Have Fun Skiing But Watch That Slope!

Kurma Murrain-Collins

Background

West Charlotte was built in 1938 and is one of Charlotte's oldest public schools with a rich history of producing widely noted public officials, athletes, and business leaders. West Charlotte High School is the only historically black high school still in existence in Charlotte, NC today. In 1955 it moved to its current site on Senior Dr., just off of Beatties Ford Road.[i]

Additionally, West Charlotte offers the following nine (9) Advanced Placement courses: English, Calculus, Statistics, U.S. History, U.S. Government and Law, Human Geography and Art. Further offerings occur for international competition in the International Baccalaureate Program which provides the opportunity for highly motivated, college-bound students to engage academic rigorous, well-rounded educational program that promotes and encourages academic competitiveness as well as an appreciation of and sensitivity to different cultures, attitudes and values.

On April 2005, West Charlotte was authorized as an International Baccalaureate World School and has successfully offered courses under the IBO program to those scholars that seek the academic rigor and cultural awareness of the international program.[ii]

The ESL Classroom

It is important to note that this unit is being designed for an English as a Second Language class. As such, I take a section to give context to my teaching and my students. Note that things have changed for ESL students who now have to perform and interact with regular students. Despite that I am helping the ESL students, this service is not exclusive to them, but I do pay more attention to this population than to the American one, as it is my job to make sure they have a clear understanding of the subject and are at the same pace with other students.

English as a Second Language

CMS provides tailored instruction in English language skills to meet the needs of the limited English proficient (LEP) students whose parents choose English as a Second Language (ESL) services.

Middle and high school ESL students receive instruction in sheltered content courses such as social studies or science in addition to specially designed English language arts courses.[iii]

Once the ESL program started, other possibilities opened to English language Learners who required having their diploma, GED or completing their graduation requirements. First was inclusion, in which students were accompanied by a TA (Teacher's Assistant) and helped in the regular subjects were they needed more assistance. Now it is called SIOP, which also involves inclusion. In this form, two teachers work collaboratively to ensure the students complete the number of credits they need to be able to graduate even if their English level is not sufficient.

I am myself an "English learner" having been in this country only for 8 years. My educational background led me to teach English as Second Language for a number of years; however, once schools started to introduce Inclusion and SIOP (Shelter Instruction Observation Protocol) so they could serve ESL students better by helping them to be successful in their school years, things started to change for ESL teachers as well as for other teachers with a bilingual background. For example, I was hired as an ESL teacher at West Charlotte High School, to actually co-teach Math in a SIOP environment, which I have been doing for the past 4 years. So I facilitate the teaching and the learning myself. Not having a lot of formal training has not been a problem at all, since the basic concepts learned at school have not been forgotten. Nevertheless, I am constantly learning, taking notes, and asking questions, as much or maybe much more than my students, mostly in order to be a voice for them.

I plan to teach this unit in collaboration with the leading Math teacher in my classroom since I am an ESL teacher working in a SIOP (Shelter Instruction Observation Protocol) class. As such, we have heterogeneous classes, not only because we can have different grades and ages in the same period class, but also because we have students from different nationalities (including American), and different English levels as well. The latter are better known as ESL (English as a Second language), ELL (English Language Learners), LEP (Limited English Proficiency) or SIFE (Students with Interrupted Formal Education in their country of origin). We are helping them develop both Mathematical understanding and English proficiency by engaging them in academic conversations. All these SIOP features will be integrated into the units developed for this curriculum.

Rationale

Entertaining with Math sounds like an oxymoron to many students. I want to explore ways students can learn algebra and understand real life applications while having fun. At the same time, students will create stories in which they can apply the knowledge acquired during the lessons. This unit focuses on slope. An overarching goal is for this

topic to be more fun for ESL students and also regular Algebra 1 students from the United States of America.

In the short period of time I have been attending this seminar I have started seeing slope in a completely different way, and that is what I want my students to see, or even beyond that. As such, this unit will contain innovative ideas, activities, tricks, and games for students to aid in their remembering slope and its applications.

It is a scientific fact that a human being can remember as many minutes of instruction as his/her age in years.[iv] As such, if our students are between 14 – 17 years of age, it is necessary to apply different techniques within a class period to keep them on task. Engaging in different learning styles can help diversify instruction and aid in keeping students engaged. One may choose to have students move around so they experience the variables and express what the slopes would look like using their bodies, their arms, and legs. Instruction should engage students in a wide array of activities that are thoughtful and short in time length.

Mathematical Background

The first four weeks of class our students learn and review simple math operations. They start with order of operations, followed by substitution (evaluating expressions), distributive property, GCF, one step equations, two step equations, multi-step equations, in which some of them have shown confusion, however other students seem to have mastered this very quickly. Students also study and review multiplying and dividing monomials and polynomials. ESL students and first time 9^{th} graders understand all of these topics on a one on one basis, nevertheless, there is a percentage of students who are SIFE (Students with Interrupted Formal Education) with whom we work almost at the same pace as with the other students but at the same time, we are reviewing elementary concepts like addition and subtraction, especially because they have not learned that numbers can be negative as well. I explain this as if they were borrowing or lending money from a bank or a friend, and it looks clearer for them this way. For others, however, is easier to imagine an exchange of goods like cows, chickens, or fruit, and that is what the variables become. How many cows do you have to pay for a house, or how many chickens you would owe if you do not have enough is more helpful than just using numbers on a number line. We use a lot of visuals, like multiplication tables, and Ms. Washington, who is an expert using the smart board has been able to teach real life examples on the big screen. Students get more motivated when technology is involved, especially if they have a computer at home. This is something that has worked really well with foreign students with high levels of poverty who do not even have e-mail accounts nor know about Facebook because this expands their horizons.

Step by step and very simple explanations and examples continue to be the best policy. With Vietnamese, Latino, African, and American students, we have found we can

pair them so one can help the other since it is their preference to work with someone with whom they can speak the same language. Nevertheless, even though numbers are universal, there are certain signs that can be different from one culture to another and that changes the perspective if you do not know about these differences.

For instance, in the United States the sign used for division is an inverted L (⌐), in Latin American countries the division sign is more like a right angle (∟). Other differences we can find in math from culture to culture are the decimal point and the thousand/million comma, which in the United States is clearly distinguished, but in Latin America is always a dot. How do they know which one is which? The quantity of numbers that follow, or the context of the situation.

We are constantly going over substitution and definitions of simple math rules such as any number multiplied by 1 equals to the same number, or the opposite of adding is subtracting, and the opposite of multiplying is dividing. By repeating these words every day, students have started to understand what they have to do when presented with these equations.

Slope is a topic that we will introduce shortly, after evaluating students' progress in the previous concepts. Students complete a drill to access their abilities in adding and subtracting positive and negatives numbers. They have 5 minutes to answer as many problems as they can. They are asked to put their calculators away in order to test their logical thinking:

1. -4-(-2)
2. -1-2
3. -8+7
4. -5-2
5. 1+-5
6. -6+-2
7. 4-2
8. -3-7
9. -7+6
10. 3-(-7)
11. -5+-6
12. 8-(-7)
13. -8-6
14. 7-(-7)
15. 3+(-3)
16. 2-(-2)
17. -4+-3
18. -2+-6
19. 2+-6
20. 8+-6
21. 1-(-2)
22. 7+(-2)
23. 4+-4
24. 6-(-2)
25. 5-(-2)
26. -3+2
27. -7-3
28. -1+-2
29. 5+-7
30. 4-1
31. -6-3
32. -5-3
33. 2+-1
34. 2—5

These as well as practicing how to reduce fractions will be the main elements we will use for ordered pairs and slope. The class is now ready to learn slope, which is a fundamental concept in the curriculum. First, let's define this idea in English and Spanish.

Language adapted to ESL students:

Slope: Slope is the 'steepness' of the line, also commonly known for the rule *rise* over *run*. We can calculate slope by dividing the change in the x-value between two points over the change in the y-value.[v]

Slope- intercept form: The equation of a straight line in the form $\boldsymbol{y = mx + b}$ where *m* is the slope of the line and *b* is its y-intercept.[vi]

Order pair: (See Figure 1)

For Spanish speakers:
Pendiente: Pendiente es la "inclinación" de la línea, también conocida como *elevación* sobre *desplazamiento*. La podemos calcular al dividir el cambio en el valor de la x entre dos puntos sobre el cambio en el valor de la y.

Forma pendiente-intersección: Ecuación de la forma $y = mx + b$, donde *m* es la pendiente y *b* es la intersección y.

Par ordenado: Conjunto de dos números arreglados en un orden particular, normalmente escritos como (1er número, 2ndo número), en donde tanto el orden como los valores tienen significados acordados. Por ejemplo, las coordenadas de un punto en un plano de coordenadas Cartesianas se escriben como (*x*, *y*), en donde *x* es la coordenada horizontal y *y* es la coordenada vertical. [vii]

There are some prerequisite skills students should know before learning how to find the slope of a line and find equations of the line, as well as making sure they know how to use a graphic calculator. After this, students should know which number in the formula y = mx + b is the slope and which one is the y-intercept. But how can you find the slope (m) of a line in the first place?

The slope of a line characterizes the general direction in which a line points. To find the slope, you divide the difference of the y-coordinates of a point on a line y the difference of the x-coordinates. (See Figure 2)

Formula to find the slope, *m*, of a line between the points (*X1, Y1*) and (*X2, Y2*) is:

$$m = \frac{Y2 - Y1}{X2 - X1}$$

Slope / Distance / Midpoint and Systems of Equations all deal with variables and order pairs. The variables *X* and *Y* become tangible when you have to figure out how much to pay for two different items, how long it takes to get to a place, the inclination of a ramp that you need to design and build for wheelchair access. And these same lines are

the basis for computer program animations, which many students are interested in and would like to know more about.

Slope is not limited to parallel and perpendicular lines, or a slope or point-slope formula. It can be summarized in one line, but its applications go much further. It is tangible and applicable in almost every aspect of life. For example, consider a student's life after graduation, which may involve working in a company or starting one's own business. Analyzing the success of the business can involve all kinds of charts (see Figure 3).

On this graph, for example, we see the sales of gold for Dubai in millions and the variation they have had between January and December 2002; using this same graph can help the company make projections of sales for future years.

Talking about classroom applications lets us turn to feelings and emotions. Can you think of a way of expressing ongoing emotions using positive and negative slopes, or using a little drama in the classroom in order to activate that kinesthetic intelligence? In the search for emotions, for example, we could find the intersection point between the feelings of two human beings. Using a slope for one and another slope for the other, we will be able to see where their emotions meet. This will have applications when talking about friendships, romantic relationships, and also parent-children or teacher-student relationships. Where is the point where the emotions of two people meet? That would be a new perspective on how to look at two points in the space when these points meet, and find out what kind of reactions this "meeting" can cause.

On the other hand, it is very useful for ESL, ELL, and SIFE students to grasp the concept of slope through a visual, at the same time, emotional connection. ☺=positive slope, ☹=negative slope.

Activities

Below are activities designed for ESL ELL, and SIFE students on the subject of slope.

Pairing up to learn slope

Students must start by recognizing lines with positive, negative, zero, or undefined slopes.

For this there are some recommendations in a rally robin structure. Student A tells student B whether the slope of problem 1 is positive, negative, zero, or undefined, and explains why.[viii] Then student B tells student A whether the slope of problem 2 is positive, negative, zero, or undefined, and explains why; repeat this until all problems are done.

Another simple way to check for understanding is pairing students; student A states the slope and y-intercept, while the other student gives a specific praise (i.e.: You did a nice job of remembering that the slope is found next to x), and then states the slope and y-intercept for the next problem. Student A gives specific praise. Alternate roles.

Let us move on to a more entertaining activity.

Draw what I say

Each student has a sheet of graph paper. A barrier is set up between each pair of face to face partners. One student is the sender and his partner is the receiver. The sender draws a figure with six segments on a coordinate plane with both x and y axes ranging from ± 12. The sender tells the receiver to plot one of the points on his/her figure. From that point the sender gives directions using slope so the receiver can draw segments to match the sender's figure. When they finish, students check to see how similar their figures are (or how different!). Roles can be reversed.

Acting out Points

Students plot an emotion. For instance, if 5 is being as happy as you can be and -5 is being as sad as you can be. We would then discuss, what might make you feel like a 4 on this scale. What might make you feel like a -3? What about a 0?

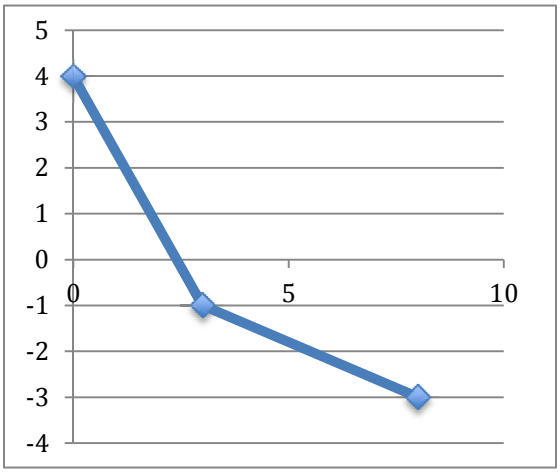
Then, I would ask the students in a count of 5 to take a position that looks like a -3 in this scale. Then, I'd ask them to look like a 2 in this scale. Again, we'd talk about what might be happening to make us feel this way.

From here, we are ready to walk around, look at others and plot their emotions.

Plotting emotions
How do you plot emotions? If -5 is most negative an emotion could be and 5 is the most positive, what numbers would you give the emotions shown in two photographs (baby crying vs. baby laughing)? (See Image 1)

This gives us only one coordinate. Let's use time as another coordinate. For instance, we have three 3 faces - happy, bored, and sad. Suppose their values on the -5 to 5 scale are 4, -1, and -3. (See Image 2)

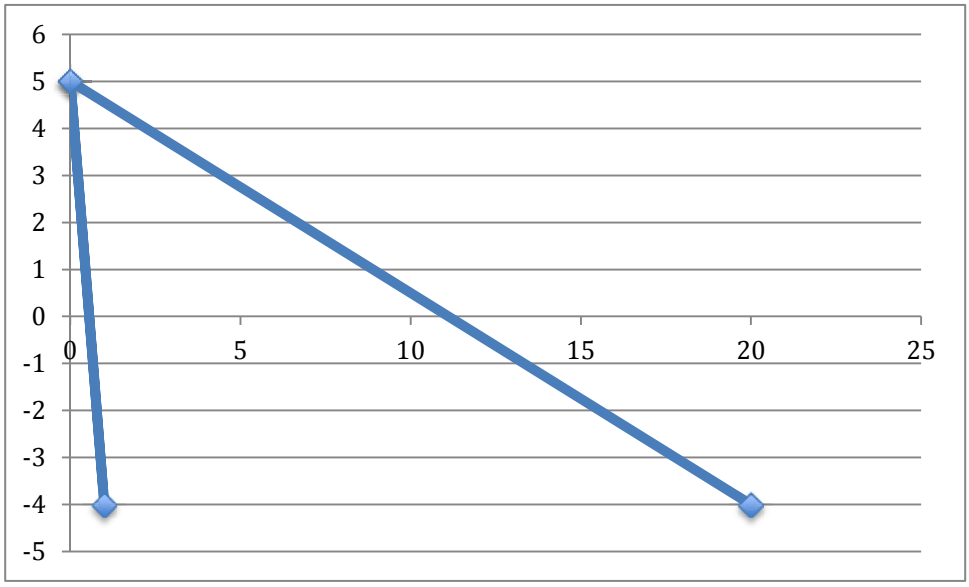
Suppose these emotions occur at times 0 (beginning), 3 and, 8. Then, we can be representing these emotions at the given times as the points (0, 4), (3,-1) and (8,-3). Below we see a plot of these points.



In groups, students will work to think of a story for what might be happening to someone to have these emotions at these times. What is happening during these times to make the emotions change? Then, I would ask my students to discuss:

1. What does it mean if we change from being happy to angry in 1 minute rather than 20 minutes?
2. What if we go from angry to sad in 30 seconds rather than 30 minutes?

To help with this, students will plot the points (0, 5) and (1,-4) (which is going from happy to angry in 1 minute) and then also the points (0, 5) and (20,-4), which is the same emotional change over 20 minutes.



What do we notice about the lines? The steepness of the line gives a sense of how fast the emotion is changing. Slope enables us to analyze this.

Another way to measure slope is if a student tells another student about an interesting day; this must have at least 5 elements, which are somewhat different from each other. For example, these components could be: wake up, hurry to get dressed, mommy made my favorite breakfast, had bad news about a relative, and aced a test. While s/he is telling his/her story, the other one is drawing a broken line on graph paper. After they both have told their stories they exchange papers, find the slope of each line, and label each line with the event, for instance, y= 3x – 5 (*aced the biology test*). Paste results around the classroom.

Battleship

Battleship is another way for students to show their knowledge, create, and compete against each other.

On graphing paper they draw the x-axis, the y-axis, and little ships according to specifications. The teacher will give specifications about the ships size and how many they can draw. Then, students place their ships in strategic places on the coordinate plane; and, let the battle begin!

One way to do this is to let them work in pairs, sit in front of each other with their own graph paper facing them. They take turns to give coordinates attempting to sink the other's ships until one of them sinks all the other's ships. (See Figure 4)

On this board, for instance, the player has recorded the shots of his opponents, which means one of the ships is sunk (as you can see the 2 exes on the two squares that represent the position of the ship→(A,8) and (B,8)). The teacher can here give the students little stickers (hearts, cupcakes, or happy faces) so that instead of marking the ships that have been hit with Xs, use the stickers to make it more colorful and make them work on a more hands-on activity.

The game is played on four grids, two for each player. The grids are typically square – usually 10×10 – and the individual squares in the grid are identified by letter and number. On one grid the player arranges ships and records the shots by the opponent. On the other grid the player records his/her own shots.

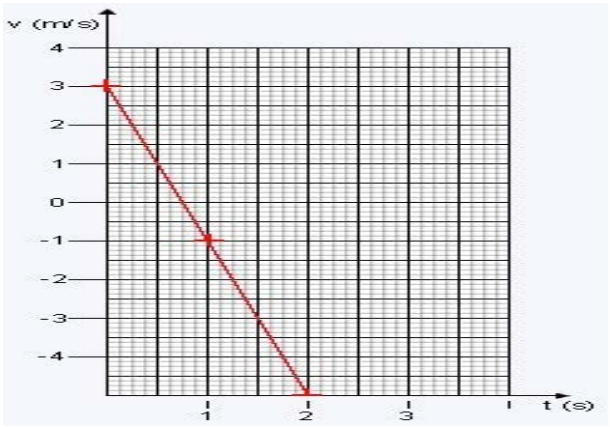
Before play begins, each player secretly arranges their ships on their primary grid. Each ship occupies a number of consecutive squares on the grid, arranged either horizontally or vertically. The number of squares for each ship is determined by the type of the ship. The ships cannot overlap (i.e., only one ship can occupy any given square in the grid). The types and numbers of ships allowed are the same for each player. These may vary depending on the rules. (See Figure 5)

There are two typical complements of ships, as given in the Milton Bradley version of the rules:

Type of ship	Size	Type of ship	Size
aircraft carrier	5	aircraft carrier	5
battleship	4	battleship	4
submarine	3	submarine	3
destroyer	3	cruiser	3
patrol boat	2	destroyer[ix]	2

After the ships have been positioned, the game proceeds in a series of rounds. In each round, each player's turn consists of announcing a target square in the opponent's grid which is to be shot at. If a ship occupies the square, then it takes a hit. The player's opponent announces whether or not the shot has hit one of the opponent's ships; the opponent then fires a shot at the first player's ships. When all of the squares of a ship have been hit, the ship is sunk. After all of one player's ships have been sunk, the game ends and the other player wins.

Another version, possibly more entertaining, is to play a “Laser Battleship” in which it is allowed to sink ships if they are in “the line of fire” or *slope*. For instance, on the coordinate point (0, 3), I shoot my laser from there to the coordinate point (2, -5), if my enemy’s ship happens to be on this line, let’s say (-1, 1), it will be hit, and eventually sunk.



Cartooning

Likewise students will be creating almost anything they want to. They can make their own fonts, comics, or games. These students might not be interested in knowing about

math equations but their eyes widen with excitement when the class work is about drawing.

Following that trend the easiest step would be to create something which lines are clearly defined, seen, and measured with the naked eye. Here is an example of what this would look like. (See Figure 6)

Next, we can plot points that will define a letter. Can you tell which letter is given? (See Figure 7)

It is easier if you know which points are connected by lines. For instance, you can now clearly see the underlying letter. Next, connect the points with lines, and determine the location of the origin. (See Figures 8 and 9)

Now, we will write the equation for the lines in our font. Let's take the line between (0, 9) and (0, 0). Next, let's consider the line from (0, 9) to (3, 6). Will this have positive or negative slope?

To find the equation of the line between (0, 9) and (3, 6), we first determine the slope between the points, m = (9-6)/ (0-3) = -1. Now, we need a point and are given two. We choose one and use the point-slope formula to find:

$$y - 9 = -1\ (x - 0) \text{ or } y = -x + 9$$

Continuing this for every line in the font allows us to know all the lines that comprise our font.

Now, there is an important trick to a font. Fonts are compactly stored in very small files on our computers. Yet, no matter what size you make them, they are in perfect resolution. Suppose you have a 6 pt font and want the letter to be twice as big. How can we double the size? You simply multiply each coordinate of every point by 2. So, we had the points (0, 9), (0, 5), (0, 0) and (3, 6). Therefore, the font that is twice as big becomes (0, 18), (0, 10), (0, 0) and (6, 12).

In a similar way, if we want this letter to be 3 times bigger. The new points would be (0, 27), (9, 18), (0, 15), and (0, 0). In fact, we could apply this same method to enlarge or shrink the robot from the earlier activity!

Find yourself in the universe

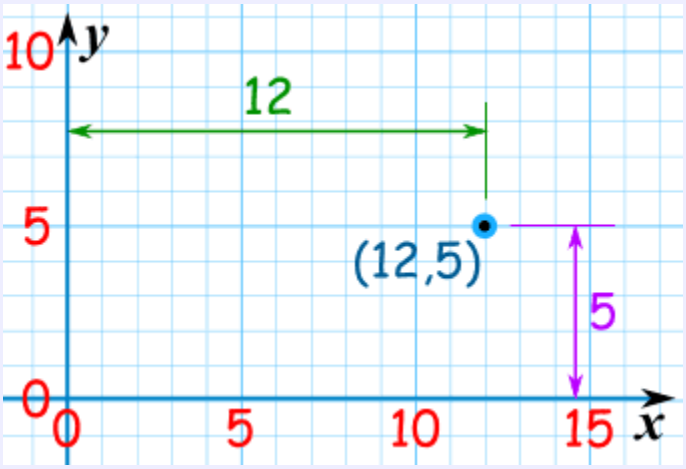
This activity could be lots of fun for those hyper competitive students who enjoy moving around.

How do you prepare for this activity. You will need at least 30 minutes to get ready for this game. Previous to students coming to the classroom the teacher will use duct tape to make four big coordinate planes on the floor, preferably using the four corners. Write the numbers on your x-axis and your y-axis with a permanent marker. Ask students to make groups of four (the groups can be bigger or smaller depending on the class size). Every member of the group will be number from to 4, or as far as the number of members. The teacher will call out an order pair first then she will say a number, and the student whose number the teacher says has to jump quickly to the coordinate plane. Nobody can move. Then the teacher checks who is on the correct point. Repeat the activity until every member of the group has jumped to at least one point. The group who wins is the one with more correct "jumps in." The prize is determined by the teacher.

Math is everywhere; you can find it in graphic arts, businesses, games, cartooning, and animation. The whole world is made of numbers, like the pumpkin π we eat on Thanksgiving. The entertainment is infinite!

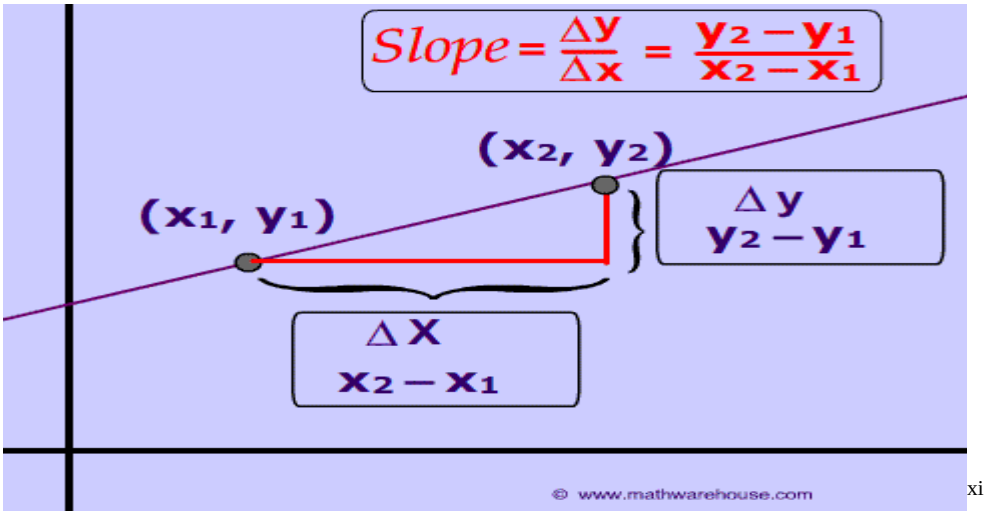
Figure 1

Ordered Pair



Two numbers written in a certain order. Usually written in parentheses like this: (4,5) Can be used to show the position on a graph, where the "x" (horizontal) value is first, and the "y" (vertical) value is second. Here the point (12, 5) is 12 units along and 5 units up.[x]

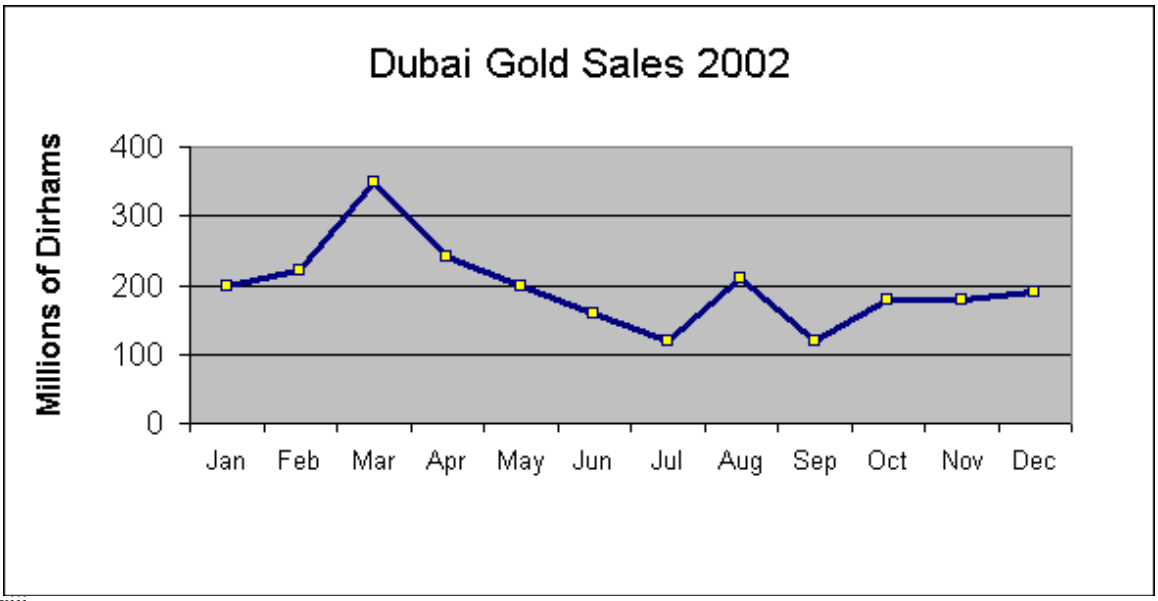
Figure 2



[xi]

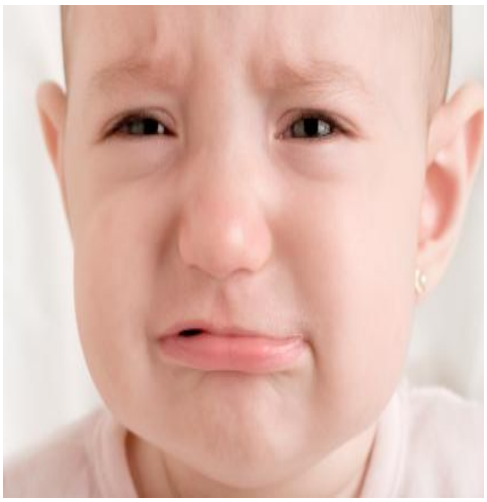
$$\text{Slope} = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta Y}{\Delta X}$$ [xii]

Figure 3



[xiii]

Image 1



[xiv]

[xv]

Image 2

4

-1

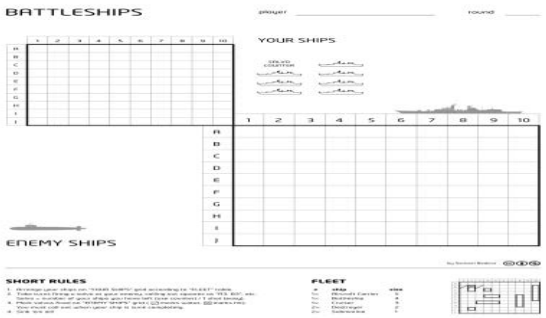
xvi -3

Figure 4

	A	B	C	D	E	F	G	H	I	L
1	■	■		■			■	■	■	■
2				■						
3	■			■		■	■	■		■
4	■		X							■
5	■					X	X			
6	■	X				■		X		X
7				X		■				X
8	X	X						X		■
9										
10					■	■	■	■	■	■

xvii

Figure 5



xviii

Figure 6

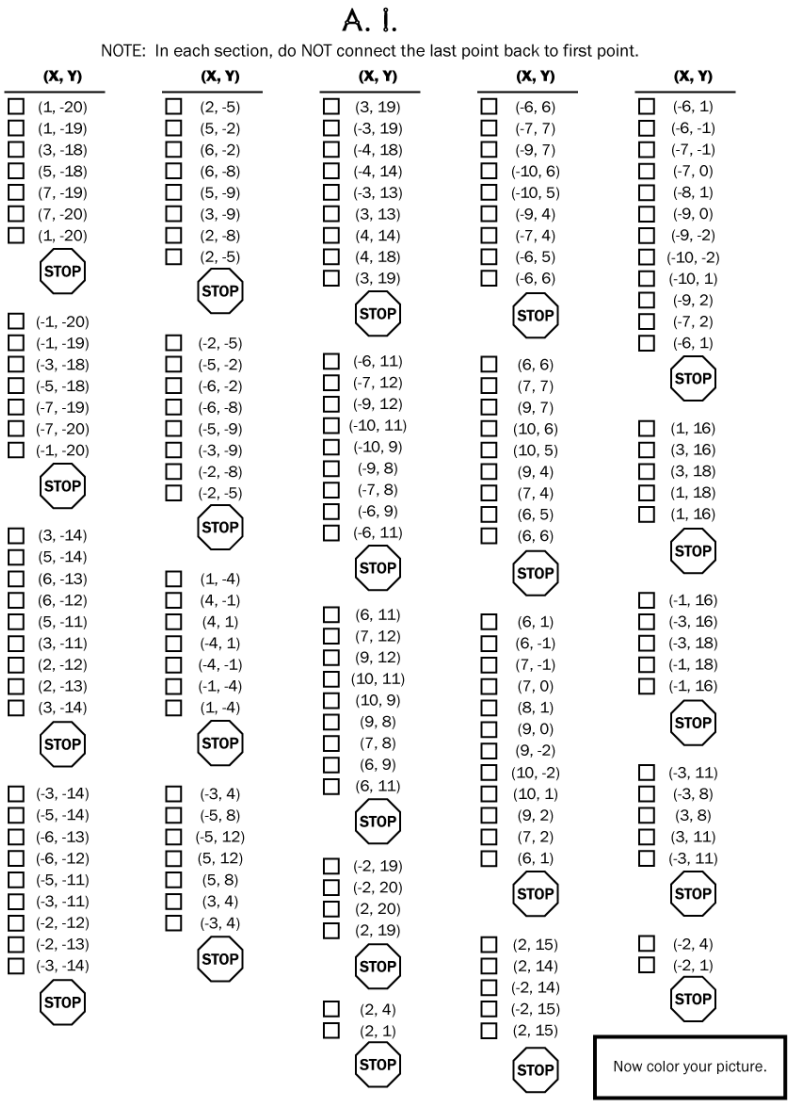


Figure 6 (part 2)

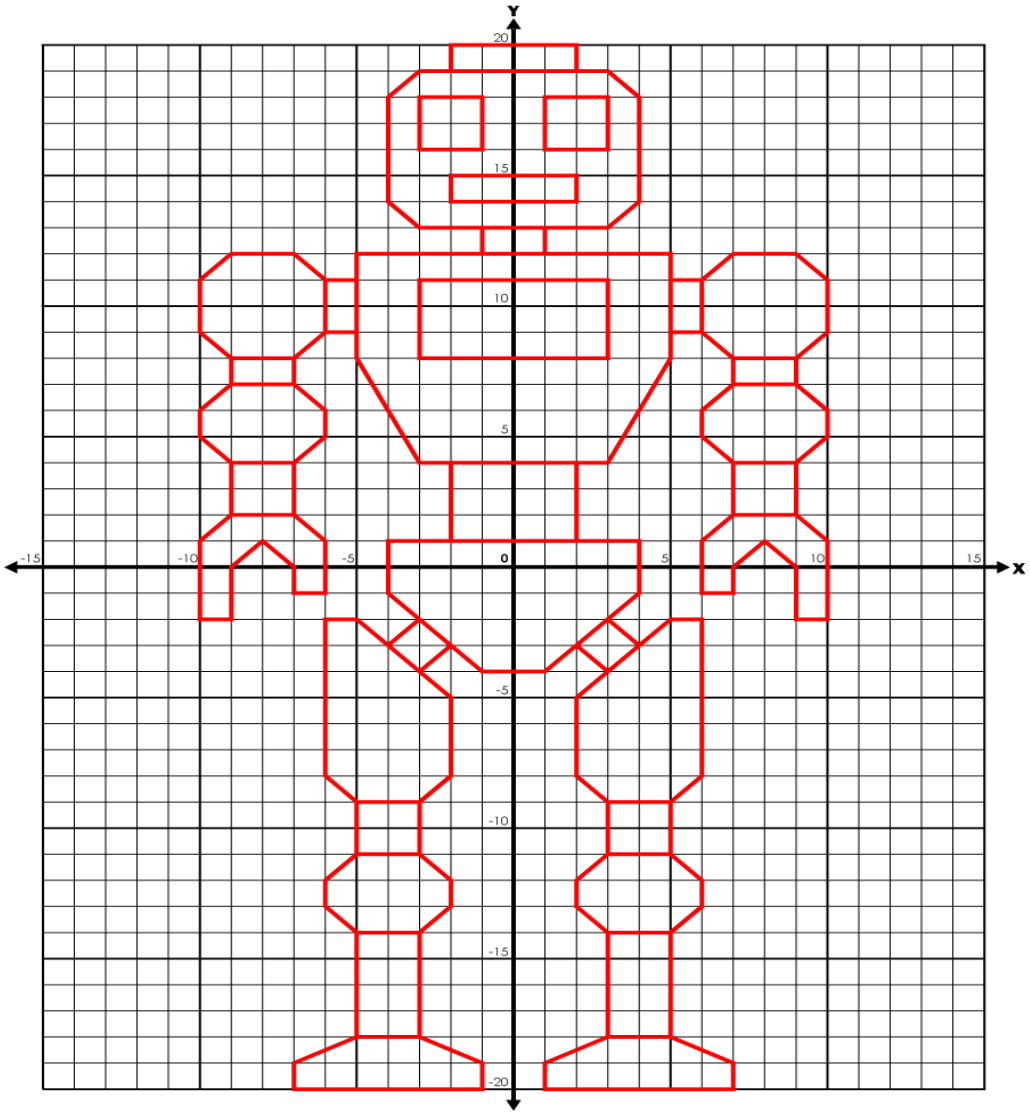


Figure 7

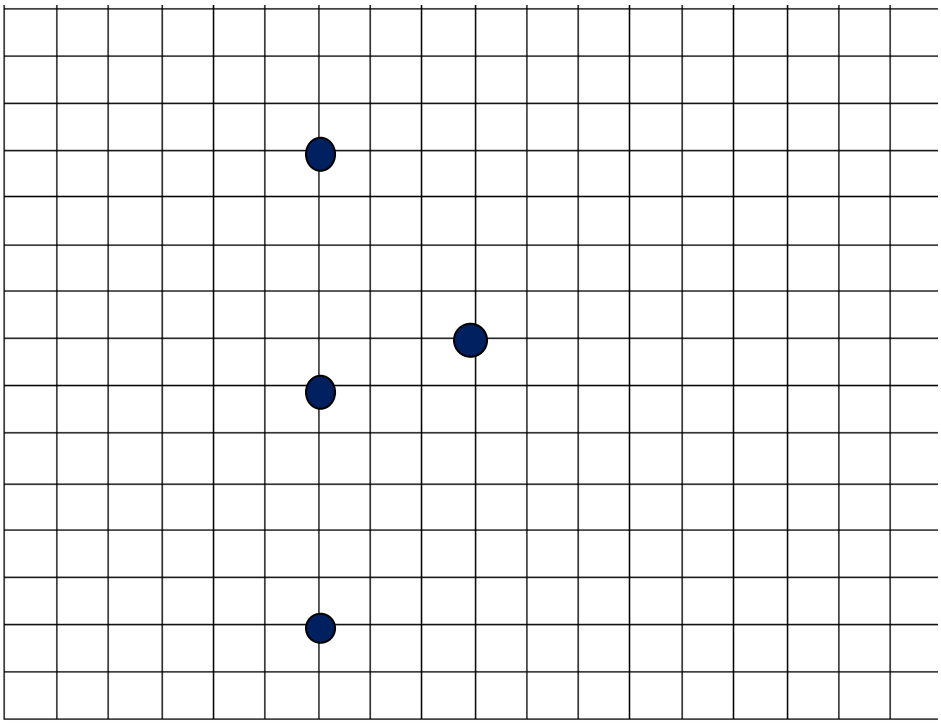


Figure 8

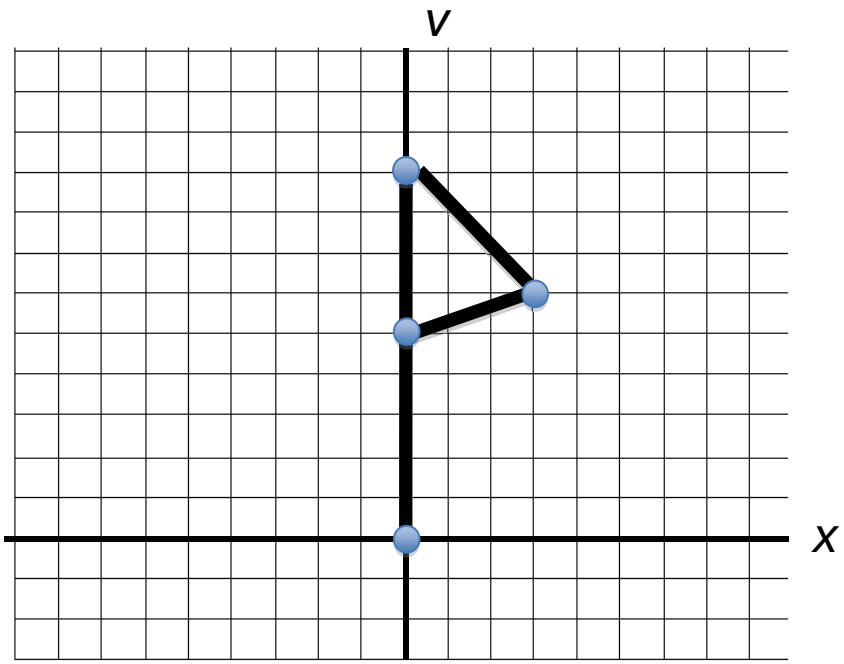
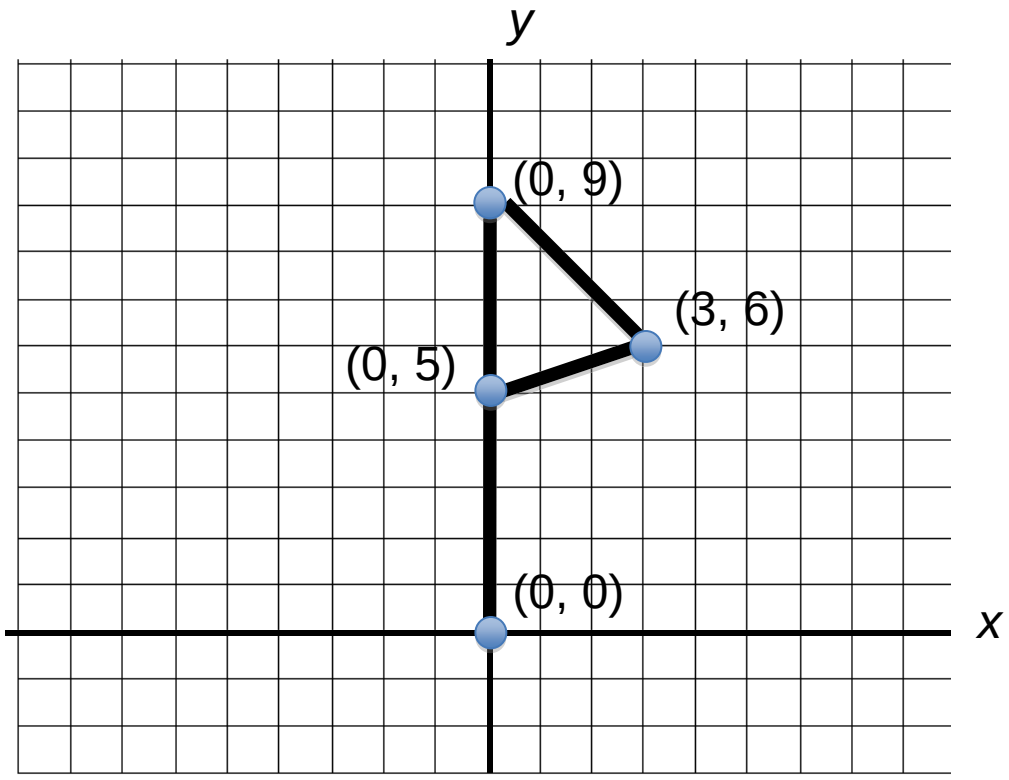


Figure 9



Bibliography for Teachers:

The books listed are full of fun activities for students as well as descriptions and explanations for teachers to be able to explain the concept to a diverse group. The Algebra 1 book is especially useful because of its bilingual glossary in one of its final sections.

- Cooperative Learning and High School Geometry. Becky Bride. Kagan Publishing. 2002.

-Algebra 1 North Carolina Teacher Wraparound Edition. Holliday, B., Cuevas, G., Moore-Harris, B., Carter, J., Marks, D., Casey, R., Day, R., Hayek, L. McGraw Hill. 2004.

- More Than 100 Brain-Friendly Tools and Strategies for Literacy Instruction. Kathy Perez. Corwin Press 2008.

Reading List for students:

This book is very useful for students to follow along with the teacher and has lost of practice or them to understand and internalize all the concepts. The glossary is helpful for Spanish speaking students.

-Algebra 1 North Carolina Student Edition. Holliday, B., Cuevas, G., Moore-Harris, B., Carter, J., Marks, D., Casey, R., Day, R., Hayek, L McGraw Hill. 2004.

List of Classroom Materials:

-Graph paper

-Duct tape

-Stickers

-Pictures or slide show of people showing different emotions

-Writing utensils, such us markers, color pencils, and crayons.

[i] http://www.cmstory.org/about.asp#site.
[ii] http://schools.cms.k12.nc.us/westcharlotteHS/Pages/AboutOurSchool.aspx
[iii] http://www.cms.k12.nc.us/cmsdepartments/ci/esl/Pages/default.aspx
[iv] More Than 100 Brain-Friendly Tools and Strategies for Literacy Instruction. Kathy Perez. Corwin Press 2008.
[v] http://www.onlinemathlearning.com/slope-of-line.html
[vi] http://www.wordcentral.com/cgi-bin/student?book=Student&va=slope-intercept%20form
[vii] http://www.mathematicsdictionary.com/spanish/vmd/full/o/orderedpair.htm
[viii] Cooperative Learning and High School Geometry. Becky Bride. Kagan Publishing. www.KaganOnline.com.
[ix] http://en.wikipedia.org/wiki/Battleship_%28game%29
[x] http://www.mathsisfun.com/definitions/ordered-pair.html
[xi] http://www.mathwarehouse.com/algebra/linear_equation/slope-of-a-line.php
[xii] http://www.mathwarehouse.com/algebra/linear_equation/slope-of-a-line.php
[xiii] http://learnwell.thelanguagemenu.com/show_file.php?id=163
[xiv] http://www.free-funnypictures.cc.cc/2012/04/sad-baby-face-sadbaby-face-baby.html
[xv] http://picturesofbabies.org/Happy-baby.html
[xvi] http://www.123rf.com/photo_14323403_student-with-books.html
[xvii] http://en.wikipedia.org/wiki/Battleship_%28game%29
[xviii] http://en.wikipedia.org/wiki/Battleship_%28game%29
[xix] Super Teacher Worksheets - www.superteacherworksheets.com