Without Geometry, Life is Pointless: A Focus on Transformations, Circles, and Probability

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Introduction and Rationale

Students of today are classified as 21st century learners. My goal for this unit is to encourage students to think about how and why the geometric concepts we learn work together, as well as how they work to apply to the real world. Essentially, I want students to understand the "math behind the math". In many cases, teachers have a vast amount of content that they have to cover and oftentimes we run out of time to help students understand in detail why things work. For example, we may give them the slope formula and tell them to use it to find the slope of a line, but we may not take the time to explain the meaning of slope and why the slope formula works or how it denotes a rate of change when applied to is worldly purposes outside of the math classroom. Students often shut down because they are not able to see the value behind what they are learning. Through this unit, we will work to unveil some of those relationships and descriptions that have been missing as part of the curriculum to help students find the value in learning the content. This unit will include some hands on group activities as well as a culminating project in which students will advertise a new product by describing the math behind creating their product. This unit will be designed to last 12 days.

Geometry is a content area that always comes as a surprise to students because of its strong variance from the popular algebra based math content areas. Students that struggle with verbal expression and abstract thinking often have a difficult time with the subject, even though geometry is such a visual math. In this unit, I will strive to provide meaningful activities that promote student abstract thinking by helping students to first master the basic understanding of the content. Teachers should understand that this unit will take a true hands-on approach that allows students to internalize why things work in geometry. This will take place through constructions, proofs, and collaboration with classmates.

The focus of this unit is transformations, circles, and probability. These are all topics that are generally taught at the end of a geometry class. While this unit has three major topics, it also works to incorporate many of the major topics covered throughout the duration of the class. Students will have to pull prior knowledge to build understanding of the new topics, and students will gain an understanding of how all of the topics work together. In order to reinforce this understanding, this unit will include a running project that students are to develop as the unit progresses. Students will be able to begin the project immediately using their prior knowledge. As the unit continues, students will use their newly developed knowledge to continue working on their project. To make this process effective, the teacher should continuously review the project and its parts with students so that they understand how to make the new connections.

Unit Design

The rationale and structure of this unit is based on the research and teachings of the PEAK Teaching for Excellence system. This set of research and instructional strategies focuses on student centered learning in the classroom through cooperative groups, differentiation, reteaching, and engagement strategies. Concerning learning content, PEAK teaches the idea of 28/3. Research shows that a student must be exposed to a certain topic at least 28 times over a three week period in order to obtain a mastery level of 80% or higher for that particular topic[1]. I have structured the unit based on this research. Throughout the unit, I have suggestions on how to reinforce, revisit, and reteach the content to be learned in the unit, as well as prior knowledge that students must have mastered in order to be successful with this unit. Seeing the content 28 times does not mean that a teacher must reteach the content this many times over a three week period; as this would be impossible. What a teacher can do is constantly help students to recall this information amongst their knowledge bank through small activities such as the following:

- revisit content through warm ups or practice problems
- refer to word walls to revisit vocabulary or content posters to remind students of processes
- have students create word walls or content posters
- play review games to help students remember computational skills
- have students create quizzes or practice problems for a partner to complete to raise the level of thinking

All of these activities have been dispersed throughout the lesson as suggestions for increasing the level of mastery for students. Teachers will also notice that each lesson in the unit is designed with an engaging introduction that invites students to make connections between their prior knowledge and the new content that is to be discovered. Many of these are in the form of a construction or partner activity. This design is based on the research by David Sousa of the Primacy Recency Effect[2], which states that there are prime times during a 90 minute lesson that students are the most engaged. These are the times in which teachers want to introduce the new content and complete the most rigorous and meaningful activities in order to enhance student understanding. The most prime time of the lesson is the first 15 minutes of the lesson. This unit encourages teachers to use this valuable time to introduce the topic through a prior knowledge connection or through a discovery activity.

Curriculum Content

There are several topics in geometry that are actually covered in other math classes as students progress through the years, however, these topics are often scattered in with no connections made between how things work. In this unit, I have included a prior knowledge section to reinforce content that students have already seen in other math classes, however, a teacher should not assume that these topics have been mastered as some of them may have been taught several years ago with no connections to the content that will be introduced in this unit. Teachers may want to create "mini-lessons" on the prior knowledge topics if they have not been discussed earlier in the course, or teachers may want to reinforce the content by continuously reviewing through warm-up activities and exit ticket activities.

One may argue that transformations, circles, and probability could all count as separate units. Through this unit, I want to show that these topics are indeed related, furthermore, I will show that they connect to previous major topics covered in the course. The Common Core Standards stress connectivity of the content and teachers are encouraged to teach for understanding instead of just teaching for memorization. For teachers that have never taught like this, making these connections can be a daunting task. For this reason, I have designed the Culminating Project to show how the content works together. This backward design approach aids teachers in planning lessons effectively.

Prior Knowledge

Before beginning this unit, there are some important pieces of mathematical content that students must have mastered. Listed below are the main prior knowledge concepts that students should be able to grasp at a high level of mastery. Listed with each concept is a brief description and visual representation of vocabulary, properties, and definitions that students should recognize.

1. Parts of a circle: Students should understand that a circle is not a polygon and that it is represented by a set of points equidistant from a given center point. Students need to know basic parts of a circle such as the radius, diameter, chord, and central angle. Students should recognize that the sum of the central angles of any circle is 360 degrees.

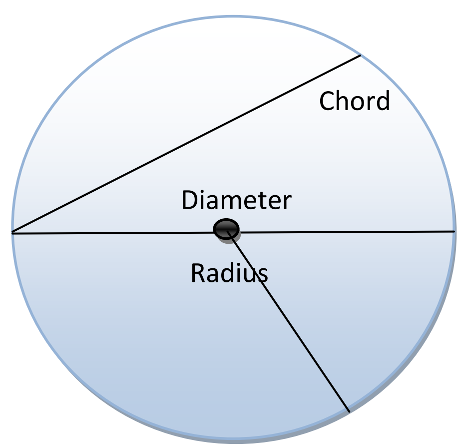
Students should be able to recognize these parts of a circle. In addition, make sure that students know the following properties of circles:

1. The radius of a circle is half the length of the diameter

2. The diameter is the longest chord in a circle

3. The circumference of a circle represents its perimeter. The circumference is πd or $2\pi r$

4. The area of a circle is represented by πr^2



2. Pythagorean Theorem: Students should understand how to use the Pythagorean Theorem to find missing legs of right triangles. In addition, students should know how to identify the parts of a right triangle as well as basic trigonometric functions used within right triangles including sine, cosine, and tangent. Students will derive a geometric and algebraic proof of the Pythagorean Theorem in this unit as more reinforcement for how geometry works. Students will use the Pythagorean Theorem on circles when identifying properties of inscribed right triangles formed by radii, diameters, and chords.

Students should be able to recognize these parts of a right triangle and how to use the Pythagorean Theorem: $a^2 + b^2 = c^2$. In addition, students should know the following properties:

1. In an isosceles right triangle, the legs are congruent and the acute angles each measure 45 degrees to create a 45-45-90 triangle

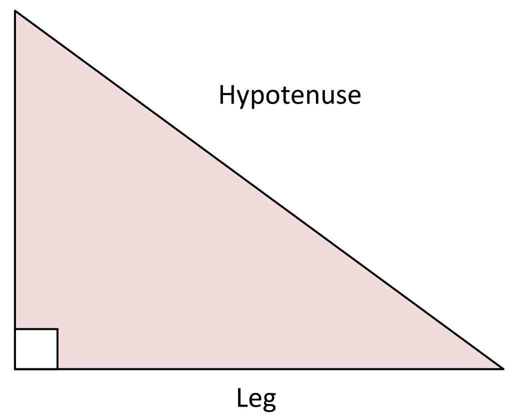
2. The hypotenuse represents the longest leg in any right triangle and is opposite the right angle in the triangle

3. Trigonometric ratios are as follows:

Sine: opposite / hypotenuse

Cosine: adjacent / hypotenuse

Tangent: opposite / adjacent



3. Distance and Midpoint Formulas: Students should understand and be able to use the distance formula to find the distance between two points on the coordinate plane. In this unit, students will make the connection between the distance formula and the Pythagorean Theorem by deriving the distance formula from the Pythagorean Theorem. Students should also be able to recognize and use the midpoint formula to find the midpoint between two points. Students should also be able to utilize the midpoint formula in finding the coordinates of a missing endpoint if given the coordinates of the midpoint and one other point.

Students should be able to utilize the following formulas on the coordinate plane:

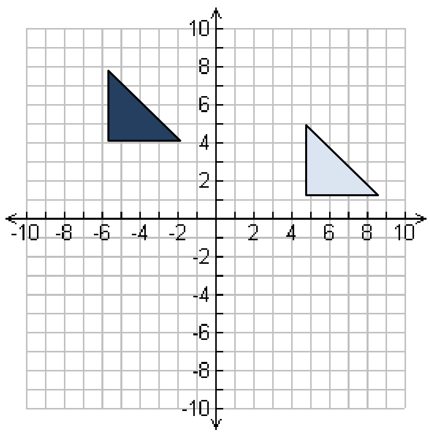
Distance:

D =

Midpoint:

Mid =

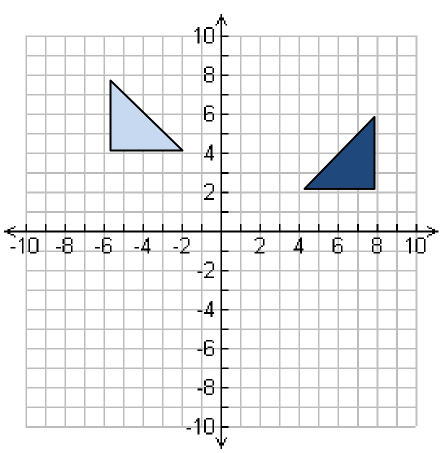
4. Transformations of figures: Students will need to know the four types of transformations and how to apply rules of each to the coordinate plane: reflection, translation, rotation, and dilation. Students will use rules of transformations to apply to triangles and other polygons inscribed in circles. Students will also use transformations to translate and dilate circles around the coordinate plane. Students have learned the basics of transformations in their previous math classes from elementary to middle school, however, transformations in the coordinate plane are new to students within the geometry curriculum. Teachers will need to make sure that students understand the various transformation rules when this unit occurs in the curriculum.



The graph to the left represents a translation. Students will be able to translate figures in the coordinate plane, identify rules in verbal notation as well as symbolic notation. Essentially, students should be able to recognize the verbal, symbolic, and graphic forms of each type of transformation. The dark blue figure is a translation of the light blue figure.

**Verbal: Move right 7, down 3*

**Symbolic: (x,y) → (x + 7, y – 3)*

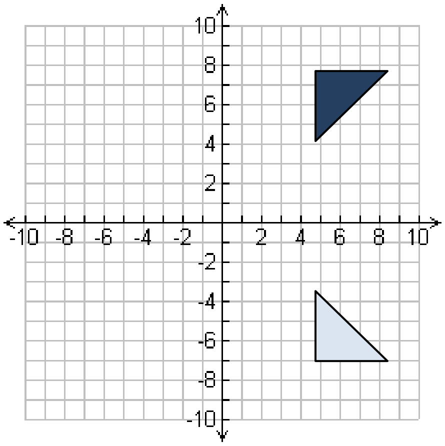


The graph to the left represents a rotation. The dark blue figure is a rotation of the light blue figure 90 degrees about the origin counterclockwise. Students should understand the rules for the following rotations about the origin counterclockwise:

90 degrees: (x,y) → (-y,x)

180 degrees: (x,y) → (-x, -y)

270 degrees: (x,y) → (y,-x)

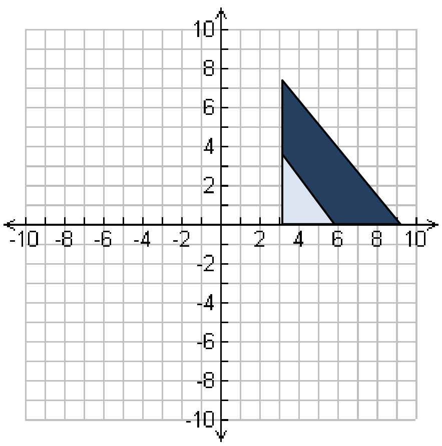


The graph to the left represents a reflection. Students should understand how to reflect over any vertical or horizontal line. Students should also recognize the following reflection rules:

Over y axis: (x,y) → (-x,y)

Over x axis: (x,y) → (x,-y)

Over line y = x: (x,y) → (y,x)



The graph to the left represents dilation. Students should recognize that dilations are formed by multiplying coordinates by a scale factor.

Order of Content

The content of this unit has been arranged so that the knowledge progresses. The first lesson is a review of transformations taken to a higher level of rigor with students proving transformations graphically by creating rules for the transformation symbolically. The next few lessons dive into the circles content and transformations help students to discover properties of circles on the coordinate plane. The last two lessons incorporate the idea of probability into many parts of the content of the course, including probability with circles. Teachers may note that there are eight lessons and this unit is designed to last twelve days assuming that teachers may want to take one day to review content and one day to have an overall assessment of the unit. This leaves some flexibility in how teachers present the content. This section of the unit is outlined with the detailed content that students should master from each lesson. There are eight lessons total designed for a 90 minute class period. Teachers with year long classes may find that they need to spend more than one day on some lessons, particularly the probability lessons. The following is an outline of the curriculum by lesson with an included objective:

Day 1: Coordinate Geometry Review

Objective: Discuss properties of triangles, quadrilaterals, and other polygons in the coordinate plane and perform translations, dilations, rotations, and reflections using these figures

Students should be familiar with writing rules and performing the following transformations: reflection, rotation, translation, dilation. Students should also recognize that a dilation is the only type of transformation that is not an isometry, which means it does not produce a congruent image when correlated to the pre-image. Teachers will want to stress the rules and properties as stated in the prior knowledge section of the unit.

As a complete overview of this lesson, teachers may want to reinforce performing transformations in the coordinate plane by completing a hands-on activity called Moving House. See Strategies 1 for more information about how to implement the activity. This activity is valuable in that it allows students to use the transformation rules in the coordinate plane and also helps students to make the cognitive connection between verbal, graphical, and symbolic representation of math. For students taking summative and end of course exams, this relationship tool can be very valuable.

To check for understanding, teachers may also want to have students perform a partner activity similar to Battleship. Teachers can give out coordinates for specific figures and ask students to use rules that they come up with to make specific transformations. Students can then give the coordinates of their new image to a partner and ask them to come up with the rule for the transformation. For example, a teacher may give out coordinates for a triangle ABC as A(3, 4) B(-2, -1) and C(2, -5). A student may perform a transformation that creates a new image of A' (1, 5) B'(-4, 0) and C'(0, -4). A partner will observe that all of the x values have been subtracted by two and all of the y values have been added by one. This is a rigorous way of reinforcing the various transformation rules with students while giving them the opportunity to collaborate with classmates.

Teachers will want to wrap up this lesson with the introduction to the unit project **Create Your Own Space**. See "Strategies: Culminating Project" for complete details of the project. This will be a great opportunity for students to brainstorm ideas about the space that they would like to design and the teacher can point out to students that transformations are involved in the creation of the project.

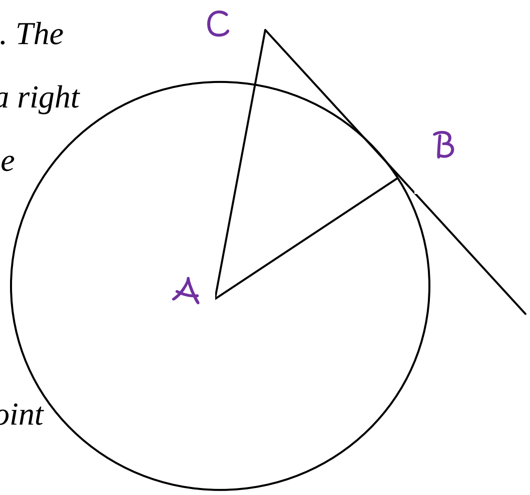
Day 2: Circles Review and Tangents

Objective: Use properties of circle tangents to solve problems involving right angle and right triangle relationships

Teachers will want to start this unit with a review of the basic parts of circles, including the radius, diameter, and chord. Completing constructions using a compass and protractor for this lesson will be a great way to show students the relationships between the parts of a circle and a tangent; teachers may want to begin with students constructing a circle and two radii. Students can then measure the central angle formed by the radii. Students can also construct the segment connecting the two radii to create a triangle. Teachers can start a discussion about the type of triangle formed and the properties of the sides and angles of the triangle as they relate to the circle. Teachers will also want to review area of circles and Pythagorean Theorem. These activities can be reviewed in a warm – up or quick activity.

The focus of this lesson is for students to understand what it means to be a circle tangent. Vocabulary includes point of tangency, Pythagorean Theorem, and radius. All of these help students to understand how to construct a tangent to a circle and how to identify if segments will be tangent to a circle. Teachers may want to introduce the following model:

In this picture, segment CB is the tangent to the circle. The tangent ,radius AB, and extended segment AC create a right triangle. Students can verify that this is a right triangle by completing the Pythagorean theorem. Teachers should also introduce examples in which students will need to use the Pythagorean Theorem to find missing measures in a picture such as this. Teachers should point out that the point of tangency is point B, where the radius and tangent meet.



Teachers can help students to understand the relationship of the tangent to the circle by having them construct tangents to circles and verify that the segments are tangent by measuring the lengths of the sides created by the right triangle formed. Students can also measure the angle created at the point of tangency to determine if it is indeed a right angle. Teachers may also want to incorporate practice skill problems so that students can show these relationships algebraically.

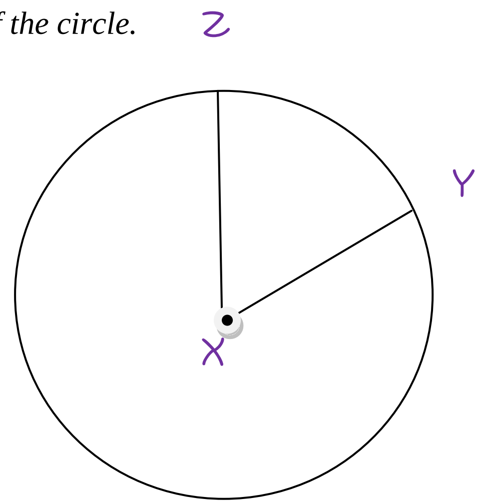
Day 3: Chords and Arcs

Objective: Use congruent chords, arcs, and angles to solve problems

This lesson helps students to understand the connection between arcs and central angles. Students should already have prior knowledge of central angles and the fact that a circle contains 360 degrees. Teachers may want to begin this lesson with a review of chords and their properties.

Teachers can help students understand the use and origin of the number pi by having students complete the Chocolate Chip Pi activity in the "strategies: introductory activities: 2" section of this unit. After completing the activity, students will recognize that the section of circle represented an arc of the circle. Teachers can introduce the relationships between the arc and the entire circle and the lesson progresses.

Teachers should explain that an arc represents a piece of the outside of the circle. The measure of an arc is its numeric value in degrees based on the portion of the circle compared to the 360 degrees total in the circle. The length of an arc is a part of the circumference of the circle represented by finding the percentage of the particular length compared to the entire circumference of the circle.



In this circle, segments XZ and XY represent radii of the circle. <ZXY is a central angle of the circle. ZY is an arc of the circle. The measure of arc ZY corresponds to the central angle <ZXY. If the measure of <ZXY is 60^o*,then the measure of arc ZY is also* 60^o*. If the length of the radius is 4, then the circumference of this circle is 8π. We want to determine the length of arc ZY by determining the fraction of the circle represented by this arc. Since the measure of the arc is* 60^0*, the fraction of the circle is 60/360, which simplifies to 1/6. This means that the length of arc ZY is (1/6) of the entire circumference, or 8π/6. Have students derive this formula by investigation through construction and thinking about the fraction of the circle represented by various central angles. This would also be a great way to incorporate algebraic problems related to the scenario. Teachers may want to have students work with a partner to discover the arc length relationship. Students can also find the measure of an arc by using a protractor and students will see that the relationship holds true with the measure of the central angle across from the arc.*

Teachers should also explain that if points Z and Y are connected, then a chord of the circle is created. Have students note the different symbolic notation used to denote chord ZY and arc ZY. Teachers can reinforce that there is a triangle created by the chord and two radii of the circle. Have students discuss the properties of the triangle created. In this case, since <ZXY is a 60 degree angle, the triangle created is an equilateral triangle. Students can prove this by showing that the triangle is isosceles since it was created using two radii. Students will then be able to show that angles Z and Y must be congruent because they represent the base angles of the triangle. <X would represent the vertex angle of the triangle. Since all three angles must add up to equal 10 degrees, angles Z and Y must be 60 degree angles as well, which means the triangle is an equilateral triangle. Have students prove this as well by measuring angles Z and Y using a protractor.

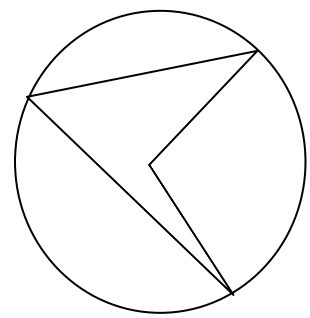
In this lesson, teachers will also want to distinguish between major, minor and semicircle arcs. Students should recognize that the type of arc is centered on the fact that a circle contains 360 degrees. Major arcs measure more than 180 degrees, minor arcs measure less than 180 degrees, and semicircles measure exactly 180 degrees. Students may also note that this relationship is similar when discussing types of angles: acute, obtuse, and right. Teachers also want to stress other relationships formed by a semicircle, such as the fact that the segment that creates a semicircle arc must be a diameter, which means the circle is split in half. Have students practice filling in missing angles in a circle using the 360 degree relationship and properties of semicircles, minor, and major arcs.

Teachers may want to close this lesson with a ticket out the door with students showing the relationship between the three types of arcs, or teachers may want to have students explain the difference between arc measure and arc length. This is a distinction that teachers will want to continuously reinforce so that students are clear as to what each word represents for the circle.

Day 4: Inscribed Angles

Objective: Find the measure of angles in circles

Students should be familiar with what it means to inscribe a figure in a circle from discussions on regular polygons. In this lesson students will discover relationships of inscribed angles to central angles in the circle. Have students discover this relationship by constructing any central angle of a circle. Students will measure the central angle and note the type of arc formed by the central angle as well. Have students discuss the relationship between the central angle and the arc, and the rest of the circle. Use the 360 degree relationship within the circle. Then, have students create a point anywhere on the circle behind their central angle. They should then construct chords using that point to the end points of their central angle. Below is an example construction:



Have students note that since all three points of the angle created touch the circle, the angle formed is called an inscribed angle. Also have students note that the inscribed angle shares endpoints with the central angle, so this means that they intercept the same arc. Students should measure the inscribed angle using a protractor. Students should note that the measure of the inscribed angle is half the measure of the central angle or half the measure of the arc. Have students repeat this process with a new circle and new central angle to be sure that the relationship holds true. Teachers may also want to have students repeat the process of constructing an inscribed angle on the circle using a new point on the same circle. Students will then note that inscribed angles intercepting the same arc must be congruent to each other.

Teachers can conclude this lesson with example problems that encourage students to apply algebraic skills to the geometric concepts. Students will be able to understand the relationships better if constructions were incorporated throughout the lesson.

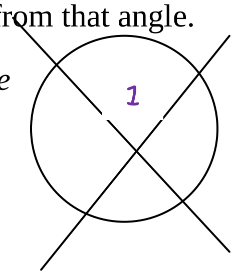
Day 5: Angle Measures and Segment Lengths

Objective: Find measures of angles formed by chords and tangents

In this lesson, teachers will introduce the last special segment in circles known as the secant. Teachers should stress that a secant is a line that represents an extension of a chord. There are three major relationships that teachers should stress:

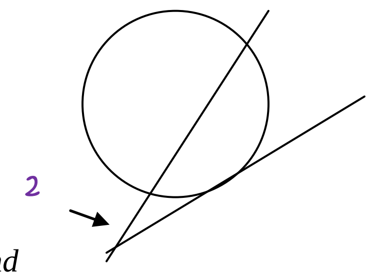
1. If segments intersect inside of a circle, then the measure of the angle formed by the intersection is the average of the arcs across from that angle.

The measure of <1 can be found by taking the average of the arcs above and below <1.



2. If segments intersect outside of a circle, then the measure of the angle formed by the intersection is half the difference of the arcs across from the angle.

The measure of <2 in this circle is formed by a tangent and a secant intersecting. To find the degree measure, subtract the measure of the arcs (positive difference) across from the angle and divide this difference by two.



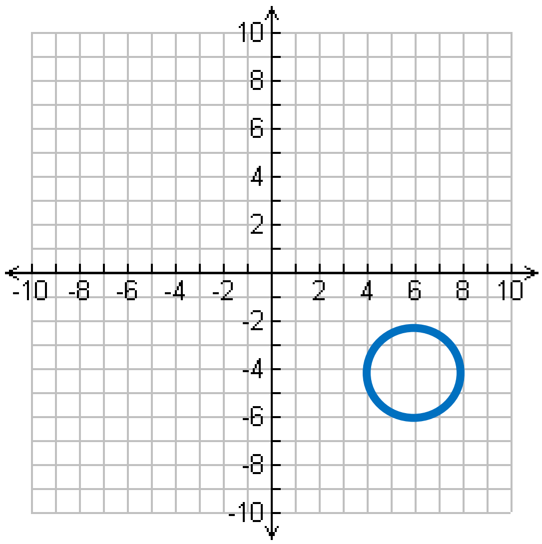
3. If segments intersect on the circle, then the measure of the angle formed by the intersection is half the arc across from the angle. You can reinforce this relationship by going back to the inscribed angle scenario. The same relationship is formed if a tangent intersects a secant on the outside of a circle.

Constructions have been a valuable teaching strategy in this unit. Teachers may want to close the unit by having students prove that these relationships hold true through constructions. Have students practice constructing tangents and creating secants to intersect in different ways on the circles. Have students verify angle measures by using a protractor. Teachers should also continue to stress the importance of symbolic notation. Students need to know how to denote the difference between segments, lines, angles, and arcs in this unit as all of these notations should show up on summative assessments and end of course exams.

Day 6: Circles in the Coordinate Plane

Objective: Write equations of circles on the coordinate plane

This lesson takes the focus of the unit back to the coordinate plane. In this lesson, students will apply what they have learned about circles to coordinate geometry. Teachers will want to introduce the formula for a circle on the coordinate plane: $(x – h)^2 + (y – k)^2 = r^2$. Explain to students the meaning of h, k, and r and how they relate to constructing the circle on the coordinate plane. Students should recognize that (h,k) is the center of the circle and r represents the radius of the circle. Students can plot the center and use the radius to plot a point north, south, east, and west of the center in order to create enough points to construct the circle. Teachers should also discuss with students that if there is no h or k value presented in the equation of the circle, then this represents a zero as the coordinate. Teachers should also discuss that if the formula reads (x + h) then this represents a negative coordinate for the x value of the center point. The same holds true for the y coordinate.



In the picture, the circle graphed has the following equation: $(x - 6)^2 + (y + 4)^2 = 4$. Students should note that the center of the circle is (6, -4) and the radius of the circle is 2 units. Graphing circles on the coordinate plane will also be a great way to bring back properties of transformations. Discuss with students the types of transformations that can be performed with the circle on the coordinate plane. Teachers should focus on translations and dilations of the circle. Students will recognize that for a dilation, the center point of the circle will not change; only the radius will be dilated. Students will note that with a translation, the new image graphed represents an isometry.

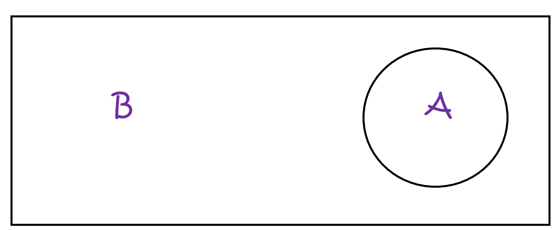
Teachers can have students practice utilizing the circle formula in the coordinate plane by giving them examples to graph as well as giving them graphs to write examples. Students should be able to use the parts of the formula to convert to the standard form of a circle as well. Have students use algebra to manipulate between the two.

Day 7: Geometric Probability

Objective: Use segment and area models to find probabilities of events

This lesson uses geometric relationships within circles and with area to determine probability of certain events. Students should be familiar with the formulas for area of basic figures prior to this lesson. The teacher will want to stress in this lesson that the relationship of probability is part to whole. The various types of probability surface when students distinguish between what represents a part and what represents a whole.

When introducing probability with area, teachers may want to begin with a simple example such as the following:



The teacher should discuss the individual area of each figure and the formula used to find area. Then, the teacher should introduce the idea of *randomization. A point can be chosen at random anywhere in the figure. After discussing the meaning of probability as part to whole, the teacher can introduce many scenarios such as the following:*

1. What is the probability that a point will lie in region B?

Students will discuss the fact that region B represents the entire rectangle, so the probability is 100%.

2. What is the probability that a point will lie in region B but not in region A?

Students will discuss how to find the area of region B not including region A. After discussing this verbally, students should recognize the mathematical relationship of region B – region A. This relationship represents the "part" in the probability, and region B represents the "whole".

3. What is the probability that a point will lie in region A?

Students will discuss region A as representing the "part" in this scenario, and region B again represents the "whole".

Part two of this lesson involves probability with circles. This is an opportunity for teachers to reinforce the concepts of central angle, types of arcs, and properties of triangles formed in circles. Teachers can introduce simple probability examples such as finding the probability that a point will lie in a central angle based on the relationship of the angle to the 360 degree of the entire circle. Teachers can introduce similar examples using arc measure and arc length by inquiring about the probability of a point falling on the outside rim of a circle. Teachers can also ask students questions about the probability that a point may lie in an inscribed figure as opposed to outside the figure inside of a circle. This takes students back to the fact that area probability involves finding the area of each individual part in relationship to the area of the whole circle.

Teachers can close this lesson with several algebraic examples that reinforce the skill of setting up probability for different scenarios. Explain to students that tomorrow's lesson will also involve probability and that the focus will be on probability of events occurring.

Day 8: Probability

Objective: Find the probability of an event using various methods

This day will be a continuation of the previous, but with more focus on basic probability involving permutations and combinations. Students should have some prior knowledge of the content from middle and elementary school math, but have not discussed probability in Algebra I or Algebra II. Students may not have discussed the meaning of a factorial in earlier math classes. Teachers may want to reinforce this concept through a warm up introduction or as an introduction to deriving the formulas for permutations and combinations.

Teachers will want to stress in this lesson the difference between a permutation and combination when discussing probability: if the order matters, you are performing a permutation; if the order does not matter, you are performing a combination. An engaging way to introduce this relationship would be to use the Deck Of Cards. See "strategies: introductory activities: 3". Students will analyze different scenarios to determine the probability of various events. After introducing the difference between permutations and combinations, teachers will want to introduce the following formulas:

- A factorial is a whole number multiplied by all of the whole numbers less than the number. Example:
    - 4! = 4 x 3 x 2 x 1
    - 9! = 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
- The number of permutations of n objects taken r at a time is
- The number of combinations of n objects taken r at a time is

One way to check for student understanding in this lesson is to have students come up with their own probability scenarios. Have students work with a partner to design a scenario that can involve both permutations and combinations. Students can create questions to go along with their scenario and have their partner or another group answer the questions. Have students create content posters of their scenarios so that you can continuously revisit them as a review of the concept.

Classroom Strategies

In addition to the discovery constructions, proofs, and partner activities described in this unit, I have included major activities that will help to reinforce the content of the lesson and add rigor to the unit overall. This unit will not serve its purpose if the teacher is not able to keep students engaged. There is a lot of vocabulary that students will need to understand as well as many relationships, formulas, and rules that students will need to know in order to be successful. The following activities act as formative and summative assessments of student understanding throughout the unit. Teachers may choose not to use all of the activities, however, it is strongly encouraged that teachers utilize the culminating project to help build student understanding.

Culminating Project

The center of this unit is the running project that students will develop as they learn the new content. This project will help students to prepare for the end of the course by constant recall of prior knowledge as well as incorporation of new content. In order for the project to be effective, the teacher should introduce the project at the beginning of the unit. This way, students understand their ultimate goal for the unit. This also gives the students a chance to truly build on what they are learning as the unit progresses.

Create Your Own Space

In this project, you will put your creative minds to use by becoming an interior designer. You will be responsible for designing a space of your own. Your space can be any of the following: bedroom, back yard, restaurant, entertainment room, business office, etc. You will decide what type of space you wish to create and get confirmation of your idea from your teacher.

Consider real life use of your space as you create your design. Would the space make sense if it were recreated in real life? For example, should an end table be the same size as a couch, or should you put a table in front of your door? Make sure that your space is livable when you are finished.

Your space will have the following requirements:

1. Created on a rectangular grid 11 x 17 paper size. Determine the scale for your space. Does 1 block represent 1 foot or 2 feet, etc?

2. At least ten individual figures of your choice. Your space must include transformations of the figures stated in requirement #3.

3. Two of each of the following transformations: translation, reflection, and dilation. You must include the symbolic representation of the rule for your transformation. You must transform the following figures:

*Circle: $x^2 + y^2 = 16$

*Square: (3, 3), (3, -3), (-3, 3) (-3, -3)

*Triangle: (0,0) (0,4) (3,0)

4. Construct one of each of the following as part of your design using the circles in your space: tangent, secant, arc, inscribed angle, and chord. Determine their lengths or measures using coordinate geometry: distance, midpoint, or slope formula.

5. Calculate the area of each of your figures in your space

6. Create 5 probability questions for your space: 2 circle probability, 2 area probability, and 1 of your choice.

7. Color and decorate your space so that your audience understands your design.

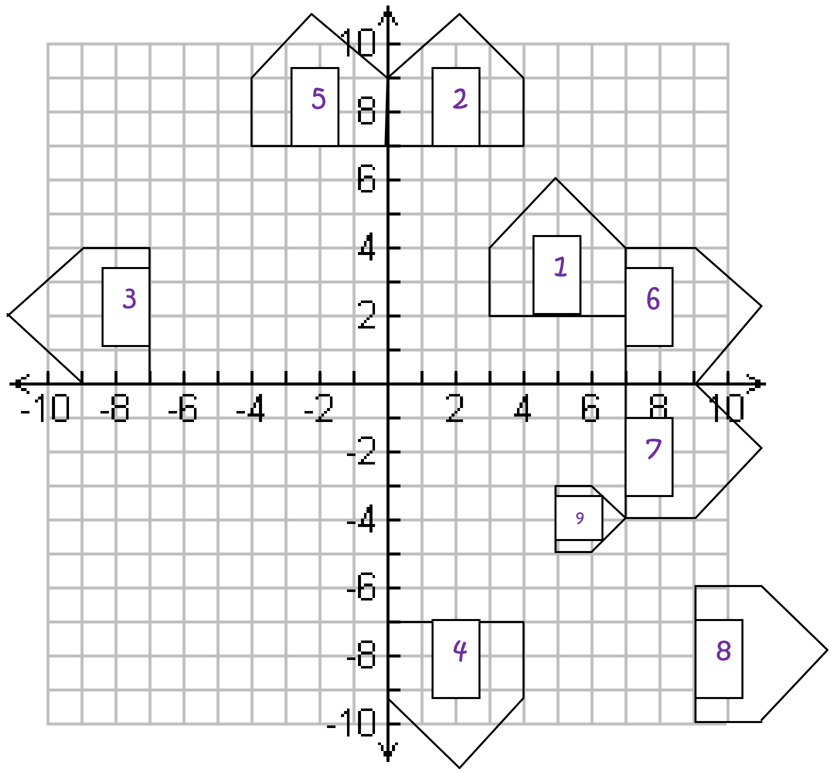
Introductory Activities

1. Moving House: Students will review properties of transformations in the coordinate plane by completing this activity. Students will be given the following: House PENTA: P(3, 2) E(7, 2) N(7, 4) T(5, 6) A(3, 4)
   Students will then perform the following consecutive transformations using house PENTA.:
   1. Move left 3 and up 5
   2. Rotate 90 degrees counterclockwise
   3. Reflect in the line y = x
   4. Reflect in the origin
   5. Rotate 270 degrees counterclockwise
   6. Reflect in the x axis
   7. Move right 3, down 6
   8. Dilate by r = ½

Teachers should remind students that they are to use the previous transformation in order to complete the next. It will be helpful if students graph as they go in order to determine if any of their houses are overlapping. Teachers should explain that the houses may share sides, but none of them will be on top of each other. In order to raise the level of thinking, teachers may want to ask students to answer follow-up questions, such as the following:

1. Which transformations represent isometries? How do you know?
2. Observe house 2 and house 5. Which coordinates do they share? How did this happen?
3. Which houses would represent a rotation of house #2? How do you know?

If done correctly, students should get the resulting graph:



Teachers may also want to increase the rigor of this activity by having students to write the verbal representation of the rules in symbolic form as well. Each rule written in symbolic notation would translate to the following:

1. (x, y) → (x – 3, y + 5)
2. (x, y) → (-y, x)
3. (x, y) → (y, x)
4. (x, y) → (-x, -y)
5. (x, y) → (y, -x)
6. (x, y) → (x, -y)
7. (x ,y) → (x + 3, y – 6)

Teachers should have already had a conversation with students about the reasoning behind the symbolic representations of each rule.

2. Chocolate Chip Pi: Students have heard of the number pi and have used it since middle school. Very few students actually know where this number comes from. We will discover the number pi by completing the Chocolate Chip Pi activity as outlined by Dr. Tim Chartier[3]. This activity will go along with the theme of discovering the math behind the math by encouraging students to understand the

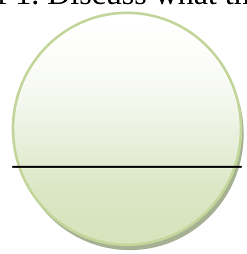
purpose and the origin of pi. This would also be a great Pi Day activity at any level. The contents of this activity are as outlined:

Objective: Students will approximate the value of pi by finding the area of ¼ of a circle under the curve on the coordinate plane.

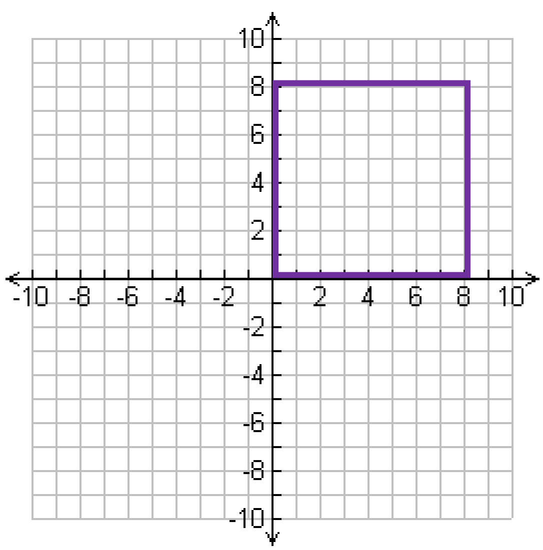
Set-Up:

1. Discuss the area formula with students. This should be review. Give students several different sized circles with various radii to determine area for practice. Then, discuss the area of a circle that has a radius of 1. Discuss what this would mean for ¼ of a circle.

If the radius of the circle is 1, then the area would be $\pi(1^2)$ which is just π. If the circle is divided into four equal parts, then the area of one fourth of the circle is $\pi/4$.



2. Discuss area on the coordinate plane starting with a square. Allow students to derive that if they count the number of squares, it will be the same as finding the area using base multiplied by height.



Students will recognize that the square is an 8x8, which means the area is $64u^2$. Students will also recognize that if they count the grid squares, there are 64 of them.

3. Transform the concept of the area on the coordinate plane to a quarter of a circle. Have students prove the relationship by calculating the area of the entire circle by

finding the radius of the circle, then dividing this area by four to represent a quarter of the circle.

Chocolate Chip Pi

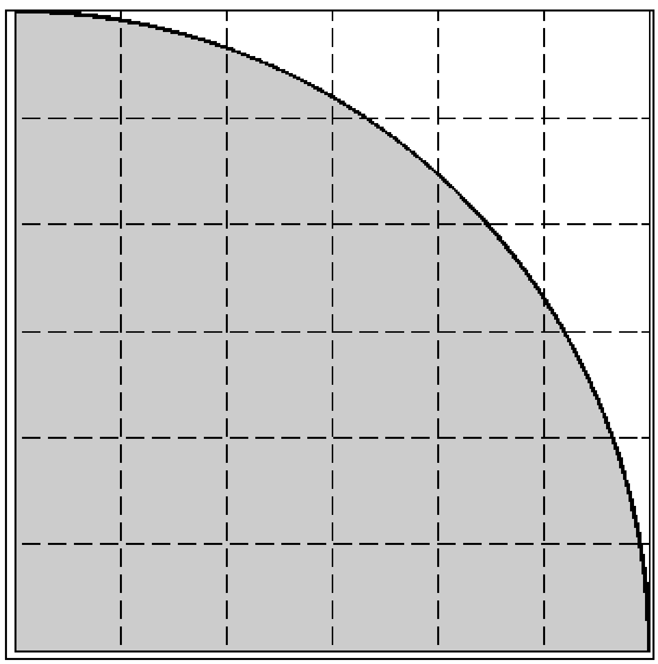
Mathematics through Popular Culture **Tim Chartier – CTI 2011**

Let's estimate the area (gray shaded region) under the curve using chocolate chips.

Algorithm: For each square, if the upper right-hand corner is in the shaded gray region, place a milk chocolate chip; else place a white chocolate chip in the square.

Estimate = 4(number of milk chocolate chips)/(total number of squares)

How close did you get?



This activity will give students an opportunity to see the relationship between pi and the circle as just an number. One problem that students often have is recognizing that pi is just a number, and not a variable. For more resources and enhancement for students, you may want to visit Dr. Chartier's blog[4] at the following website: http://forum.davidson.edu/mathmovement/2011/09/30/chocolate-chip-pi/

This site will give students an opportunity to see how the activity works complete with pictures. You may also create a larger grid with the same curve and have students repeat the activity. Students will recognize that the larger the grid, the better the approximation of pi.

3. Deck of Cards: In this activity, students will complete permutation and combination activities using cards from a deck. This activity allows students to collaborate and distinguish between how to perform a permutation and how to perform a combination. The activity involves a worksheet from the following website[5]: http://www.teachengineering.org/collection/drx_/activities/drx_probability/drx_probability_activity3_worksheet.pdf

Teachers may want to have students work with a partner or in small groups in order to complete the activity. To prepare in advance, teachers will want to have a deck of cards available for each group so that students will be able to perform the permutations and combinations by hand in order to verify the work algebraically. This will again help to reinforce the connection between symbolic and graphical representation.

Conclusion

This unit does not include a summative assessment other than the culminating project, however, teachers may want to also include some type of assessment or smaller formative assessments throughout the unit in order to test student understanding of the content algebraically and geometrically. A focus on vocabulary is also critical to this unit.

As I teach geometry from year to year, I continue to consider ways to help students see how this math works. Geometry teachers should always keep in mind that this math is a completely new way of thinking for most students. Following the Common Core Standards helps teachers to dive into the mathematical practices that students should exhibit while learning the content. This is useful when considering the best teaching practices for enhancing student success in the geometry content. Throughout this unit I kept a continuous focus on student centered activities that I know will engage students in the content and help to add value to what they are learning. I chose to create this unit because of its level of rigor, and the ability to incorporate all of the content of the course based on what students should have in their prior knowledge bank. I hope that through my thoughts and ideas I am able to inspire other teachers to think about the way that students learn best.

Notes

1. Rogers, PEAK Teaching for Excellence, 2008
2. Sousa, How the Brain Learns Mathematics, 2008
3. Chartier, Math Through Popular Culture: Chocolate Chip Pi, 2011
4. Chartier, Math Movement: Chocolate Chip Pi, 2011

5. TEACH Engineering Activities: Probability, 2011

Resources

Bibliography for Teachers

Chartier, Tim. "Chocolate Chip Pi." Math Through Popular Culture Seminar, Charlotte Teacher's Institute. September 29, 2011

Dr. Chartier implemented this activity with the Charlotte Teachers Institute seminar. As a group, we constructed the Chocolate Chip Pi to see how close of an approximation we could derive from the formula for calculating the area under the curve.

Chartier, Tim. "Chocolate Chip Pi" Math Movement. http://forum.davidson.edu/mathmovement/2011/09/30/chocolate-chip-pi/ (Retrieved November 18, 2011)

Dr. Chartier shares background information on his blog on how the Chocolate Chip Pi activity works. The blog breaks down the logistics of the activity by first showing how the area approximation works with a chocolate bar and then showing how the approximation error decreases as the size of the grid increases.

Rogers, Spence. *PEAK Teaching for Excellence*. Conifer: PEAK Learning Systems Inc., 2008.

This teacher's manual is a set of instructional strategies and research based on the principles of PEAK. The book discusses assessment, engagement strategies, specific math strategies, differentiation, and cooperative learning strategies.

Sousa, David. *How the Brain Learns Mathematics*. Thousand Oaks: Corwin Press, 2008.

This book focuses on how the brain develops math concepts as students grow from birth to adolescents. The book presents strategies for teaching math at each level of growth as well as strategies for helping student to retain content based on research.

TEACH Engineering. "Probability Activities."Teach Engineering. http://www.teachengineering.org/ (November 18, 2011)

This website provides resources for teachers to implement various lessons. The website is specifically designed for k – 12 math and science classrooms. The website provides research on best practices for teachers as well in the classroom.

Reading List for Students

Chartier, Tim. “Chocolate Chip Pi” Math Movement. http://forum.davidson.edu/mathmovement/2011/09/30/chocolate-chip-pi/ (Retrieved November 18, 2011)

Have students view and discuss this blog by Dr. Chartier as it will help to reinforce the purpose of the Chocolate Chip Pi activity. Teachers may want to have students do the activity with the Hershey’s bar before they do the activity with the area under the curve grid. The blog also gives students more opportunities to see how math works in the real world through other blog posts by Dr. Chartier.

Classroom Materials

Culminating Project: Create Your Own Space

11 x 17 paper

Construction paper

Compass

Protractor

Ruler

This project involves constructions. Teachers should help students to refresh their memory of constructions and coordinate plane graphing through review and warm up activities.

Introductory Activities: Moving House

8.5 x 11 graph paper

Ruler

Teachers should remind students that they will need to complete the transformations from the previous new image in order to produce consecutive images. Students should graph their images as they complete the activity to make sure they do not overlap.

Introductory Activities: Chocolate Chip Pi

8.5 x 11 grid with shaded curve (see activity)

White and brown chocolate chips

Teachers may want to have students complete this activity in groups in order to conserve materials. Teachers will also want to create various size grids in order to compare pi approximations.

Introductory Activities: Deck of Cards

Card decks

Activity sheet

This activity can be completed in partners or groups to conserve materials and for better discussions.