Math is Beautiful! Creating mathematical art

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Introduction

All human beings desire to create beauty – it is what makes us human – and yet, what is beauty? Are there specific components that all things beautiful possess? If so, why or how does it make it beautiful? According to Kant, a "judgment that something is beautiful or sublime is made with the belief that other people ought to agree with this judgment - even though it is known that many will not." [1]

Art is a field of study that scrutinizes "beauty." Objects of art contain lines, shape and color. These components combine to create form, texture, and a sense of space wherein there are sections of light and dark. There is a sense of rhythm, balance, and unity within the object that is both emotional and mathematical. "Light was considered as the most beautiful revelation of God" during the Middle Ages. [2] Without light, there would be no color, as manifest in this rose window at Notre Dame.

Many artists intuitively apply mathematical concepts of symmetry, balance, color, proportion and perspective without realizing they are using math. Mathematical proofs also have a sense of rhythm, balance and unity. There is a satisfying sense of completeness in the line of thinking and the shape of the argument. This elegant proof by Pythagoras has "beauty in method"[3] because of its simplicity. It would seem, then, that logic, as well as emotion guides the creative process. It all depends on how you look at it.

Background information

What is beauty? Beauty is a concept within the philosophy of aesthetics used to describe an emotion coupled with visual or aural perceptions that give pleasure. The word, *aesthetics,* derives from the Greek *aisthetikos*, meaning "of sense perception." (About.com) and became a philosophical topic under the tutelage of Immanuel Kant. "Beauty" is not relegated solely to the realm of art, though pleasing visuals are often "beautiful" and for many artists, their purpose for creating art is to make something beautiful. When one uses the word "beautiful", it is the "aesthetics" of the object being considered, something beyond the physical presence or components, that requires a dialectic of shared perceptions. There is often no consensus or definitive explanation. Hegel comes closest in identifying the aesthetic aspects of beauty in art.

> "Art proper, for Hegel, is the sensuous expression or manifestation of free spirit in a medium (such as metal, stone or color) that has been deliberately shaped or worked by human beings into the expression of freedom…. Classical art, Hegel contends, fulfills the concept of art in that it is the perfect sensuous expression of the freedom of spirit. It is in classical art, therefore—above all in ancient Greek sculpture (and drama)—that true beauty is to be found. "
> (http://plato.stanford.edu/entries/hegel-aesthetics/#IdeBeaSuc)

Some common mathematical relationships (such as "golden ratio", repetition, symmetry) that regularly occur in Greek sculpture and Moorish tilings are perceived as "beautiful" due to their perfect rendering of sensuous expression. These art forms give people pleasure on many different levels of perception. Therefore, they set the soul free and the viewer experiences transcendence. Let your eye or your finger trace the gold overlay in this tiling from the Alhambra; it is mesmerizing and soothing.

What is the ultimate arrangement of shapes combined with hue/brightness/contrast considerations that makes a work of art "beautiful?" For Hegel and other classical philosophers, it would be the use of the "golden ratio." A golden ratio is denotated by the Greek letter "phi," which is an irrational number. The ratio is determined by the relationship of a divided line segment wherein the longer portion of the segment compared to the shorter portion is equal to the relationship of the whole line segment to the longer portion of that segment. Golden polygons (rectangles, triangles, pentagons) would have this ratio present either in the edges of the polygon or in the diagonals. Idealized sculptures of people would have this ratio apparent in the grouping of facial features or lengths of their limbs. In visual 2d arts, the ratio would be apparent in the grouping of objects of interest, relative sizes of the groups/objects, color/hue patterns according to the color wheel and other relationships the artist wishes to emphasize.

Rationale

Gardner's Multiple Intelligence approach to classifying intellectual strengths and proclivities is often used in educational settings to help individuals have success in communicating ideas to others. Having strong spatial abilities is an advantage as this intelligence spans many disciplines such as math, engineering, computer science, visual arts, dance and sports. Many of my students are into computer games and simulations on their Wii. In order to create these entertainments, a programmer must envision a 3d world and write a mathematical code to represent the image on a two dimensional screen through the use of a long and complex series of binary code – a programming language. My students are also very involved in fantasy novels and imaginary worlds. How did George Lucas go about creating life-sized models of worlds or costumes for a character in his movies? I wondered how my students could design a "set" for a play from one of their favorite scenes in a book? What would they need to be able to do to accomplish this?

Visual Literacy (Jo Anne Vasquez, Michael W. Comer, and Frankie Troutman) is a growing field in education that encourages the use of pictures, drawings, constructs, and concept maps to enable students to synthesize information and create a new product or new interpretation of an idea.

> "In a recent report on skills needed in the American workforce, the National Center on Education and the Economy (2008) stated that although so-called basic literacy skills are still necessary for adult workers, *they are not sufficient* for people to become "knowledge workers" in a globally competitive marketplace. Rather,
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> > this is a world in which comfort with ideas and abstractions is the passport to a good job, in which creativity and innovation are the keys to the good life, in which high levels of education—a very different kind of education than most of us have had—are going to be the only security there is. (pp. 6–7)
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> Teachers must come to grips with this growing emphasis on what students can *do* with knowledge rather than on just what units of knowledge they have… To achieve this high level of cognition requires the abilities to process, organize, and assimilate new knowledge." (2010 NSTA, p. xii).

The concept of "visual literacy" is evident in Daniel Pink's best-seller, *A Whole New Mind*, announcing the Machine Age of the Industrial Revolution is out and the Conceptual Age is here. Outlining six fundamental abilities essential for professional success in any field, his focus is on the development of "right-brain" thinking strategies that tap into creative synthesis and complement the left-brain's factual and logical data bases. The left-brain capabilities recreate what is already known. Right-brain capabilities add innovation and personalization to existing products, making them more desirable and "pleasing." Left-brain thinking in learning and economic activities can be replicated by machines – most notably, computers and robotics. In order to thrive, the human populace must develop the underutilized potential of the right side of the brain. This is where developing visual literacy can help with a very *left-brained* subject, mathematics. The visually literate artist leaves the computations and proofs for computer programs and uses technology to create simulations and visualizations of the universe, molecular structures, and consumer products that satisfy a need to realize the imagined, the impossible or even, the ridiculous. Using the *right brain*, the mathematical concept, "phi," is more than just an irrational number, it can be applied to products that have a form and function which is beautiful.

Participants and Pacing

This unit is developed for the 4th grade Horizons class where the students are functioning two or more years beyond their age mates in mathematics. The class text is the Holt Middle School Course 1 Math, essentially a pre-algebra text. These students need horizontal enrichment in geometry and hands-on activities as their spatial abilities are not fully developed at this age. This unit on Plane Geometry also eliminates the need for rote memorization (at which my students excel) and focuses on problem-solving with divergent and open-ended solutions. The unit incorporates the math concepts of ratios, fractions, proportion, rotation, symmetry, translation, reflection, similarity, and congruency. The unit should last about four weeks with 1 hour of instruction and construction, daily. Two of those weeks will be math concept intensive. The final two weeks will be the artistic application of the math concepts learned, the end product being an acrylic painting or fabric wall hanging along with a portfolio of paper and pencil geometric constructions and tilings.

My students attend an all-county magnet school for the gifted and generally represent all races, religions, and genders. The biggest challenge is their fine motor skills. My students can barely write in cursive and have just gotten beyond stick-figured drawings. Intricate, detailed drawings may be beyond their physical capabilities. As such, I have enlisted the help of my school's Art teacher to aid in technique and I have also secured the additional support of a local Art outreach program – ArtStart – where a resident artist comes to my classroom for an hour to help my students create modern art in the style of

Sol LeWitt or Picasso while I aid with the mathematical reasoning for their color, shape, and other composition decisions.

Objectives

The main objective of this unit is to create a portfolio of work in geometric designs and one acrylic painting (or fabric wall hanging) that utilize a majority of the math and art concepts introduced in this unit of study. The students will learn the vocabulary of plane geometry and apply it in the context of patterns, tiling and the golden ratio. Students will use pattern blocks, architectural templates, a ruler and protractor to construct monohedral and dihedral tilings that demonstrate rotation, translation, and/or reflection of a prototype shape and incorporate a conscious application of proportion in shape, color and/or frequency. They will use the properties of a regular pentagon to create aperiodic tilings, as well. It is further hoped the student will understand the underlying rules for topological disks to explain why their particular prototype "works" as a tiling piece. My students will also examine abstract art such as cubism, minimalism, and neoplasticism as an extension of their Golden Ratio project with their Art teacher and ArtStart resident artist. Students will learn how to construct "golden" polygons using compass and straight edge for use in their "Minimalist installation" and master the art of writing explicit installation directions for others to follow. They will also apply the concept of translation on coordinate grids to create cubist art. In addition, they will visit a local art museum's exhibition on Modern Art to incorporate the ideas of artistic composition: color and pattern, intensity of hue, proportion, and unity in their final products of acrylic paintings or fabric/ paper wall hangings, proving "Math Is Beautiful."

Strategies

The main strategy I will employ is Learning Immersion. The students will learn by doing. I will teach them how to use a compass and straight edge to create golden polygons. The Art Teacher will show them how to apply these shapes in an artistic manner. Having used the concepts of proportions and ratios in other disciplines (architecture and engineering in Math, positive/negative spaces in Art class) the students will be working in small ad hoc cooperative groups playing with shape and composition and then, sharing their knowledge and skills to help each other justify their choices mathematically. The students will visit an Art Museum to see where and how mathematics was applied in famous artwork and take these new techniques and apply it to their own work. I will employ the "red filter" technique in quilting to enable students to see light and dark values objectively. Students will be immersed in a variety of art forms that have mathematical principles as a design element. Moorish tile designs, Penrose tessellations, and "Modern" Art will be employed. A Learning Immersion Approach is optimal at this age as the students have few preconceived ideas of what they can and cannot do. Students at the early adolescence stage are interested in learning new skills that will expedite and enhance their entry into adulthood.

A second strategy I will employ is "Writing Across the Curriculum." Students will keep a Math/Art journal of their thoughts and feelings towards the project and their product. Students that can articulate their ideas on paper are better able to express themselves in a public medium. I like the idea of a dialectic journal as an extension of our Writer's Workshop where students get feedback from peers on how well their math and art ideas translate and are understood by the viewing public. I would collect these journals after daily self reflections, for review, and have a weekly peer review entry for the dialectic sections. Journaling will allow the student to work through design and math issues without judgment. The peer dialectic will give the students needed grounding as to the efficacy of their attempts to merge mathematical principles with visual beauty.

Another venue for practicing their writing skills would be short, detailed instructions, a la Sol LeWitt, in the creation of a color installation executed by someone other than the artist. This task fits in nicely with North Carolina's emphasis on "content writing," a necessary skill to be mastered in 4th grade, showing that the student can write explicit, sequential, and clear directions for a task.

Unit Ramp-up

I needed a 'hook" to capture my students' imagination and give them a chance to be successful in creating something real from an imaginary source. The Walch Roth Math series, *Build It!* gave me the opening I needed. We were reading *Call it Courage,* by Armstrong Sperry, wherein a young boy must survive on a deserted island and find the means to get home. My students loved the adventure story. They also loved how, suddenly, they were stranded on a deserted Pacific island due to a story from their math and engineering unit! They had to draw up plans and build a structure (no tools other than rulers and scissors, scavenged materials from home) they all could fit into. My students would activate the right side of their brains through concrete applications of their spatial intelligences in this engineering process. Now is the time to use this can-do spirit and creativity for a more aesthetic product.

I will begin the Math Is Beautiful! unit with a few classes in how to construct golden rectangles and golden cuts. Students will make several paper templates of this concept and move on to the golden triangles and pentagons. Students will cut out tag board renditions of their “golden shapes” and save them in a re-sealable plastic bag to use as tracing patterns for their art project. This will take 2 hour-long lessons.

Teacher background knowledge –

To construct a “golden” rectangle:

- Draw a 2” line segment with points *A*, *B* on opposite ends.
- Place the sharp end of the compass on *A* and open the compass so the pencil tip is at point *B*.
- Keeping the sharp point in place at *A*, swivel the compass pencil to mark a small arc (*in red*) directly above and directly below the line segment.
- Move the point of the compass to point *B*, making sure the pencil tip touches point *A*, swivel the pencil to mark directly above and below the line segment. These new arcs should intersect with the previous markings to make curved “x’s” above and below the line segment.
- Connect the markings above and below using a straight edge. You have made a perpendicular bisector of line segment *AB*. This will be one side of the “golden” rectangle. Label the intersection of the bisector with the original line segment, *m*.
- Using the compass, take the measure of segment, *Am*. Place the point of the compass on *m* and make a mark directly above *m* along the bisector. Label this point of intersection *C.*
- Using the measure of *Am*, place the point of the compass at *B* and make an arc above *B.* Measure the distance between *A* and *C* with the compass. Then place the point at *m*, to mark an intersecting arc (*blue)* directly above point *B.* Label this point *D*.
- Connect points *B* and *D*, then *D* and *C* to make a square.
- Take the measure of distance *AC.* Placing the point of the compass on *B*, mark the distance, *AC*, along the original segment of *AB*. Label this *E.* Mark the same distance from point *D* along segment *CD* and name this new point, *f*.
- Connect *Ef* and *fC.* The resulting rectangle, *EfDB* is “golden.” The length of A*B* over *EB is* equal to *phi* (or approximately 1.6:1.0).

A “golden” right triangle, to demonstrate the “golden cut,” follows steps 1-7 without connecting *D* to *C*. Instead, connect *C* to *A*. The distance from the midpoint, *D,* to *B* is used to measure the distance from point *A* along *AB* to point *E.* The arc drawn from point *A* starting at *E* should curve through *AC* at the midpoint, *D,* creating the “golden cut.”

“Golden” triangles are isosceles triangles wherein the ratio of the side to the base is *phi*. An acute golden isosceles has interior base angles of 72^o. An obtuse golden triangle has an interior angle of 108^o. Both of these are easily discovered using the diagonals of a regular pentagon (see figure 1).

Figure 1

Teacher background on artists and examples of art used for exemplars -

Sol LeWitt: “He reduced art to a few of the most basic shapes (quadrilaterals, spheres, triangles), colors (red, yellow, blue, black) and types of lines, and organized them by guidelines he felt in the end free to bend. Much of what he devised came down to specific ideas or instructions: a thought you were meant to contemplate, or plans for drawings or actions that could be carried out by you, or not.

Sometimes these plans derived from a logical system, like a game; sometimes they defied logic so that the results could not be foreseen, with instructions intentionally vague to allow for interpretation. Characteristically, he would then credit assistants or others with the results. “ http://www.nytimes.com/2007/04/09/arts/design/09lewitt.html

Pablo Picasso: By 1910, Picasso and Braque had developed Cubism into an entirely new means of pictorial expression. In the initial stage, known as Analytical Cubism, objects were deconstructed into their components. In some cases, this was a means to depict different viewpoints simultaneously; in other works, it was used more as a method of visually laying out the FACTS of the object, rather than providing a limited mimetic representation. The aim of Analytical Cubism was to produce a conceptual image of an object, as opposed to a perceptual one.

http://www.artchive.com/artchive/P/picasso_analyticalcubism.html

Piet Mondrian: Neo-Plasticism is a Dutch movement founded (and named) by Piet Mondrian. It is a rigid form of Abstraction, whose rules allow only for a canvas subsected into rectangles by horizontal and vertical lines, and colored using a very limited palette.

http://www.artcyclopedia.com/history/neo-plasticism.html

Theo Van Doesburg (1883 - 1931): Dutch painter, designer, and architect, Theo van Doesburg adopted an early style that was highly influenced by Impressionism, Expressionism, and Cubism. In 1915, he met Mondrian and switched his approach to that of total abstraction. He co-founded the De Stijl movement in 1917 and devoted his career to spreading the group's ideas. During the 1920's he traveled across Europe promoting his artistic beliefs, settling only for a short time from 1922 to 1924 to teach at the Bauhaus. In the same decade, he switched from his horizontal and vertical line compositions and began incorporating diagonals in his series, Counter-Compositions. Doesburg titled this new style, Elementarism. He moved to Paris in 1929, set up a studio in the suburbs and published a manifesto of Concrete art a year later.

Roger Penrose - The first Penrose tiling (tiling P1) is an aperiodic set of six prototiles, introduced by Roger Penrose in a 1974 paper,[14] but is based on pentagons rather than squares. Any attempts to tile the plane with regular pentagons will necessarily leave gaps, but Johannes Kepler showed, in his 1619 work *Harmonices Mundi*, that these gaps could be filled using pentagrams (viewed as star polygons), decagons and related shapes. Penrose subsequently reduced the number of prototiles to two, discovering the kite and dart tiling (tiling P2) http://en.wikipedia.org/wiki/Penrose_tiling

Activities – Applying the concepts of Proportion and Balance

Previous to this curriculum enrichment activity, I would have covered Chapter 7 and 8 in the Course 1 Math text concerning topics on similarity, proportions, ratios, and percentages and tessellations.

Teacher resources – Use the URL for the artists for examples of cubism, neoplasticism, elementarism and minimalism. I used my Art teacher's SRA texts and the internet for information on color theories. I also used my Art teacher's SRA texts on Art, that were developmentally appropriate for my students' fine motor abilities, to gauge the skill and quality of possible products I might expect. If accessible, call your local Art Museum to see if they have artist residency programs that place local artists in schools as an outreach program. Also ask if they have visuals of the art on display to use as teaching materials.

Beginning – "Math and Art together!"

Part of my curriculum enrichment is to teach geometric constructs. So, as an opening task, I would introduce my students to *Neoplasticism* and *Minimalism* art forms using the works of Theo van Doesburg and Piet Mondrian. I would ask them to identify the shapes in the artwork.

Piet Mondrian, 1921. Neoplasticism. Composition with Large Blue Plane, Red, Black, Yellow, and Gray. Oil on canvas 60.5 x 50 cm (23 3/4 x 19 5/8 in) Dallas Museum of Art

Writing Prompt #1 - We would informally talk about the use of color and whether or not the piece was pleasing to look at. We would find elements that were the same in each artist's work, such as rectangles, lines, and primary colors. Then I would have my students describe how they thought the artist created the piece and have them write "directions" to go along with the art, focusing on specific math terminology and directional language as part of the "writing across the curriculum" regional initiative. I would post these vocabulary words and directional terms as an anchor chart. Student volunteers would read their directions and the class would guess which art work was being described. Students would edit their papers as needed.

Theo van Doesburg , 1917 *Composition VII (the three graces)* Neoplasticism

Writing Task #1: Organize the sequence of steps to create this art work. How many different ways can you organize the shapes – size, color and their directionality?

Teacher information - *<u>Neoplasticism</u>* - the belief that art should not be the reproduction of real objects, but the expression of the absolutes of life. To the artist's way of thinking, the only absolutes of life were vertical and horizontal lines and the primary colors. It is a rigid form of Abstraction, whose rules allow only for a canvas subsected into rectangles by horizontal and vertical lines, and colored using a very limited palette. To this end neoplasticisism only used planar elements and the colors red, yellow, and blue.

Theo van Doesburg,1925. *Counter-Composition XVI in dissonances*. Elementarism

Question: How is this different from Mondrian's piece? Is the difference worth a friendship?

Writing Task #2 – Write a persuasive essay on why the change in the art represented a change in allegiance, or not.

Teacher information - *Elementarism*: A modified form of Neo-Plasticism propounded by van Doesburg in the mid-1920s, notably in a manifesto published in the journal *De Stijl* in 1926. Whilst maintaining Mondrian's restriction to the right angle, Elementarism abandoned his insistence on the use of strict horizontals and verticals. By introducing inclined lines and forms van Doesburg sought to achieve a quality of dynamic tension in place of Mondrian's classical repose. Mondrian was so offended by this rejection of his principles that he left De Stijl.

Math Lesson 1 - "What's Golden about this rectangle?

In this class I will teach compass and straight edge construction of a "golden" rectangle starting from the construction of a perpendicular bisector of a line segment, to the creation of a square from this bisector, and then discovering the "golden" cut to create a parallel line segment to the bisector and connecting the four corners to form a "golden" rectangle. (See diagram in Ramp-up notes). Students would trace this on tag board, cut out as a template for future use, and save in a plastic bag. Students then would use the golden ratio to find whole number lengths for rectangles of similar shape to trace and cut.

Art Lesson 1- "What is artistic license? Does everything have to be exactly the same?" The next class would have the students create a simple design using lines, squares, and rectangles (10 mins), then write directions on how it was composed (30 mins). (See Figure 2, examples of student work) This is the idea behind Sol Le Witt's *Conceptualism Art.*

Then, have each student pass their directions to another student (hiding the original design) to recreate using acrylic paint and a straight edge (25 mins). The next class, students compare results/ discrepancies.

Figure 2

Sol LeWitt – *wall drawing #995*

A current installation is located at the Bechtler Museum, Charlotte, NC

Writing Task #3 – have students practice using specific vocabulary for a minimalist conceptual installation that a classmate will have to create. For example: What kinds of polygons and prisms do you see? Where are they located, how many, what colors?

Math Lesson 2 – "What to group, how to group to create proportional relationships?" Next, I will extend their knowledge of Sol LeWitt's *conceptual minimalism* to create an installation blueprint using "golden" polygons and primary colors arranged in groups that are proportional in size *and* arrangement. This would require my students to use specific and precise mathematical language and thinking processes in the directions given to the craftsmen who must recreate the artist's vision. Not all of my students could attempt this complex a task, but sometimes they do surprise me with what they think is "not *too* difficult!" Students trace equivalent golden rectangles and triangles on tag board using the base ratio of 1.6::1 and cut out. Then arrange them in proportional groupings on 12"x14" construction paper, trace their design, and then mathematically justify their proportional arrangements. This would take about an hour. (see student example, figure 3). Figure 3 Students will paint their drawings in a subsequent Art class.

Art Lesson 2 -
Students will learn about monochromatic shadings in Art class. They will take new drawings of "golden polygons within a golden field" and shade the objects using monochromatic values from light to dark.
Resources/materials: overheads of art work created by Mondrian, van Doesburg, Vasarely, and Le Witt. (or, saved downloads from URLs in a file for a SmartBoard), 3" wide strips of red cellophane.
Art Lesson# 3 – Coordinated Cubism
Students use a photo of themselves to sketch their portrait onto heavy drawing paper that has been lightly lined with a 1" coordinate grid. Center the face at the origin. Draw a second portrait on another gridded paper keeping the main features of the face as close as possible to the same coordinate points in both drawings. Color- wash one portrait in cool colors and the other in warm colors. Allow to dry. Slice the cool portrait into strips along the horizontal gridlines. Cut vertical slits along the vertical gridlines in the warm portrait – be *sure not to cut* through to the edge of the paper. Weave the cool strips over and Figure 4 under across the warm strips according to the coordinates. Features will match up, slightly off, and have a Cubism appearance. (see student example, Figure 4). Two days, 60 mins each, to complete.
Art Lesson #4- "What about other media to show Conceptual Art?"
The students could draw or use mixed media (such as collage) to create a proportional design according to their blueprint. To minimize student time, I would cut out an assortment of golden rectangles and triangles to trace and color (with colored pencils and markers) or to cut out of construction paper, newspaper, or fabric and paste into place. This would take 2 hour-long class sessions.

Part II Activities – The Concepts of Similarity, Translation, and Tiling
Continuing with math lessons in Chapter 7 on tessellations, I would introduce the works of Penrose and Escher and examples of monohedral tilings from Creative Publications' book of *Tessellations.*
Math Lesson 1 - What is "tiling"?
In this introductory lesson, as an extension to Chapter 7-9, tessellation and patterns, Students will look at the properties of "successful" tilings. Teacher discusses common uses of tiles – shower walls, flooring, etc. Are more than one shape used in these tile designs? Students are given an assortment of pattern blocks or use the website, mathforum.org or Science U geometry center, in the computer lab to create a pattern that will tile the plane with no gaps or overlaps. The edges can be irregular as it is assumed the pattern would go out in all directions indefinitely. Students are asked to identify the

center point of adjoining blocks and determine the type of angles that meet there and their measure. Through discovery, students will soon deduce (or you can coach) that the meeting point of the tiling group must add to 360^o. Students trace their proto-type group onto isometric dot paper (or appropriate isohedral grid) and label their first effort #1. Question: What other block shapes will tile? Is there a pattern or chart you can make to show at a glance what will work? What are the fewest number of different combined shapes to use as a prototype? Is there a maximum number?

Art Lesson 2- Balance and Proportion Between Light and Dark Values

Students will view famous modern art paintings, using a red cellophane strip, to "see" which colors are considered dark and which are light to the eye's receptors. They would number the sections of the painting, as a class, and figure out the ratio of light to dark shapes within the compositions. The class would take out their tiling design from Math class and now, color the objects to also show a proportional distribution of light and dark values. This lesson would take approximately an hour.

Math Lesson 2 – What is a regular and irregular Tessellation?

Time to introduce students to vocabulary. Regular tilings or tessellations all use the same regular polygon shape, repeatedly. Semi-regular tilings or tessellations use two or more regular polygons to create a proto-type shape. There are 8 variations of this that work:

http://mathforum.org/sum95/suzanne/whattess.html

Notice the numbering system refers to the number of sides per polygon and describes each shape in the order of its placement around the shared vertex. Triangles are 3, squares are 4, etc. Students will use templates of non-pattern block shapes to find the "optimal eight" and create tessellations. They may discover that one arrangement has a "center".

Math Lesson 3 – Mono or Di!

Tilings that use only one shape are called monohedral tilings. Dihedral tilings use two shapes. Periodic tilings have elements of translation, as well as rotations or reflections. Task: Have students create mono or dihedral tilings using pattern blocks. Trace the shapes onto grid paper – use color to emphasize a pattern. Or, use an online source to create tilings at the computer lab.

Teacher Resources: online interactive tessellations can be found at Science U: Geometry Center [5]; Google "tessellations" to find sites not blocked by your district.

Art Lesson 3 – Stained Glass

Students can create a stained glass effect for their tiling patterns by cutting out geometric shapes from cellophane and sticking them onto clear Contact Paper. Students create a tessellation and transfer their design onto the right side of a 12" x 12" piece of clear

Contact Paper laminate. Students outline the edges of the shapes with black permanent marker and then, let the teacher use an exacto knife and straight-edge to cut the paper backing according to the traced design. (Be careful not to cut through the clear laminate). Students peel up one shape at a time and stick their pre-cut pattern pieces in place. There should be a 2” border around the edge of the design so that black matting can be applied. Hang finished “glass” on the windows. 2 hours.

Math Lesson 4 – Is it a glide, a flip, or a spin?

Introduce the geometry terms that describe movement on a plane: rotation, symmetry, translation. Have students identify which terms apply to their tessellation from the previous lesson.

Math Lesson 5 - How is a “Penrose” tiling different?

Vocabulary: a penrose tiling is aperiodic. This means it has no translational properties. The design cannot slide over on top of another section of the design and match it perfectly. The example is a P1 tiling using pentagons and three other shapes. http://en.wikipedia.org/wiki/Penrose_tiling

Introduce the use of concave polygons in creating this tessellation. Notice the use of the “dart” shape. Does the rule “angles sharing a vertex must add up to 360^{o}” still apply? Notice, also, that the combined dart and kite shapes create a “fat” rhombus.

Task: Have students create their own “fat rhombus” by tracing their regular pentagon shape on an index card, cutting it out, and drawing two sets of diagonals. A “fat” rhombus appears at the top by combining the two obtuse golden triangles, two golden acute triangles appear on each side with an small obtuse golden triangle at the base. Using the rhombus shape and a protractor, create the angle bisector for each of the wider angles on the rhombus (see figure 5). Students can then cut out their “dart” and “kite” shapes to create a Penrose 2 tessellation (figure 6).

Figure 5

Figure 6

http://www.uwgb.edu/dutchs/symmetry/penrose.htm

Art Lesson 4 – Who is MC Escher?

Introduce Escher’s tessellations of irregular polygons. How did Escher create his interlocking shapes? http://www.mcescher.com/Gallery/symmetry-bmp/E67.jpg

Do you see brown horsemen or white? What was the beginning polygon for this tiling shape?

Task: Have students create their own irregular shape to tessellate starting from a square or hexagon, cutting out a piece from one side and taping it, straight edge to straight edge,

onto the opposite side of the square. Cut-outs can occur on every side as long as the piece is reattached to the opposite or adjoining side.
Question: How many shapes are used in this design? Describe the placement of each shape in relationship to another of the same kind – was it flipped, rotated, or made to glide? http://www.mcescher.com/Gallery/symmetry-bmp/E67.jpg
Task: Have students create an irregular, "natural" shape and trace it several times, but still touching, using flips, glides or rotations. Can the "gaps" be filled with another "natural" shape?

Math Lesson 6 - Fun with heptiamonds!
Vocabulary: a heptiamond is a shape created from 7 equilateral triangles (hence the *hept* in the -iamonds name). Students can use rhombi (2 triangles), trapezoids (3 triangles), triangles, or a combination of these to create the design, However, the proto-type disk must have an area of 7 triangles. There are 21 known combinations and only one will not tessellate.

Review the rules for a tessellation. Hand out red (3 triangles), blue (2 triangles), and orange (1 triangle) pattern blocks. Challenge students to find all 21 configurations of a heptiamond. Trace and label color sections with each construct on grid paper for templates. Create four sections of each heptiamond to test if it will tessellate.
Teacher Math Resources **- Tilings, Patterns, and Symmetry**

For background information, I found Keith Devlin's book, *Mathematics: the Science of Patterns* and Grubaum's & Sheppard's book, *Tilings and Patterns* to be most helpful in understanding the math concepts. Devlin's is more geared to modern applications of math concepts and theorems, whereas Grunbaum's is more focused on the actual math and construction behind the concept of tiling. The Creative Publication's classic, *Tessellations*: *the geometry of patterns,* is very teacher friendly with reproducible isometric grids, tiling prototypes, and teaching basic tiling concepts. *Math and Art: an introduction to Visual Mathematics,* by Sasho Kalajdzievski is very helpful for the teaching of golden ratios, golden cuts, compass constructions, and tilings. This book and David Farmer's *Groups and Symmetry: A guide to discovering mathematics* are also useful texts geared for more advanced students who would have access to computer programs to aid with their research and calculations. For art ideas using tiling, patterns and perspective I also used quilting books and wiki sites.

Unit wrap-up - Art Show!

Collect a portfolio of art work created during this unit. Then make a display at school and host a Gallery Crawl or, ask a local coffee shop to display student work to show that Math is Beautiful!

Notes:

[1] http://en.wikipedia.org/wiki/The_Critique_of_Judgment

[2] http://en.wikipedia.org/wiki/Beauty#cite_note-phrase-0

[3] http://en.wikipedia.org/wiki/Beauty

[4] http://www.google.com/images?oe=utf-s&rls=org.mozilla:en-US:official&client=firefox-a&q=Alhambra-mosaics&um=1&ie=UTF-8&source=uni&ct=tile&resnum=4&ved=0CDcQsAww&biw=1280&bih=831

[5] http://www.scienceu.com/geometry

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Examples of student art:

"Coordinate Cubism"

Conceptual Minimalism – a la Sol LeWitt

"Directions and Implementation" using golden figures, lines, primary and secondary colors.

"Writing Across the Curriculum"

Bibliography of Works Cited

Web sources

Aesthetics and Beauty

Houlgate, Stephen, "Hegel's Aesthetics", The Stanford Encyclopedia of Philosophy (Summer 2010 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/sum2010/entries/hegel-aesthetics/>.

Abstract Art

http://www.abstractart.20m.com
http://neoplasticism.com/index2.html
http://en.wikipedia.org/wiki/Theo_van_Doesburg
http://www.artcyclopedia.com/artists/mondrian_piet.html
http://en.wikipedia.org/wiki/Cubism
http://en.wikipedia.org/wiki/Georges_Braques

Color Theory

http://www.worqx.com_ - color palette, color studies, color vocabulary
http://en.wikipedia.org/wiki/Johannes_Itten - facts and links

Tiling and Tessellations

tiling/symmetry
http://www.uwgb.edu/dutchs/symmetry/penrose.htm
http://en.wikipedia.org/wiki/Penrose_tiling
http://www.mcescher.com/Gallery/symmetry-bmp/E67.jpg
http://mathforum.org/sum95/suzanne/whattess.html
http://www.google.com/images?oe=utf-8&rls=org.mozilla:en-US:official&client=firefox-a&q=Alhambra+mosaics&um=1&ie=UTF-8&source=univ&ei=WjZnTIifMYT78AbT9sCzBA&sa=X&oi=image_result_group&ct=title&resnum=4&ved=0CDcQsAQwAw&biw=1280&bih=831
Math in art and architecture
http://www.math.nus.edu.sg/aslaksen/teaching/math-art-arch.shtml

Books

Bezuszka, S, M Kenny, and L Silvey. *Tessellations: the geometry of patterns.* CA: Creative publications. 1977. Print.

A great source for basic pattern block shapes and design ideas.

Devlin, Keith. *Mathematics the Science of Patterns: the search for order in life, mind, and the universe.* NY: Scientific American Library. 1997. Print.

A good source for the mathematics behind tessellation designs.

Grubaum, B and G C Sheppard. *Tilings and Patterns*. NY: W.H. Freeman & Co. 1987. Print.

A good source for terminology and descriptions of mathematical patterns and shapes. There are problem sets for the advanced student to ponder.

Kalajdzievski, S. *Math and Art: an introduction to Visual Mathematics*. 2008 . Print.

Useful source for all kinds of visual math products. It gives helpful examples of modern applications of visual mathematics.

Pink, D. *A Whole New Mind*: *why right-brainers will rule the world.* NY: Berkley Publishers, Penguin Group. 2005. Print.

A light read on the ways technology is changing society and the need to be more creative in order to succeed. Current applications for visual math and “right-brain” thinking is stressed as necessary for future employment.

Turner, R.M. *State of the Art Program: portfolios.* TX: Barrett Kendall Publishing, Ltd. 1998. Print.

Examples of visual math applications.

Vasquez, J A, M W Comer and F Troutman. *Developing Visual Literacy in Science, k-8*. Arlington, VA:NSTA Press. 2003. Copyright 2010. p.xii. pdf. Online.

Justification for the use of visual/spatial thinking activities in elementary school.