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Entertaining in Math

CTI 2012

Multiplication Makes Me Sick!

Introduction

Multiplication Makes Me Sick! This is a common thought for any student in grades 3-5. Sure, when they first hear they are going to learn multiplication (and then division) they get super excited that they will begin "hard math" or "grown up math". However, as soon as the students realize that they must know their facts or memorize them, it's almost like they hit a mental wall. More and more students upon entering fourth grade do not know their facts. This becomes extremely frustrating for two things: 1. When I was a student we were expected to know all of our facts and take timed tests and 2. It seems that if students don't know their multiplication facts, they tend to struggle when it comes to division (and don't even get me started on multi-digit multiplication and division!). Through the required Math Investigations curriculum, students are shown a variety of strategies for learning and solving multiplication and division facts, and with those different strategies, they are expected to choose the one that works the best for them. It seems though, that with all the different strategies they STILL DON'T KNOW THEIR FACTS! Have I mentioned this is exasperating for a fourth grade (and then fifth grade) teacher! I am writing this unit to try to find more fun and exciting strategies to teach students to remember and recall their facts so that they are most successful in "harder" math problems.

I am a fourth grade teacher at a large urban school in North Carolina. There are currently approximately 1,000 students enrolled in the school, and this enrollment increases daily. Of these students, approximately 6% are Asian, 59% are African-American, 10 % are Hispanic, and 18 % Caucasian. 52% of these students live in poverty and 66% of the students are on free and reduced lunch. In the fourth grade, there are 7 fourth grade teachers. I have 27 students in my classroom, 1 student has Individualized Education Plan (IEP) and 4 students have 504 plans for behavior.

Rationale

When students come to fourth grade they will learn to: interpret a multiplication equation as a comparison (multiplicative comparisons),multiply or divide to solve word problems involving multiplicative comparison, multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Students will also, illustrate and explain the calculation by using

equations, rectangular arrays, and/or area models, find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Students also need to apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

Content Background Knowledge

With the new North Carolina Math Common Core standards, students are expected to come to fourth grade being able to: Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each, interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each, use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, determine the unknown whole number in a multiplication or division equation relating three whole numbers, apply properties of operations as strategies to multiply anddivide, understand division as an unknown-factor problem, fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division and by the end of Grade 3, know from memory all products of two one-digit numbers.

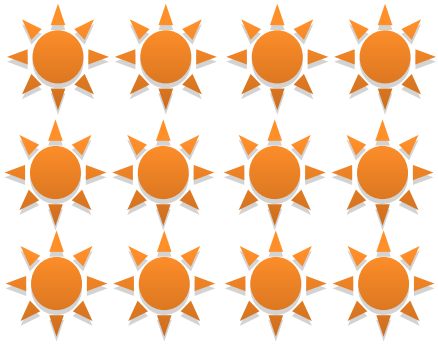
History of Multiplication

Multiplication has been around for thousands of years. The earliest known form of multiplication has been dated back to approximately 2600 BC in Ancient Sumerian times. The Ancient Egyptians of 1600 BC used hieroglyphics as their system of multiplication. The Ancient Chinese documents their multiplication tables in a book called Zhou Bisuan Jing. These tables were very similar to the multiplication tables that we use today. The Ancient Indians developed a system similar to what we know as Lattice Multiplication.

The multiplication sign "X" has a different meaning depending on the speaker (or the culture/language). For example in Japanese, the "X" means "multiplied by". If they were solving the problem 7X4= (for example), it would be stated as "seven multiplied by four". This would mean that there would be 4 groups, with 7 items in each group. In English, the "X" has two meanings. We say "times" or "multiplied by". This terminology has opposite meanings when it comes to the number of groups and the number of items in each group. For example, if we were solving 7X4, we could say "seven times four". This would indicate there are seven groups with four in each group. We could also say "seven multiplied by four". This would indicate there are four groups, with seven in each group.

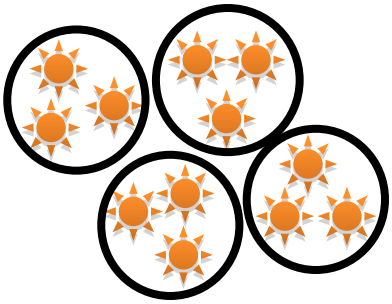
As elementary teachers, we teach our students a variety of ways to solve multiplication problems, and as stated before, we encourage the students to try each method, and choose the one that works best for them. We teach students that multiplication can be solved by:

- Arrays: Arrays are used in multiplication (and division). It is a great visual to show how multiplication can be shown as repeated addition (and division can be shown as fair shares).



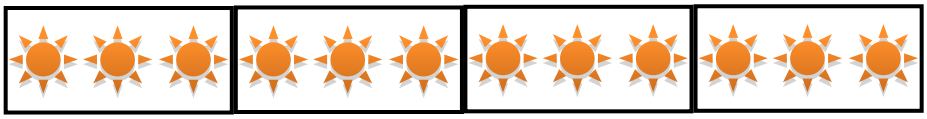
3 x 4 = 12 2 x 3 = 6

- Groups - For instance, below we see 4 groups of 3 or 4 x 3 = 12.



- Repeated Addition

  Suppose we multiply 4 and 3. This is like adding 4 groups of 3 as seen below.



  In other words, we have 3 + 3 + 3 + 3 = 12.

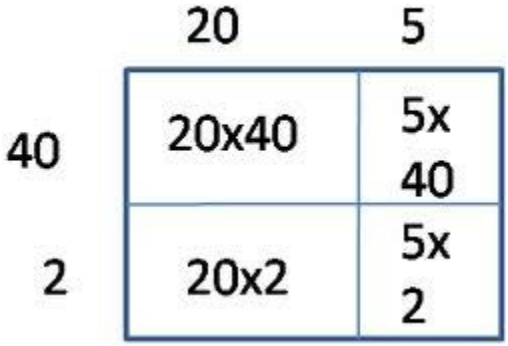
Fun Fact: British mathematician Keith Devlin (the "Math Guy" on National Public Radio (NPR)) firmly does not believe that multiplication is repeated addition. In an article that he published entitled "It Ain't No Repeated Addition" Keith states "Multiplication of natural

numbers certainly *gives the same result* as repeated addition, but that does not make it the same. Riding my bicycle gets me to my office in about the same time as taking my car, but the two processes are very different. Telling students falsehoods on the assumption that they can be corrected later is rarely a good idea. And telling them that multiplication is repeated addition definitely requires undoing later"[1] Keith strongly believes that we as teachers are steering students in the wrong direction, and in us telling them that multiplication is repeated addition gives students the impression that numbers are an additive system and nothing else. He says that part of the reason that students do not conceptually understand math is because we continuously change the "math rules" on the students and thus they never have a full understanding of math. Keith believes that we need to change our math vocabulary around and explain to students that in math, you can either multiply or add numbers (and states that division and subtraction are just the reverses of these two processes).

While I agree with Keith Devlin's explanation of his reasoning, I do feel that when starting off teaching basic multiplication, it is okay for teachers to teach students repeated addition. I was previously a second grade teacher, and it was appropriate for students to learn this method. However now, when we teach our multiplication unit, I do not touch on the "repeated addition" but instead teach and model strategies to multiply larger numbers.

- Jumps on a number line (all jumps are the same size; you repeat a number a certain amount of times).

3. Breaking apart the numbers (or as we call it in my classroom: "The Magic Box")



Math Magic and the benefits of playing games (especially in math) in the classroom

Throughout my years of being a math student, and being a teacher of math (and no, I am not any kind of expert) I have discovered that there are two kinds of math students: the students who hate it, and the students who love it. The students who love math, I have found, will understand math no matter how it is taught. They pick up on concepts right away, they are able

[1] http://www.maa.org/devlin/devlin_06_08.html

to figure out problems on their own (sometimes without being taught "tricks" or strategies for solving), and some discover math without being in the classroom. I envied and hated those students when I was in school, and I envy those students in my classroom now. There are some days when I feel like saying to them "you know, why don't you come up and teach the lesson" because they are that good. I don't worry so much about the successes in math with those students (I still worry). It's the students (like me when I was in school) who hate math that really concern me. These students just don't get it no matter what. I have more of these kinds of students in my class than the later. These are the students who this unit is written for. The ones who struggle with their multiplication facts (no matter how many times they practice flash cards, do timed tests, etc). These students are the first to be frustrated, and the first to give up. We don't give up in my classroom. These students need extra motivation to keep them engaged in tasks they hate.

So, why not make math magical? Even the simplest of multiplication can be solved magically and many magic tricks are based on multiplication. Dr. Art Benjamin, of Harvey Mudd College, is a great example of a magical mathematician (Mathemagician). He has performed his show: "Mathemagics" throughout the United States. In his performances, Art is able to perform mental math calculations quicker than a calculator. I have shown my students many of his multiplication performances where he mentally multiplies enormous numbers (faster than a calculator) and it is quite a motivator. After they watch him perform these amazing tricks, they then want me to give them all kinds of math problems to solve (and they even try to act like him). It's amazing to watch.

Another "magical" way to get students excited about math is through Maths Busking[2]. Maths Busking is a group of street performers that show the surprising and fascinating side of mathematics through their performances. It is designed to help the public become more aware of the importance of mathematics. These 100 performs perform all over Europe. They currently have ten shows: Mind Reading (math skill: binary numbers), Zeeaman's Ropes (math skill: Topology- the study (in geometry) of all different sorts of shapes in shapes. For example if "something" can be stretched, or squashed together to form a different shape, these shapes would all be equivalent.), Cubic Root Wiz (math skill: cubic numbers), Divine Remainder (math skill: division of three digit numbers), Emergency Pentagon (math skill: how to make a perfect pentagon), Knot Handkerchief, Handshakes (math skill: permutations and combinations), Waist Coat and Handcuffs (math skill: Topology), Twenty Quid Game (math skill: multiples), and Magic Square (math skill: algebra).

Why is this important? Research shows that students, who are engaged in math, will improve their comfort in math. Playing (math) games has large and lasting benefits for students' understanding of numbers. These kinds of math activities offer an alternative way of reinforcing math concepts, instead of the ever-so-popular "drills" and practice problems. Math games

[2] www.mathsbusking.com

naturally differentiate themselves as well give opportunity to present "high level" math concepts. You will find in this unit, that there is more math "games" than lessons because in my classroom, we work a lot in hands on center groups (which we call "Math Club") instead of practice worksheets to help reinforce these math concepts. I find that while observing my students "play" these games, they are engaged, good math conversation is flowing, and without them knowing it, they are learning.

Classroom Strategies

As I mentioned above, you will not find Lesson Plans. Instead you will find strategies for my unit of Multiplication. This is how I teach in my classroom. We have a 15 minute mini lesson(our curriculum uses Investigations[3]) and then the students break up into their "Math Club" Groups (these groups are leveled, so there is an "above grade level group", "on grade level group", and a "below grade level" group).

1. **Factor Pairs**

   In this Investigations Game, the students will work with multiplication combinations that have products up to 12 x 12. The students will also need Array Cards (these also come in the Investigations kit, but you can find many templates online). The students work in pairs and spread out the Array cards out with the dimension side up. The students choose an array card (randomly, I have my students close their eyes and choose one, or else they tend to continuously choose only the Array cards they know, which doesn't give them practice with the facts they don't know). They must say the number of squares that are on the array card. For example if their Array Cards dimensions are 3 X 6 (6 X 3) they must say there are eighteen squares. After they have made their guess, the partner will turn the card over. If the student is correct, they get to keep the card. If they get it wrong they must write down the Array card they did wrong. The next partner then goes. The students keep playing this game until all the cards are solved. While they are working, they need to keep a list of all the multiplication combinations they are working on. For practice purpose, these are the multiplication problems I have my students make flash cards for and quiz them weekly on until they get them all correct.To make this more challenging for the above grade level students, I tell them they must take out all the array cards through 5 x 12. I also tell these students that they are not allowed to count the squares in the array cards.

   This game is usually played in the beginning of the one-digit by one digit multiplication unit. Students in fourth grade must be fluent with their multiplication facts through 12 X 12. Students should know the facts quickly and mentally. Factor Pairs helps and gives students a visual representation of math facts using the array strategy.

[3] http://investigations.terc.edu/

2. **Multiple Turnover**

This is another game that comes from Investigations Unit. Students really seem to struggle with understanding the difference between the vocabulary words "Factor" (the number you count by when you skip count) and "Multiple" (the numbers you land on when you skip count). I tell my students that it is: "FACTOR times FACTOR equals MULTIPLE. We write this over and over again. However, students still seem to get these two concepts confused. This game is to help reinforce the means of the terms factor and multiple. The materials needed in this unit are decks of multiple cards (there are many templates online, or you can just put numbers on index cards) and a recording sheet. Students play this game as well in pairs. The students each get ten multiple cards (dealt out by random). They arrange their ten multiple cards face up in a line (all players should be able to see each other's multiple cards). The player with the smallest multiple card gets to go first. The student can call out any whole number they want, except for the number one. Each partner writes the number on their paper. Both students must search in their multiple card pile for multiples of that number. They must write down their multiples on their recording sheet and turn those cards over. Players continue in this way until the first partner turns over all ten of their multiple cards.

Before playing this game, we play a couple of rounds as a whole class. To differentiate the lesson, students who are below grade level only get multiple cards two thru twenty. They can also use a calculator if needed. On grade level students use multiple cards numbers two through eighty, and above grade level students use multiple cards numbers two thru one hundred thirteen. With the below level students I usually play with them in a group (in my classroom there are six students who fall under this category). As they gain more mastery of this game and their understanding of factors and multiples, I will start adding more multiple cards to their decks. As students start understanding the game more, they begin to develop strategies to help them become the first player to turn over all their cards. This game helps build fluency with multiples as well as builds numerical reasoning

3. **Munch**

Students have a game board and 4 dice (2 player game). On each game board there are 10 products. Student A calls a "target product" on the game board. Both players roll their dice. Students must manipulate the dice (any way they want) to get closest to the target product. The player who gets the closest to the product gets to put a chip on it.

This game is naturally differentiated. To play this game, I would have homogenous partners. Students have the opportunity to choose any combinations with their dice and they can create any combinations of multiplication, addition, subtraction, and division. The only concern I have with the students playing this game this way, is that the students MUST be aware of the third grade skill of Order of Operations. If a student rolls a 4, 2, and 6 and they arrange this as such: 4+2x6=36 they would be wrong, because according to the rules of the Order of Operations, they would have to multiply six times two first. The students would have to know that if they want to arrange the dice as shown above, they would have to tell their partner (or write it down for their partner) that they would put parenthesis around the four plus six. This could really get confusing

for the students.**Note**: at the end of the "Classroom Strategies" section of this unit, there is an example board that I use with my students for this game. You can also create your own.

4. **Burns' rendition of Munch**

I have created my own multiplication version of the game Munch as mentioned above. The students would again need to be put in homogenous partners. Each partner pair gets a piece of blank paper and two dice. Students must create a five by five array on their blank paper (must look like a BINGO board. If you think students would have trouble creating this, make a template for them). On the top of the columns the students must spell out B-U-R-N-S. On their board the students are allowed to have one "Free Space" and the rest of the board must be filled with possible products they can get when they roll two dice and multiply the numbers together. Note: DO NOT give the students any clues as to how they should figure out their numbers. This is part of their discovery of the game. Give students time to figure out the numbers to fill in the boxes. Some students may roll the two dice multiple times and put the products in the boxes that way and some may just fill in their board with random numbers. That is OKAY. After the students have created their boards, play BINGO. (Pre-prep materials are products through six times six). Students try to be the first to get BINGO. After the game is done, have a class discussion about the numbers they put on their board. Some students won't be able to win because of the numbers they chose to put on their boards. For example, if a student had the product thirty-five. Thirty-five will NEVER be called if you are using a six-sided dice. This is because there is no seven on the dice, so there would not be a way to make thirty-five. Students can write their reflections about the game, finding they had in the game, etc in their math journals (if they keep them).*** Note:** At the end of the "Classroom Strategies" section there will be a blank template of this game.

5. **Beat the Clock**

One great and fun way to progress monitor kids on their mastery of their multiplication facts is through this website:http://www.oswego.org/ocsd-web/games/SumSense/summulti.html. On this website, the students get to choose the number of questions they think they can answer in the time they choose. I always have my students try to solve 10 problems in a minute. The computer gives the students four cards, and the students must arrange the four cards into a multiplication sentence. If they "beat the clock" they go onto the next level. If they don't "beat the clock" they stay on the level they are currently on. I use this as a motivational tool. When we set goals, I always play first and post my score up on the board. The students always want to and try to beat me. It's a great way to get them excited about knowing their multiplication facts!

6. **Buzz-Beep**

Pick a number to be "buzz," another number to be "beep" (and a third number or type of number, like squares or primes, to be "bop," if you're ambitious). Students stand in a line or circle and count up from 1 by ones - but every time they reach a number that's a multiple of the buzz and/or beep number, they say "buzz" or "beep" instead of the number. If it's a multiple of both, say "buzz-beep." If you mess up, you're out.

7. **Factor Aerobics**

Like Buzz-Beep, pick a number to be your left hand, another to be your right hand (and a third and fourth to be your feet, if you're ambitious). Then, count up from 1 by ones. Every time you say a number that's a multiple of the numbers assigned to your limbs, raise that limb. This is REALLY hard but SUPER fun.

8. Multiples on a Hundreds Chart

Give students a 100s chart. Ask them to color in all of the multiples of a number of their choice. What patterns do they see? Then pick a different number, do the same thing. Describe the different kinds of patterns that are possible, and which numbers make those patterns. Then, you can do the same with different square charts and compare. This is quite interesting.

9. Multiplication Bump

This game is another good strategy for practicing and mastering multiplication facts through twelve times twelve. The teacher makes twelve game boards (each board focuses on a specific multiplication set). Students need a twelve sided die. They must roll the dice and multiple the number they roll by the factor on top of the page. If they get the answer correct, they can put a chip on the multiple.

For differentiation purposes, above level students can use digit cards (higher than twelve) choose a card and multiply that number by the factor on top.

10. Multiplication Concentration

This is played like the typical childhood “memory” game. The students make their own cards based on the multiplication facts they need to work on (or you can give them multiplication facts to make). The students use index cards and write the numeric expression on one side, and the product on the other side. Partners mix up their cards and lay them product side face down. For differentiation on this game, the on-grade level students can create their game doing two-digit by one-digit multiplication. Above grade level students can make their game by making two-digit by two-digit multiplication cards.

11. Multiplication Monster

This is something fun for the students to do while they are practice their facts (especially around Halloween time). Students roll two dice to create a multiplication problem and use the number to give a monster body parts. The first problem they create and solve determines the number of eyes the monster has. The second problem they create and solve determines the number of horns the monster has. The third problem they create and solve determines the number of teeth or fangs the monster has. The fourth problem they create and solve determines the number of arms the monster has. And the fifth problem they create and solve determines the number of legs their monster has.

End of Unit Project: Multiplication Carnival

Many places around the world have a "Celebration of Mind" carnival to celebrate famous Mathematicians. This would be fun for the students to have their own celebration of mind as a reward for their hard work of mastering their multiplication unit. Students can work in partners or as an individual. They will either create their own multiplication game OR facilitate a game that you have done in the unit as a class. As a class, we set up the room to be easily accessible for others to move around and/or to look like a carnival. Invite other fourth grade classes to come to the carnival to play the games. This would also be fun to do as a whole grade level, where each class has their own multiplication games or do it as a multiplication and division review. What a great way for the students to show what they know and take ownership for their learning!

Name_____ BURNS Multiplication BINGO

Directions: With your partner, you will fill in the BURNS Multiplication BINGO spaces with multiples. You can use the dice to help you if you would like. Remember to leave one open space as a "Free Space".

B	**U**	**R**	**N**	**S**

Name Date

Multiple Turn Over Recording Sheet

Write the numbers of your ten Multiple Cards on the blank cards. As each factor is called, record it in the factor list. Then write which multiples of that number you have among your cards.

Multiple Cards I drew:

Factor	Multiple Cards I turned over

Multiplication Munch

Students have a game board and 4 dice (2 player game). On each game board there are 10 products. Student A calls a "target product" on the game board. Both players roll their dice. Students must manipulate the dice (any way they want) to get closest to the target product. The player who gets the closest to the product gets to put a chip on it!

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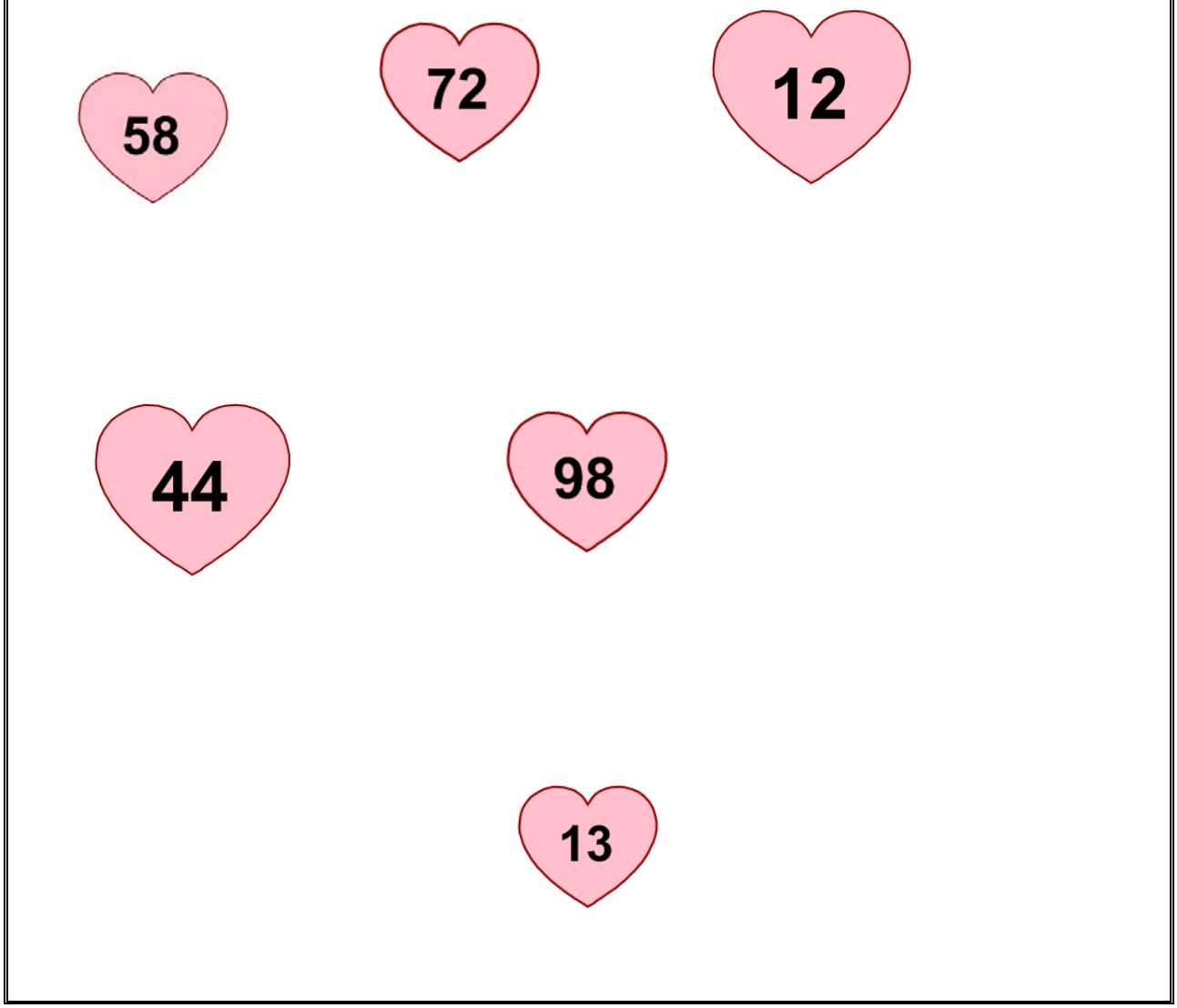
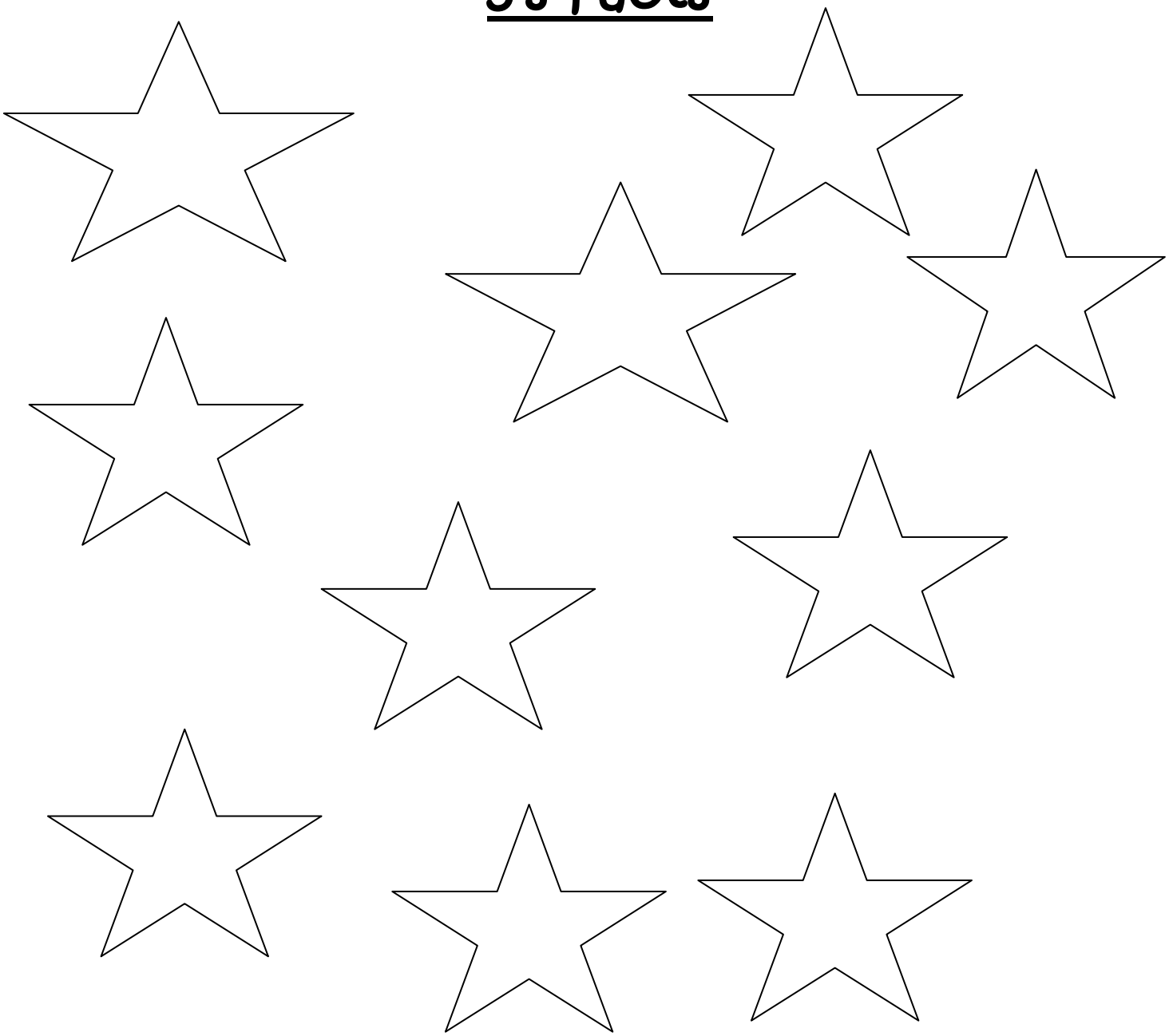


Multiplication Bump!

Teacher Note- to make game board, write in the multiples you want students to work on.

5's Facts



Annotated Resources

Bibliography for Teachers

1. " MathsBusking ."MathsBusking . http://mathsbusking.com/ (accessed November 26, 2012). This website has wonderful games that fit into the curriculum as well as street performances of math to make it exciting for students to see.
2. "4th Grade | Curriculum By Grade Level | Investigations in Number, Data, and SpaceÂ®." Welcome to Investigations | Home | Investigations in Number, Data, and SpaceÂ®. http://investigations.terc.edu/curric-gl/g4/ (accessed November 26, 2012). This is the curriculum the students and teachers use in our school district. *Investigations* is a program with a lot of hands on, self-discovering learning.
3. "Arthur Benjamin's Home Page." HMC Math: Harvey Mudd College Department of Mathematics. http://www.math.hmc.edu/~benjamin/ (accessed November 26, 2012). This is a great resource to use in getting students excited about learning and studying math. There are great videos of Art Benjamin performing his math magics presentations.
4. "Georgia Tech School of Mathematics." Art Benjamin Magic. www.math.gatech.edu/news/art-benjamin-magic (accessed November 26, 2012). This is a great resource to use in getting students excited about learning and studying math. There are great videos of Art Benjamin performing his math magics presentations.
5. "It Ain'tNo Repeated Addition." Mathematical Association of America. http://www.maa.org/devlin/devlin_06_08.html (accessed November 26, 2012). This is a good resource to use when first starting to research strategies to teach multiplication. It gives a different spin to the normal way teachers teach students multiplication.
6. "Keith Devlin." Stanford University. http://www.stanford.edu/~kdevlin/ (accessed

November 26, 2012). Keith Devlin writes many articles about the pros and cons of certain strategies in teaching mathematics. This is useful when teaching math and trying to find the best strategies.

7. "Multiplication Tool: About." Multiplication Tool: Learn Standard Multiplication, Partial Products Multiplication and Lattice Multiplication. http://www.multiplicationtool.org/about_multiplication.php (accessed November 26, 2012). This is another great resource for teachers for finding strategies in teaching multiplication.
8. Rosenfeld, Alvin. "The Benefits of Board Games | Scholastic.com." Scholastic, Helping Children Around the World to Read and Learn | Scholastic.com. http://www.scholastic.com/resources/article/the-benefits-of-board-games (accessed November 26, 2012). Many parents wonder why students go to school to "play games". This is a great resource in helping to explain why math games (and board games as well) are important to the development of students brains.
9. "Ginger Snaps." Ginger Snaps. http://gingersnapstreatsforteachers.blogspot.com/ (accessed November 26, 2012). This website is a resource for teachers who like to teach through funny lyrics and songs.

Student Reading

Here is a list of good readings and resources the students can use when learning multiplication:

1. www.multiplication.com –the website has many fun and engaging multiplication online games. Most of the games on this side are good for students to learn, memorize, and practice their basic multiplication facts. It is differentiated as well because students (and

teachers) can choose the difficulty ("easy", "medium", "hard") to meet the students needs.

2. http://www.oswego.org/ocsd-web/games/SumSense/summulti.html - this website is used in one of the teaching strategies in my unit. This is another great site that I use with my students to motivate them to learn and know their multiplication facts.
3. Hershey's Chocolate Multiplication Book by Jerry Pallotta- this book is a cute book to introduce simple multiplication facts to children.
4. Times Table the Fun Way: A Book for Kids: A Picture Method of Learning the Multiplication Facts by Judy Liautaud- this book has cartoons and a collection of stories to teach students multiplication facts.
5. Corkscrew Counts: A Story About Multiplication By Donna Jo Napoli- this book is about different characters who have to arrange themselves into groups to play birthday party games.

Classroom Materials

1. Factor Pairs game: This game needs Array Cards (which comes with the Investigations kit, but templates can also be found online) and notebook paper for students to record facts they are still working on (the ones they don't know by memorization yet).
2. Multiple Turnover- This game needs a Multiple Turnover Recording Sheet (which is attached in this document), and digit cards (if you don't have these, playing cards will work fine).
3. Munch- Students will need a game board and 4 dice. An example game board is included in this lesson, but teachers can make the game board to fit their own students' needs.

4. Burns' Rendition of Munch- you will need a BINGO card template (I have attached one with my own last name on it. Teachers may change this attachment), and 2 sided dice.
5. Multiples on the Hundred's Chart- students will need Hundreds boards (you can find these on various math websites) and colored pencils.
6. Multiplication Bump- needs a game board. There is one attached, however teachers should re-create these boards based on the students' in their class' needs. Students will also need a 12-sided dice. If you do not have these, you can find templates to make paper ones on line.
7. Multiplication Concentration- students will need index cards and markers.
8. Multiplication Monster- students will need dice, construction paper, and markers.
9. Multiplication Carnival- the materials for this game will vary due to the types of games you can have the students choose from as well as the games students will create for themselves.

Works Cited

" MathsBusking ."MathsBusking . http://mathsbusking.com/ (accessed November 26, 2012).

"4th Grade | Curriculum By Grade Level | Investigations in Number, Data, and SpaceÂ®." Welcome to Investigations | Home | Investigations in Number, Data, and SpaceÂ®. http://investigations.terc.edu/curric-gl/g4/ (accessed November 26, 2012).

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