Modeling Pop Culture

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Background

The primary goal of this curriculum unit is to encourage students in Advanced Functions and Modeling (AFM) classes to transition from focusing on strictly skill-based, computational exercises to applying what they've learned to real-world situations. My goal is to get students to discover "the power of reasoning" (Burger &Starbird, p. 54) and not to simply focus on getting to the "right" answer to a problem or exercise. If they can see that math is not always about "plug and chug," if they can have opinions and make conjectures about problem situations and back up those conjectures with evidence, data, and computations, then this unit will have achieved its goal.

I teach in Charlotte Mecklenburg Schools (CMS), an urban school district which serves approximately 135,000 students in grades K-12. The high school in which I teach is a large, suburban school located in Cornelius, NC, in the northern part of Mecklenburg County. The school is in its second year of operation, having opened in August 2010, and currently serves about 2200 students. The student body consists of 76.6% Caucasian students, 11.1% African-American students, 2.2% Asian students, 7.0% Hispanic students, 0.4% American-Indian students and 2.8% Multi-Racial students. The AFM classes are comprised of a mix of juniors and seniors. The school draws from a population of relatively high socioeconomic status, and has a very active and well-funded PTSA. Therefore, resources not normally available through the school system may be procured with the assistance of the PTSA.

The classes I teach are on a 4 x 4 block schedule. All lessons and activities take place during a 90-minute class period. Due to budget difficulties in our school system, class sizes are extremely large this year; two of my classes have over 40 students in them and the third "small" class has 26. This has led me to focus on lessons and activities that encourage students to work together in a collaborative environment and not to rely so much on teacher-centered instruction and input. Discovery, collaboration, and resourcefulness are my key concepts for the year.

The lessons and activities in this curriculum unit are designed to be taught in any classroom. Desks can be arranged in a traditional rows and columns layout, or, as I have mine, in groups for cooperative learning. Some of the activities will require students to have access to computers connected to the Internet, either in the classroom or in a computer lab. New to my classroom this year is a Promethean board for interactive instruction. I am still developing activities and lessons that take advantage of this new technology, but all the lessons and activities in this unit can be taught in a low-tech classroom with a blackboard or whiteboard and maybe an overhead projector.

Objectives

The North Carolina Essential Standards for Advanced Functions and Modeling have an emphasis on modeling and problem solving. Of the five essential standards, four of them use the phrase, "to solve problems," and two of them use the phrase, "to model and solve problems." The standards apply to "a diverse array of functions," including linear, polynomial, exponential, trigonometric, power, logarithmic, piecewise-defined, and recursively-defined functions. One essential standard is specifically to "construct models of bivariate data to solve problems." Other standards focus on analyzing univariate data and using both theoretical and experimental probability to model and solve problems.

To teach to these essential standards, I divide the AFM course into six units taught in this order: functions; exponential and logarithmic functions; sequences and series; trigonometry and trigonometric functions; data analysis and statistics; combinatorics and probability. In terms of basic mathematical and computational skills, the first two units are a review of topics students should have mastered in previous courses (Algebra 1, Geometry, and Algebra 2). The final four units build on skills learned in earlier courses and add new material that most students have never seen before, such as graphs of trigonometric functions, normal distribution, and using permutations and combinations in probability and odds.

Because this curriculum unit is about modeling, and because modeling is a recurrent theme throughout the entire AFM course, I have structured this unit as an ongoing series of lessons and activities that will be integrated into the course over the entire 90-day semester. Rather than providing a one- or two-week single-topic unit to be taught all at once, I have developed a curriculum unit that has elements to be woven into the course in places that will serve to best enrich the students' understanding and mastery of three of the five major topics. Due to space and time limitations, I will focus on three of the six units: functions, exponential and logarithmic functions, and sequences and series. For a unit on trigonometry and trigonometric functions, I refer the reader to my 2010 CTI curriculum unit entitled, "The Art of Trigonometry." Each topic has lessons and activities that teach and develop the basic mathematical and computational skills, but those skills are not part of this curriculum unit. This unit provides activities and projects that emphasize the modeling aspect of each topic.

Strategies

Functions

The Functions unit covers polynomial functions. A few exercises deal with linear functions, but the emphasis is on higher-degree quadratic, cubic, and quartic functions. In Algebra 2, students learned how to find the solutions of quadratic equations by completing the square, factoring, using the Quadratic Formula, and graphing. With an understanding of the idea of zeroes, maxima, and minima, students can use scatter plots, graphs and regression analysis to model real-world data with polynomial functions of higher degree. The following are two examples of problems students are expected to be able to solve in this unit.

Polynomial Modeling Example 1

Boyle's Law states that, at a constant temperature, the pressure of a gas is inversely proportional to its volume. The results of an experiment to explore Boyle's Law are shown.

Volume (liters)	Pressure (atmospheres)
1.0	3.65
1.5	2.41
2.0	1.79
2.5	1.46
3.0	1.21
3.5	1.02
4.0	0.92

1. Create a scatter plot of the data.
2. Determine a power function to model the pressure P as a function of volume v.
3. Based on the information provided in the problem statement, does the function you determined make sense? Explain.
4. Use the model to predict the pressure of the gas if the volume is 3.25 liters.
5. Use the model to predict the pressure of the gas if the volume is 6 liters.

Polynomial Modeling Example 2

The numbers of laptops sold each quarter from 2005 to 2007 are shown. Let the first quarter of 2005 be 1, and the fourth quarter of 2007 be 12.

Quarter	Sales (thousands)
1	423
2	462
3	495
4	634
5	587
6	498
7	798
8	986
9	969
10	891
11	1130
12	1347

1. Predict the end behavior of a graph of the data as x approaches infinity.
2. Use a graphing calculator to graph and model the data. Is the model a good fit? Explain your reasoning.
3. Describe the end behavior of the graph using limits. Was your prediction accurate? Explain your reasoning.

Exercises such as the two above are meant to help students to understand the difference between a mathematical model of reality and the actual reality. While most of the students will end up with a low-level understanding of this difference, many of them will have difficulty applying it. And having textbook examples with data that they find dull, boring, and uninteresting does not serve them well. I will provide two lessons/activities for this unit to help build students' mastery of modeling with polynomial functions. I am hopeful that using more current, pop-cultural examples will help hook them and allow them to then apply what they've learned to other, less thrilling exercises and problems.

One lesson (adapted from Chartier, Clayton, Namas and Nobles) will teach students how translations and dilations of polynomial functions can be used in the creation of scalable fonts. Many of the AFM students have an affinity for visual arts. I have actually overheard students discussing the use of fonts in signs and reports, so I believe enough of them will be interested in scalable fonts that this example will capture their interest.

A second lesson will demonstrate to students how polynomial functions, specifically quadratic functions, are integrated into the currently popular “Angry Birds” game for smart phones and computers. Dr. Tim Chartier, Associate Professor of Mathematics at Davidson College, has created a webinar which can be found at http://davidson.mediasite.mcnc.org/mcnc/SilverlightPlayer/Default.aspx?peid=0291948b5d7e495c894e753659fbf5e41d&playFrom=12000. Through watching this video and doing the subsequent classroom activity, students will learn how quadratic functions can predict the path of an angry bird as it launches itself toward the pigs upon which it is trying to exact revenge.

Exponential and Logarithmic Functions

The mathematics covered in this unit includes inverse relations and functions, and the inverse relationship between exponential and logarithmic functions. The modeling in this unit focuses on exponential growth and decay and compound interest. The textbook includes exercises dealing mainly with population and investments. An enrichment exercise explains the “rule of 72” as used by many finance professionals. I believe that the financial examples and exercises used in this unit are valuable for all students. Financial literacy is an important life skill for everyone.

The following two exercises are examples of the type students are expected to be able to solve in this unit.

Exponential and Logarithmic Modeling Example 1

A student researched the average cost per gallon of gasoline in two different years.

Average Cost per Gallon of Gasoline	
Year	Cost ($)
1990	1.19
2007	3.86

1. If the average cost of gasoline increased at an exponential rate, identify the rate of increase. Write an exponential equation to model this situation.
2. Use your model to predict the average cost of a gallon of gasoline in
   a. 2011
   b. 2015
3. When will the average cost per gallon of gasoline exceed $7?
4. Why might an exponential model not be an accurate representation of this data?

Exponential and Logarithmic Modeling Example 2

In chemistry class, Brandon learned that, due to differences in atmospheric pressure, water boils at lower temperatures at high altitudes than it does at sea level. The table shows the boiling point of water for several different elevations above sea level.

Elevation (1000 m)	Boiling Point (°C)
0	100
1	99.29
2	98.81
3	98.43
4	98.1
5	97.8
6	97.53
7	97.28
8	97.05
9	96.83
10	96.62

1. Use a graphing calculator to create a scatter plot of the data.
2. Based on the shape of the scatter plot, what type of function can be used to model this data?
3. Write an equation to model the data, and then use the model to predict the boiling point of water at 25,000 feet above sea level.

Example 2 above illustrates one area with which students struggle when modeling exponential decay. The idea that the function used to model exponential decay will have values that approach zero, but will never equal zero is often difficult for students to reconcile with real-life applications. When talking about population decay, the difference between theoretical and actual decay is challenging for students to understand. I have my students do an activity in which they model exponential decay with M&Ms or Skittles candies. Eventually they get to a point in the activity where they have no candies remaining – or where the value of the function equals zero. But the exponential model, when graphed, has $y=0$ as an asymptote, and the function used to model the situation will never have a value of zero. Why will the function never equal zero when in reality the number of candies will equal zero? To help students get a better grasp of the concept of infinitesimal quantities, I will show students a video clip of Tim Chartier's infinite rope mime sketch. (http://vimeo.com/20079804). In this sketch, Dr. Chartier visually represents the concept of infinity by performing as a man picking up a rope and trying, to no avail, to find the end of the rope. He keeps pulling and pulling and pulling on it, but never finds the end. Then, through his frustration with his inability to get to the end of the rope, he has the bright idea of cutting it to force it to end and just get rid of the darn thing. But he discovers the concept of infinitesimal quantities as he keeps cutting, and cutting, and cutting, but never gets to where there's no rope left – there's always a piece left that he can cut in half. Although most of my AFM students have a pretty firm concept of infinity in terms of ever increasing quantities, they have a harder time imagining ever decaying quantities and infinitesimal amounts. I hope this sketch will help them to understand why exponential decay theoretically never goes below zero.

The other lesson I will teach also uses exponential decay. I will use the parlor trick of being able to pick a name out of a phone book to help students see that a seemingly unmanageable amount of information can be culled down rather quickly by cutting it down exponentially (adapted from Greenbaum and Chartier.) In this trick, I will have a student select a name from a local phone book and tell them I can find the name they picked in 20 guesses or fewer. By continually dividing the book into halves, I can quickly isolate the page, then the half-page, then the quarter-page, etc., until it is a choice between two entries and one of those two will be the entry the student selected.

The math behind this trick is based in the exponential function, . Figure 1 shows how the number of available names diminishes to one name if we start with 1,048,576, or 2^{20} names in the phone book.

Figure 1

Guess	Number of Names Available	Exponential Form
0	1,048,576	2^{20}
1	524,288	2^{19}
2	262,144	2^{18}
3	131,072	2^{17}
4	65,536	2^{16}
5	32,768	2^{15}
6	16,384	2^{14}
7	8,192	2^{13}
8	4,096	2^{12}
9	2,048	2^{11}
10	1,024	2^{10}
11	512	2^{9}
12	256	2^{8}
13	128	2^{7}
14	64	2^{6}
15	32	2^{5}
16	16	2^{4}
17	8	2^{3}
18	4	2^{2}
19	2	2^{1}
20	1	2^{0}

Students can now understand that by expressing the starting quantity in base 2 (or "two to some power") we can predict exactly how many guesses it will take to pinpoint someone's choice regardless of how many possibilities there are to begin with. I will then extend this example to a theoretical phone book of the entire world and ask students to predict how many guesses it would take to find someone's choice. Given a current estimate of the population of the world of 7 billion people (http://www.worldometers.info/world-population/), we can express 7 billion in base 20 by calculating . Students will want to use "guess and check" by raising two to different powers until they get closest to 7 billion. I will then tie in logarithmic functions to show them that is the same as . They will use the change of base formula to calculate the power as approximately 32.7. Rounding that number up to 33, the students will learn that they can name someone's choice from a world phone book in 33 or fewer guesses.

Sequences and Series

This is the unit in which I have the most difficulty teaching modeling. The math in this unit deals mainly with nth terms and sums of arithmetic and geometric sequences. We touch briefly on the Fibonacci Sequence, and use Pascal's Triangle for binomial expansion. The examples and exercises in the text often have students use an arithmetic sequence to find the seating capacity of an auditorium in which the number of seats in each row increases by a constant number from the front to the back of the auditorium. And they use geometric sequences to determine how many people may receive a chain email message. In teenage circles, email and instant messaging have been all but replaced by texting and by social networks such as Facebook and Twitter. So it seems relatable to students to find examples that use these current technologies rather than the relatively outdated technology of email.

I will use an example of a heavily-promoted movie that, despite all the hype, fails at the box office in the opening weekend and effectively falls off the face of the earth until it shows up at video rental kiosks six months later. If I go see the movie on its opening day and hate it, how long would it take to spread the word that it's not worth the $10 admission price? What if 200 people see it at the same time I do, and they all hate it as well; then how long would it take to spread the word?

If I post on Facebook, or tweet that I hate the movie and two of my friends share this information, then two of each of their friends share it, and then two friends of each of those people share it, and so on, how many levels of sharing will it take to spread the word to enough people to cause the movie to flop? What if it's three people who share the information at each level? What if 20 other people start a thread at the same time I do, or 50 other people, or 150 other people? How many degrees of word of mouth does it take to cause a movie to tank? Can we use a geometric series to model this?

The simplest geometric model to start with would be a doubling model. We'll start with one person seeing the movie, hating it, and spreading the word to friends. And we'll assume that each friend spreads the word to two more friends each time. The general formula for the sum of the first *n* terms of a geometric series is . In this case, a_1 equals one since only one person hates the movie, and *r* equals 2, since the number of people spreading the word doubles each time. So our explicit formula turns out to be , or, completely simplified,

Using different values for *n*, we see that even with only one person panning the movie, it takes relatively few iterations of spreading the word to do damage to the movie in its opening weekend. It is difficult to find data on numbers of tickets sold for successful movies; the more readily available data are about the gross sales in dollars. But using the example of "Harry Potter and the Deathly Hallows: Part II" which had the largest ever total gross one-day ticket sales of $91,071,119 on July 15, 2011 (http://www.the-numbers.com/movies/records/#wideopening), we can estimate that a very successful movie will sell at least 5 million tickets nationwide on its opening day. So how many iterations of bad word of mouth will it take to make a potential mega-hit become a flop? Well, if only one person pans the movie, we can use our formula, , plug in 5,000,000 for S_n and then solve for n. Doing so, we find that it takes about 22 iterations to get to 5,000,000 people hearing the movie is bad when only one person publicly pans it. If more people pan the movie, or if the word gets spread at a faster rate, or if the reality is some combination of more people and a higher rate, it becomes easy to see how quickly a movie can go from potential blockbuster to complete failure in very little time due to social networking.

Lesson Plans and Classroom Activities

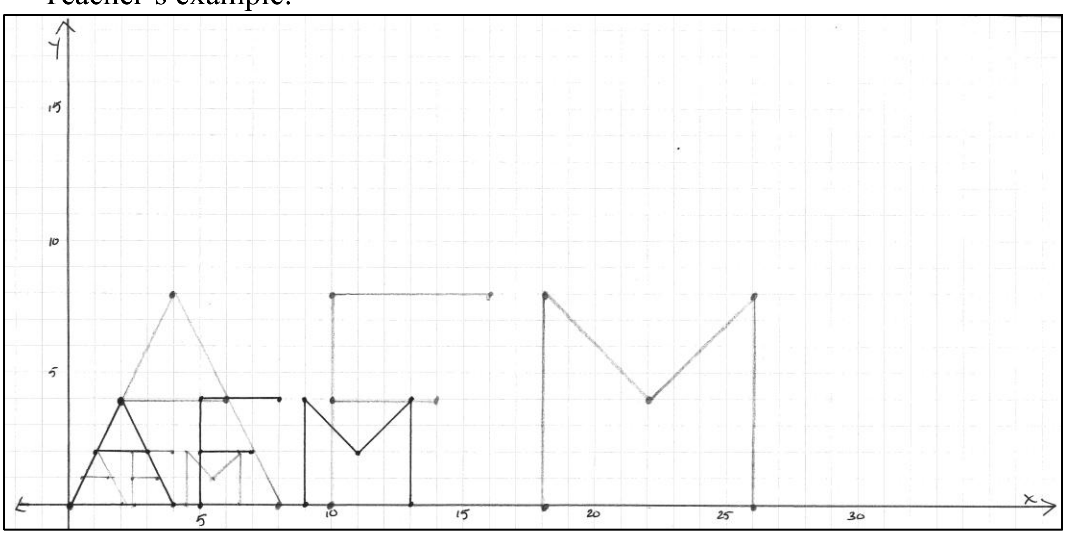
Functions

Lesson 1: Scalable Fonts

1. Choose a three letter word or your initials.
2. Create your own simple font by sketching your word or initials on graph paper, using only straight lines.
    a. The first letter of your word should have the origin as one of its endpoints.
    b. Make sure to use at least one of each type of line: horizontal or vertical, positive slope, negative slope.
3. Label the coordinates of the endpoints of each of the segments used to create each letter.
4. On a separate piece of paper, write an equation for each segment.
    a. Find the slope using ——.
    b. Use one of the segment's endpoints and the slope you just calculated and plug into point-slope form [ ] to get the equation of the segment.
    c. Repeat these steps for each segment of each letter.
    d. Think about the constraints on each of these equations.
        i. Are you drawing the entire line for each equation, or just a segment?
        ii. What determines where to start and where to stop drawing each segment?
        iii. Include these constraints in your equations.

5. Make sure your list of equations doesn't specify what actual letters you have sketched. It should just be a list of equations and the x- or y-values to sketch them between. Now exchange equations with a classmate.
6. On a blank sheet of graph paper, follow your classmate's equations and constraints and see if you can sketch their word.
7. Now go back to your own original word. You're going to perform two dilations on your word:
    a. Multiply all the coordinates by some scale factor (I suggest using 2 to keep it simple) to expand the letters.
    b. Divide all the coordinates by some scale factor (again, I suggest using 2 to keep it simple) to compress the letters.
8. Using two different colored pencils, sketch the expansion and compression on the same set of axes as your original word.
9. Choose at least 3 of the expanded segments and 3 of the compressed segments and write the equations for these segments (using the method from step 4 above.) Make sure your choices include one positive slope, one negative slope, and one horizontal or vertical line.
10. Did the equations change when the letters were expanded or compressed? If so, how did they change? If not, why not?

Teacher's example:



	Original Points	Expanded	Contracted
A	(0,0)	(0,0)	(0,0)
	(1,2)	(2,4)	(0.5,1)
	(2,4)	(4,8)	(1,2)
	(3,2)	(6,4)	(1.5,1)
	(4,0)	(8,0)	(2,0)
F	(5,0)	(10,0)	(2.5,0)
	(5,2)	(10,4)	(2.5,1)
	(5,4)	(10,8)	(2.5,2)
	(7,2)	(14,4)	(3.5,1)
	(8,4)	(16,8)	(4,2)
M	(9,0)	(18,0)	(4.5,0)
	(9,4)	(18,8)	(4.5,2)
	(11,2)	(22,4)	(5.5,1)
	(13,0)	(26,0)	(6.5,0)
	(13,40	(26,8)	(6.5,2)

Equations for A:

Equations for F:

Equations for M:

Notes to teacher: After students have completed this activity, you can demonstrate how the curved parts of fonts could be modeled with sections of polynomial functions. This will lead into the lesson on modeling real-world data with power and polynomial functions.

Lesson 2: Angry Birds

This lesson requires very little preparation, since it has already been created as a webinar. Simply play for the class the 10-minute webinar "Algebra 4 Angry Birds" (http://davidson.mediasite.mcnc.org/mcnc/SilverlightPlayer/Default.aspx?peid=0291948b5d7e495c894e753659fbf5e41d&playFrom=12000) created by Dr. Tim Chartier of Davidson College. In this webinar, Dr. Chartier has done a wonderful job of illustrating how quadratic functions can be used to predict the path of an angry bird launched from one of three common angles (30°, 45°, and 60°.) This is an excellent alternative to the common quadratic examples using the flight of a rocket or a ball thrown across a field.

Using the simplified equations derived by Dr. Chartier for the webinar, , , and , have students predict situations such as:

1. The best angle to launch to reach a target that is approximately
   a. 7 units away horizontally, and 5 units off the ground.
   b. 9 units away horizontally and 3 units off the ground.
2. Will the bird hit its target if:
   a. It is launched at 30° and the target is 2.5 units away horizontally and 3.2 units off the ground?
   b. It is launched at 45° and the target is 3.5 units away horizontally and 3.5 units off the ground?
3. What height of target will the bird hit if it is launched
   a. From 60° with the target 5 units away horizontally?
   b. From 30° with the target 5 units away horizontally?

This lesson can be followed up with a practice worksheet of exercises and word problems involving other situations modeled by quadratic functions.

Exponential and Logarithmic Functions

Lesson 3: Infinite Exponential Decay

This lesson may come right after an activity modeling exponential decay with candy (see http://serc.carleton.edu/quantskills/activities/MandMModel.html or do a web search to find one of many good examples of such an activity.) Show the video clip of Dr. Chartier's "Infinite Rope" mime sketch (http://vimeo.com/20079804.) The purpose of this short lesson is simply to open up a class discussion on the concept of infinitesimal quantities, and the concept of numbers that get smaller and smaller and smaller but never reach or go below zero.

Lesson 4: Tricks with Exponential Decay

Using a telephone directory from a large city (I use Charlotte, NC) give the book to one student and ask them to choose one listing from that book. Tell the student you will guess their choice in 20 guesses or fewer.

Start by dividing the book approximately in half, holding each half in one of your hands. Then ask the student, "Is your choice in my left hand or my right hand?" Based on their answer, take that section and divide it approximately in half, then ask them again, “Is your choice in my left hand or my right hand?” For each of their answers, take that section and divide it in half, asking the left hand/right had question each time. Eventually you will narrow it down to one page. Then begin breaking the page in half. Phone books usually have several columns of listings, so first ask the student if their choice is on the left half or the right half of the page. Once you’ve isolated the column, then break the column in half by asking if their choice is in the top half or bottom half of the column. Keep breaking that down to halves until you have isolated two listings. Then ask if their choice is the first listing or the second listing.

Now have students research the current population of the world. Once they have found that number, ask them to calculate how many guesses it would take to find someone’s choice if there was a phone book of the entire world. They will need to use logarithms to calculate how to express the population in terms of base 2. Many will want to use “guess and check” by raising 2 to various powers until they come up with a number that is close to the population number. Since the number of guesses is a whole number, this method is actually ok, but won’t tie in to logarithms. For example, if the population is 4 billion, student will have to solve or

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This lesson can be followed up with practice in solving problems using exponential decay and using logarithms to solve exponential equations.

Sequences and Series

Lesson 5: How to Make a Movie Fail and the Box Office

Ask the students to think about a movie about which they heard a lot of hype, but by the time they decided to go see it, it was already gone from the theaters. What could cause that to happen? It was supposed to be a big blockbuster, but it ended up “tanking” at the theaters. Could word of mouth make that happen? How could it possibly happen so quickly?

Once the students have come up with the idea that electronic word of mouth, or social networking, could have something to do with it, it is time to get them thinking about it as a geometric sequence or series. One could correctly argue that this scenario could be modeled by an exponential function, and there could be an extension exercise in here to help students see the connection between geometric sequences and series and exponential functions.

Ask students some leading questions such as:

1. If I go see the movie on its opening day and hate it, how long would it take to spread the word that it’s not worth the $10 admission price?

2. If I post on Facebook, or tweet that I hate the movie and two of my friends share this information, then two of each of their friends share it, and then two friends of each of those people share it, and so on, how many levels of sharing will it take to spread the word to enough people to cause the movie to flop?
3. What if 200 people see it at the same time I do, and they all hate it as well; then how long would it take to spread the word?

Next, guide the students to discover the terms to use to model this situation with a geometric series. Have them start with a doubling model, with only one person seeing the movie, hating it, and spreading the word to two friends. Assume that each friend spreads the word to two more friends each time.

Remind students of the general formula for the sum of the first n terms of a geometric series: . Ask more directed questions:

4. What will we use for, a_1? Why? (We use one since only one person hates the movie).
5. What will we use for r? (Two, since the number of people spreading the word doubles each time)
6. Can you write an explicit formula for this sequence? (, or, completely simplified,
7. What value of n will give us a sum large enough to do damage to the movie in its opening weekend

Have students research data on numbers of tickets sold for successful movies. They may discover that the more readily available data are about the gross sales in dollars rather than the number of tickets sold. Help them estimate ticket sales using those numbers and assuming an average ticket price of $8-10.

Finally, students will use the formula, , plug in their estimated number of ticktes for S_n and then solve for n. Doing so, they will find how much iteration it may take to get to a “critical mass” of people hearing the movie is bad when only one person publicly pans it.

Works Cited

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