validates-project-based-learning> (Accessed September 12, 2011). The Edutopia website is full of free resources for teachers and students. This article summarizes research that stresses the importance of authentic learning instead of rote learning. On this website, there are also links to other studies, as well as actual projects. I use Edutopia for my classroom all the time.

Reading List for Students

Purple Math. "Slope of a Straight Line." Purplemath.com
<http://www.purplemath.com/modules/slope.htm> (Accessed November 15, 2011). This website explains what the slope of a line is in terms of a graph and an equation in slope-intercept form. It provides images of linear equations, and it also explains the "slope formula".

Wikipedia. "Linear Equation." Wikipedia.com
http://en.wikipedia.org/wiki/Linear_equation (Accessed November 15, 2011). This Wikipedia page provides a detailed explanation on what a linear equation is. It summarizes eight different ways to write an equation of a linear function. This page gives examples of the difference between a linear equation that has a positive slope and a linear equation that has a negative slope. It also talks about the difference between a linear and nonlinear function.

Math Warehouse. "Dilations in Math. How to perform dilations." Mathwarehouse.com
<http://www.mathwarehouse.com/transformations/dilations/dilations-in-math.php> (Accessed November 15, 2011). This is an excellent website that summarizes the properties of a dilation in the coordinate plane. It has explanations, examples, practice problems, and even a computer simulation of an object getting dilated. The vocabulary terms "pre-image", "image", and "scale factor" are explained, and this webpage also shows the difference between a dilation using a scale factor greater than one, and a dilation using a scale factor less than one. Students can even post questions to be answered.