Seeing Geometry Through Art

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Introduction

I wrote this unit to help students more readily visualize the concepts that are the foundation of high school geometry. This will facilitate a greater comfort level and depth of understanding of basic geometric principles and their applications. I will weave an art strand throughout my entire curriculum. My geometry course is divided into four large sections/units. Using art as a tool, we will explore the big geometric concepts in each of these four sections. Exploring geometry through art will promote the success of both standard-level and honors-level students. It will provide students strategies for visualizing geometric figures in nonstandard positions (rotated, flipped, and compiled with other figures in complex diagrams). Students will also gain a more multi-dimensional perspective, as we explore 2D, 3D, and even 4D spaces. This unit will help all students realize that geometry is an integral part of the arts and is meaningful outside of the classroom. As they see math in the context of the arts, I hope to pique their interest and curiosity. Math will become more relevant to them. This will keep students more enthusiastic about the learning objectives and, ultimately, keep them more engaged.

Rationale for Weaving the Unit in Smaller Strands Throughout the Entire Course

Weaving an art strand throughout my curriculum greatly helps me, as the teacher, in two ways. Firstly, it gives me a wealth of tools to use in teaching my objectives. I will have a greater bank of resources, which will increase the richness of the learning environment. I can more easily assist students with weaker math foundations as they have contextual repetitions of the material. I can also more easily go in greater depth with these same contextual examples. Most of the activities included in this unit can be adjusted for time and degree of difficulty. This will give me more flexibility in appropriate pacing, improving differentiated instruction based on the specific needs of the students in each class. Secondly, this art unit gives an increased sense of continuity not only to the course, but also to its real-life applications. With an underlying theme throughout the year, the curriculum objectives will seem less random and will have more purpose.

To capture the essence of this instructional unit, one must be introduced to the four broad geometry subgroups that divide my course and some of the instructional challenges associated with each.

Geometry Subgroup 1: Point, Lines, Planes, and Angles

Students explore how theory affects reality and study points, lines and planes. The fact that a line contains an infinite number of points is something that is memorized but not always understood. Understanding that a line segment is also a set of infinite points is rarely understood. When students do not understand these concepts, they struggle with their applications. Students also struggle visualizing these concepts. Seeing the abstract, such as planes, parallel lines, and intersections, proves challenging. Students memorize that "parallel" means "not intersecting". Such an inability to grasp these concepts prevents students from correctly visualizing them. Students have concluded that a point not on a line is parallel to the line. Their memorized definition, "not intersecting" lead them to the wrong conclusion. Understanding the concepts would have prevented the incorrect deduction. Working with angles also proves challenging. Students cannot always see that an angle bisector creates three angles. Learning that a linear pair is a set of adjacent supplementary angles is simple to most students. Identifying linear pairs when rotated (turned) and reflected (flipped) is not.

Geometry Subgroup 2: Triangles and Proofs

In the second broad section of the course, students study triangles, the properties of similar triangles (same shape, proportionate size) and congruent triangles (same shape, same size), and write proofs. Students justify answers and prove conjectures. Students begin to experience basic concepts within the foundations of more complex problems. Most problems consist of several adjacent triangles (triangles that share a common side). Students must prove or disprove congruent triangles, and justify their conclusions with math laws. Again, the challenge comes as students memorize concepts without understanding. They are therefore unable to identify particular parts of these complex diagrams. For instance, in a compilation of many triangles, students may not even see the individual triangles, but just a "jumble" of lines. Add the rotations to a compilation of several figures and students are ready to give up before they have started.

Geometry Subgroup 3: Quadrilaterals and Circles

In the third section of my course, we study quadrilaterals and circles. Students explore how to break down the broad by examining the simple smaller parts of larger more complex diagrams. It is important to note that many of the quadrilaterals and circles we evaluate contain the angles and triangles we have previously studied. It is also the time of year to start really emphasizing re-looping of the entire curriculum, and final exam review becomes a big part of instruction. Studying quadrilaterals and circles in conjunction with cumulative review is quite helpful. While studying these chapters, we revisit congruent triangles, right triangles, vertical angles, angles formed by parallel lines, and trig ratios. Students that have struggled with this material previously, will typically continue to do so. Defeatist attitudes are an obstacle as students do not feel they can get past those complex figures with their elusive "simple parts".

Geometry Subgroup 4: Area, Surface Area, Volume, and Translations

In the fourth and last section of the course, we explore the concepts of space and motion in space. We study area, surface area, volume, and translations. Students typically do well in determining area of regular figures, but no irregular figures. Many students struggle to break the irregular figures back down into simple shapes. As we continue with volume, students explore the relationship between prism and pyramid, as well as cylinder and cone. Students also complete basic graphing on the x,y,z axis, three-dimensional space. Students finish up the year with transformations – rotations(turning figures), reflections (flipping figures over various axis or lines), translations (sliding figures right, left, up, down, and in combinations of those), and dilations (enlarging or reducing the size of a figure by a specific scale factor). Students seem to struggle with composition transformations (more than one move at a time). These are usually difficult to visualize, and students will have to take the time to draw them, as best they can. Rotational symmetry (turning a figure such that it appears in its original position) is another visualization challenge in this section. Students struggle to turn the figures in their minds and drawing more complex figures is not always an option.

Connecting the Four Geometry Subgroups

A common instructional challenge within each subgroup is helping students identify basic figures after being transformed (rotated, flipped, or compiled into a larger figure). Some students do not have a strong spatial aptitude and become frustrated when they are unable to model the abstract. Students are easily discouraged when a simple figure is presented in a different setting. Using art within each unit will improve students' comfort level with these transformations. This will, in turn, improve students' willingness to attempt complex problems. Using art to help the students "see" geometry will keep them invested in the curriculum. The contextual references will make the course more significant to students. Art will also increase my enjoyment in teaching the course. My enthusiasm for art in conjunction with my enthusiasm for geometry should improve the quality of instruction in the classroom.

Background

I teach high school geometry to standard-level and honors-level students. This is a tenth grade math course. Algebra I is a prerequisite for this course. Approximately 56% of my standard class is composed of students who have repeated courses. Approximately 56% have entered my geometry class with a D in their previous math class. The ages range from 14-18 years old. The typical class size is around 30 students. The typical maximum is around 38 students and the minimum is around 25 students. My honors classes are primarily freshmen. All students have entered the honors course on or above grade-level. Only about 7% of the honors students scored below grade level on state tests in previous years. The ages range from 13-15 years old. The class size ranges from 23-38 students. I teach in a large metropolitan area. Our school has very few math resources. Standard school supplies are the tools with which we are expected to teach. Considering art applications without the need of extra supplies is tremendous. Generally speaking, asking students to spend extra money on supplies for projects is not welcome. The activities within this unit require basic supplies; paper, pencils, colored pencils, rulers, and scissors. I am able to supplement these supplies if necessary, and I typically do. For little to no extra monetary investment, there should be a significant return.

Section 1 out of 4: Seeing Geometry through Art Unit (Points, Lines, Planes, Angles)

Objectives of section 1

As previously mentioned, in the first section of the course, students study points, lines, planes, and angles. Aside from learning the basic symbols and characteristics of the undefined terms (point, line, and plane), students have to apply them to defined terms such as angles, parallel lines, and polygons. Students use these various types of angles to problem solve and to determine parallelism. The art-unit objective associated with this section of curriculum is to give students contextual repetitions in working with these geometric concepts. In doing so, students will be able to better visualize them. Students will take the basic principles associated with these geometric terms and apply them to perspective drawings. Within the art activities, students will also define two-dimensional space and three-dimensional space, and reflect on the relationship between the two. Furthermore, students will reflect on how three-dimensional representations in two-dimensional space can seemingly contradict reality. In perspective drawings, real- life parallel lines are drawn as non-parallel lines. Students who do not know this will make this discovery, within the activity, as they create their own perspective drawings. Bringing art into this section will help students see the concepts of parallel and non-parallel, as well as see the angles formed by these lines. As students use art as a tool to see geometry in action, they will be more readily able to see these same geometric concepts in other contextual problems, and in complex diagrams in a test setting.

Strategies of section 1

Basic Perspective Drawing: A Discovery Lesson

Perspective drawings can be very basic like a road or a house. As a strategy for understanding parallel lines in a plane, students will be asked to take two pencils and place them on their desk so they represent parallel lines. Students will then be asked to take two pens and model a road. Usually students will duplicate the model they created with the pencils. Now that students have modeled two parallel lines (pencils), and a road (pens), students will be asked to draw an illustration that matches each. They will draw (or construct) two parallel lines below their pencil model. Students will then be asked to draw an illustration that matches the pens; to illustrate a road. Some students will only make their road as long as the pens. They will be directed to lengthen the road. As a class, we will reflect on what would make the road look more realistic. At first students will edit their illustration by a dashed line down the middle, indicating a passing zone. Some students will know to use perspective, and start to converge the lines, and some will not. As students evaluate the accuracy of two parallel lines representing the road versus two converging non-parallel lines representing a road, they will start to discern parallel versus nonparallel naturally. Students will be asked to reflect on why they think that two nonparallel lines on paper best represent parallel lines in our world. This will lead to a discussion of two-dimensional (2D) space and three-dimensional (3D) space. Students will define parallel lines and give contextual examples. Student will also define both 2D and 3D space. (See Appendix B for The Road Activity.)

Vanishing Points

Our next strategy will include a discussion about the vanishing point in the perspective drawing of the road. Students will define vanishing points. In perspective drawings, parallel lines are represented with non-parallel lines. The point at which these nonparallel lines intersect is called a vanishing point. We will look at several basic three-dimensional drawings and students will be shown vanishing points in each setting. As a class, we will look at examples of one-point perspective, with one vanishing point (Fig 4); two-point perspective, with two vanishing points (Fig 5); and three-point perspective, with three vanishing points (Fig6) INCLUDEPICTURE "http://www.khulsey.com/perspective_basics_line.jpeg" * MERGEFORMATINET

Students will then be given several different three-dimensional drawings and be asked to extend lines to find the vanishing points. Once a basic understanding has been established, students will complete two perspective-drawing exercises that focus on vanishing points. In the first exercise, students will be asked to create a perspective drawing of an empty room using one vanishing point. The simplest additions that students will make will be basic shapes, like windows and a table. For more detail, students will also add wooden floors, and texture to the ceiling. I will use a YouTube website to help students see exactly how to complete the exercise. Students' final product will most likely be more simplistic than the figure below; however, as artistic ability and creativity allow, students can make additions.

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As students are completing the perspective drawing, they will respond to questions associated with the connections of geometry and art. They will answer when parallel and non-parallel lines are used in this context. They will make conjectures about how these concepts would be applied to 2-point perspective drawings and 3-point perspective drawings. Lastly, students will explain how to prove whether lines are parallel or just appear to be parallel. They will now have drawn and applied points, lines, planes, and angles. (See Appendix B for activity sheet.)

In the final activity, students will view a picture of a two-point perspective cube as shown on the next page. While looking at the cube, students will be instructed to draw only the top plane and one of the vertical edges (line segments). Then the illustration will be taken from view. Students will complete the drawing using vanishing points. This exercise will be the most challenging. In many ways it will also offer the most flexibility. This can be a teacher-directed activity for a standard class.

INCLUDEPICTURE "http://www.sheamusburns.com/wp-content/uploads/2009/03/cube_tutorial_2.jpg" * MERGEFORMATINET It will be a great opportunity for a student with stronger artistic ability to lead the math class. This, of course, is a great confidence boost to the student who does not typically feel successful in math and is now a leader in the math classroom. This exercise can also be sent home as independent practice. It also could be bundled with several similar exercises (in basic perspective drawing) for part of an extra credit assignment or as part of a mini-project. There is an opportunity here to choose whether to make a min-project more art driven or more math driven. It may even be appropriate to write up both options and let students choose based on their strengths or based on their needs.

Perspective and Optical Illusions

There are several other considerations for using art to see geometry in the first chapters of the course. Once students have an understanding of perspective drawing, they can start to evaluate when the rules of perspective have been violated and what the implications are. For standard students, this portion of the art unit would most likely be completely teacher guided. Honors students could be introduced to the idea of optical illusions, and then asked to complete independent research and report how and why optical illusions work. (See Appendix B for activity sheet.) Whether teacher driven or student driven, the answer includes the laws of geometry. Parallel and non-parallel INCLUDEPICTURE "http://upload.wikimedia.org/wikipedia/en/thumb/0/02/OpticalIllusionStJohnLateran.jpg/220px-OpticalIllusionStJohnLateran.jpg" * MERGEFORMATINET lines, similar and congruent figures, dilations (change in size), etc are just a few of the geometric concepts included in creating optical illusions. Seeing the optical illusion of the tile floor in the Basilica of St. John Lateran in Rome, as shown above, would be good follow up to the perspective drawing. It gives the illusion of 3D space while in 2D space. Another great optical illusion that would be an excellent follow-up to the perspective drawings is the “Ponzo Illusion”, as shown below. That is a railroad tack drawn in perspective. Two colored line segments of the same length are shown in the tracks. Due to the perspective of the drawing, they do not appear to be congruent segments INCLUDEPICTURE "http://upload.wikimedia.org/wikipedia/commons/thumb/0/02/Ponzo_illusion.gif/220px-Ponzo_illusion.gif" * MERGEFORMATINET

After looking at these optical illusions as a class, students will reflect on the specific math applications that would be necessary to create them. Depending on the students’ needs associated with the pace of the class, it will be easy to bring in other famous optical illusions and ask students how geometry helps to create them. Two more good examples are: Ebbinhaus Illusion (Fig 1) and the Café Wall Illusion (Fig 2).

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Figure SEQ Figure * ARABIC 1

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Figure SEQ Figure * ARABIC 2

Figure 1 gives the illusion of dilation (a change in size). Figure 2 gives the illusion of non-parallel lines when we are actually looking at parallel lines. This illusion is particularly helpful in helping students learn how to prove parallelism, which they have to do in the next section of curriculum.

Section 2 out of 4: Seeing Geometry through Art Unit (Triangles and Proofs)

Objectives of Section 2

In this section of the curriculum, students continue to identify the basic geometric terms; however, they do so in more complex settings. Students will now also use the properties of the various types of triangles and their parts to problem solve and to write proofs. Diagrams become more involved and answers to problems are more involved. It is no longer enough to state the answer, rather, students must justify answers with math laws (theorems and postulates). The optical illusions from the previous section are a great artistic transition into writing proofs. They help students understand why we have to write proofs. We all must be able to prove (or disprove) whether something is what it appears to be. Optical illusions illustrate that point immediately. The art objective, as we add triangles to our instruction, will be to take the very simple and form the more complex. Students will take basic artistic (and geometric) shapes to create a more complex piece of art.

Strategies of Section 2

Color-Coding

A common strategy that we use during instruction is to take a complex diagram and literally cover up all parts of the diagram except for <u>one</u> known simple shape or object. In covering up the entire diagram except for <u>one</u> triangle, or <u>one</u> linear pair (two angles side-by-side that form a line), or <u>one</u> pair of vertical angles (two angles formed by opposite rays.), students are able to see the very simple and fill in one piece of the complex puzzle. In continuing the process students actually see <u>one</u> step at a time. In the end we uncover the entire diagram and all of the missing angle measures and missing side measures have been filled in. As we continue to transition into a greater comfort with more complicated problems, students will work toward seeing these simple shapes without covering up all of the others. Color-coding is a great tool to complete this task. Students use the same objective as before. Students look at one simple piece of the figure and color code that piece only. For example: Find a linear pair and go over the lines that form it in lime green. Find the missing measure only associated with the lime green. Students will do the same for all of the linear pairs. This will accomplish 2 objectives: 1) It will help students identify basic geometric figures in nonstandard positions. The linear pairs will be flipped and rotated. 2) It will re-emphasize that this seemingly complicated problem is merely several simple problems. Students will continue in this manner for the other basic figures within the complex diagram. Students will label vertical angles in valentine red; perpendicular lines in purple; and triangles in turquoise. Students will further see that compiling the simple creates a larger more complex product. All too often even if students are able to grasp that base angles of an isosceles triangle are congruent, once the triangle is rotated, the “understanding” is gone. Using art as a practical simulation of compilation diagrams (diagrams created by putting several simple figures together to create a more complex figure) will be a great strategy to help students more readily see geometry concepts and apply their properties to problem solving. The color-coding is more mathematical than artistic; however, it is a powerful introduction into the true art activity in this section.

Origami Fold

Students will complete an origami fold. One side of an origami cube is a classic and good fold. Students will name every shape and geometric term as we fold. For example, instead of folding the paper in half, students will create a vertical bisector, etc. Once the entire origami fold is completed, students will unfold the paper. Yes, they will undo what they just did. However in creating the series of creases in the paper, the class will also have completed a basic geometry review. Students will be look at the many creases/lines in their paper. They will then take a pencil and a ruler and start to draw over any creases that create geometric figures and label them all. For example, they would draw over a linear pair (adjacent supplementary angles) created by the creases, and label it using the correct term. Other examples of geometric terms that students will find and draw over and label include acute angles (angles whose measure is less than 90 degrees), and obtuse angles (angles whose measure is greater than 90 degrees). Students will find over 30 geometric terms within the creases. Upon completion of this portion of the activity, students will rename these geometric figures on notebook paper with the appropriate symbols, definitions, and problem solving application (a reason for needing to know them). At this point students will have two options. One Option: Create another origami fold and then unfold it. Then take a pencil and highlight the labeled geometric figures in such a way that they are a natural part of a simple piece of art. Students will end up using simple geometry to create simple drawings like trees with no leaves, or the outline of a house. Some students will take a more creative approach and draw the outline of a “modern art” bird or cat. Students will show geometry as a natural part of art. A Second Option: Take the list of the geometric figures found on the origami paper (a minimum of 25 terms) and use it to complete a simple sketch naturally composed of the geometric terms. In the sketch, all of the terms would have to be illustrated and labeled. For example, a simple sketch of the top of a school desk, complete with a pencil, a pen, an eraser, and piece of notebook paper would produce most of the geometric terms naturally. If the objects on the desk are strategically placed, congruent and similar figures would be present as well. Students have gotten very creative with this activity. Students have recreated famous album covers, like Abby Road, by The Beatles, and highlighted and labeled the geometric terms within the scene. Other examples are lighthouses, cars, pianos, bridges, cityscapes, and the like. Students enjoy the freedom to inject their personal artistic tastes in a math assignment. They also start to look at the side of a skyscraper and see parallel lines, transversals, and quadrilateral. They start to see geometry everywhere as natural part of our world. (See Appendix B for Simple Art Activity.)

Section 3 out of 4: Seeing Geometry through Art Unit (Quadrilaterals and Circles)

Objectives of Part 3

In the third section of the curriculum we study quadrilaterals and circles. Mathematically the connection exists in seeing these figures as compilations of most of our previous geometric concepts. For example, in the quadrilateral chapter, students have relatively less new material. Rather, as diagonals cross every type of quadrilateral, all of the various types of triangles and angles we have studied are formed. If students see the quadrilaterals in this manner, they have very little difficulty with the chapter, as it is largely review. A similar connection can be made within the circle chapter. As students see the angles formed by *tangents* (lines touching the boundary of a circle in exactly one point), *secants* (lines going through a circle and intersecting it in two points), and *chord*s (line segments whose endpoints lie on the circle), they again are looking at relatively less new material. Rather, they are looking at the geometry foundations in a new setting. Students will see the complex as a compilation of the simple. Logically, this is the converse (a change in order of the hypothesis and conclusion) of the objective in the previous section (This would be a great opportunity to review the logic concepts of converse, inverse, and contrapositive, which are also covered throughout the curriculum). In the last section we looked at the simple creating the complex. In this section we look at the complex as a compilation of the simple. The art objective in this section is for students to break down complex art forms into simple figures. On the most basic level, students will look at simple works of art and describe them as the sum of their parts. The perspective drawings from the first section will be a great place to start. Rather than see "a table", students will see and describe a horizontal plane, line segments, perpendicular lines, right angles, edges, and vertices. We will apply this same objective to famous and much more complex works of art.

Strategies of Part 3

Seeing One Famous Piece of Art as the Sum of its Parts

In the third section of the geometry course, students will identify a famous piece of art as a compilation of basic geometric figures. "Guernica" by Pablo Picasso (as shown below) is a good example. INCLUDEPICTURE "http://www.pbs.org/treasuresoftheworld/guernica/images/guer_page_pix/guer_painting_large2.jpg" * MERGEFORMATINET It is an amazing piece of art with tremendous history. It also has numerous examples of the geometry concepts we study. This portion of the art unit is also an excellent place for me, as the instructor, to really dig into some great works. My own interest in many pieces of famous art will keep this section fresh and exciting to teach. It also will allow an opportunity for various levels of mathematics to be studied. Students will be introduced to several famous pieces of art, like “Le guitariste” by Pablo Picasso (Figure 3) or “Le Viaduct at L’estaque” by Georges Braque (Figure 4). These are both examples of Cubism, which is centered on geometry. Students will be asked to write a reflection on what they see. Then students will be asked to view each painting as the sum of its parts and then write a second reflection on what they see. (See Appendix B for activity sheet.) Students’ description of the overall picture versus their description of the pictures’ “pieces” will be very different. This will illustrate that looking a complex problem at a first glance is much different than looking at it as many simple pieces. After re-evaluating how to view art, students will select a famous work of art and print a copy. Students can then trace or duplicate the famous work. Very artistic students may choose to try to create their own work. Students will then identify and label the geometric terms and concepts found naturally within the work. The geometry they see should include triangles, quadrilaterals, and all of their parts, as well as similar and congruent figures. (See Appendix B for Cubist Art Activity.) In certain works, circles may not be present. However, depending on the amount of time you have and the level of students, you may want to ask students to use a Wassily Kandinsky work and include circles. For honors students, the concept of cubism and its connection to geometry can be developed much further. Students can make a product using Primitive Cubism as a guide (cones and spheres). It may also be appropriate, depending on the level of the class, to start to discuss the fourth dimension. “Crucifixion” by Salvador Dali shows the cross as a hypercube. The fourth dimension and the hypercube are complex artistic and mathematical concepts. Imagine a net of 6 squares drawn on paper, that when cut out and folded at each edge, form a cube. A hypercube exists in 4 dimensional space, which is very abstract (cannot be drawn or shown). In the fourth dimension, 3-dimensional figures fold over themselves. Take the same net of squares that just created a cube, and replace the 6 squares with 8 cubes. Imagine trying to fold the cubes around, in much the same manner you did the squares. This would form a “hypercube.” Just sketching the net of a hypercube and having students reflect on the principles of each of the four dimensions would be a great activity. For standard and honors classes, examining the hypercube in a teacher-guided discussion would work well. Following that discussion, students should evaluate the complexity of the hypercube compared with its very simple parts. Ultimately, I would like students to make a 2-D net of a cube and of a hypercube. I would like them to reflect on the 6 squares that construct the cube and the 8 cubes that construct the hypercube. I have written an activity to complete this task; however, depending on the pace of the students in individual classes, it may not be possible to complete it. While the fourth dimension is very interesting and very in-depth, it is not part of our curriculum and would be an extra activity, should time permit. (See Appendix B for hypercube activity sheet.)

Section 4 out of 4: Seeing Geometry through Art Unit (Area, Surface Area, Volume, and Translations)

Objectives of part 4

In the fourth and final section of my curriculum students study space and motion in space. Students study area, surface area, volume, and transformations (objects turned, flipped, slid, and changed in size). The art objective in this section will focus on transformations. Students will create a rotation and reflection table. In doing so, students will model the properties of each and make conclusions about a series of rotations and reflections. As students physically complete this task they will be able to better visualize less complex problems without a model. Students will also identify and create Celtic knots that possess rotational symmetry.

Strategies of Section 4

Rotation and Reflection Table

Students will construct an equilateral triangle. Students will then draw the three symmetry lines through each vertex. Below is an illustration of an equilateral triangle with one of the lines of symmetry drawn through it. Students will draw three symmetry lines, one through each vertex. Each line of symmetry should be numbered 1,2,and 3. Instructions and guided practice will be given on rotations and reflections. Students will then learn the symbols associated with no rotation, 120-degree rotation, and 240-degree rotation. They will also learn the symbols for a reflection about symmetry line 1, a reflection about symmetry line 2, and a reflection about symmetry line 3. Students will then be guided in a small series. For example, a 240 degree rotation, followed by a 120 degree rotation gives back the original position. Students know that a 360 degree turn does the same, but cannot always connect the series as being the same. Certainly, the symbolic representation of the series will be new to students. Once they have a basic understanding of that, setting up the table will be a powerful tool in helping students see translations. There is an example of a table below.

e stand for the **Do nothing** movement.

a stand for the **Rotate 120 degrees counterclockwise** movement.

b stand for the **Rotate 240 degrees counterclockwise** movement.

X stand for the **Flip about axis through the top vertex** movement.

Y stand for the **Flip about the axis through the lower-left vertex** movement.

Z stand for the **Flip about the axis through the lower-right vertex** movement.

With these definitions for **e, a, b, X, Y,** and **Z** we can form the Cayley table for the symmetry group of the equilateral triangle: (In this table the first motion performed is in the left-hand column and the second motion is in the top row.)

•	e	a	b	X	Y	Z
e	e	a	b	X	Y	Z
a	a	b	e	Y	Z	X
b	b	e	a	Z	X	Y
X	X	Z	Y	e	b	a
Y	Y	X	Z	a	e	b

Z	Z	Y	X	b	a	e

Standard classes will most likely have a teacher-led class. Honors classes will most likely be able to work much more independently or perhaps in small groups. This activity is more mathematical than artistic; however, rotations (turns) and reflections (flips) are evident in many patterns and in many pieces of art. Primarily, for the purposes of this unit, it is a good and colorful example of transformations. It is also good introduction to transformations, which is more artistically introduced in the next activity.

Celtic Knots and Rotational Symmetry

One related area that students also have a difficult time visualizing is rotational symmetry. Rotational symmetry exists when a figure is rotated. However, after the rotation, it appears the same as in its original position. Discussing symmetry using the context of Celtic Knots will be very engaging for the students. Drawing Celtic Knots will keep less motivated students much more involved in the topic. It will give all students a contextual application. Students will construct an equilateral triangle (as seen in red in Figure 5). Students will then place the "X" patterns over each line segment (as seen in figure 6). Students will then connect the parts of the "X" connecting an "over" segment with and "under" segment, to create the Celtic knot, as seen finished in Figure 5.

Figure 5

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2.gif" * MERGEFORMATINET

Figure 6

Students will complete this activity with a triangle midsegments added. Triangle midsegments add a line segment inside the triangle that is parallel to the base and half the length of the base. This increases the complexity of the Celtic knot and still maintains symmetry. Students will then either add another parallel segment to create a third Celtic knot, or create their own design. At the end of the activity students will reflect on why the knots "x" pattern works and how they relate to rotational symmetry.

Upon completion of the geometry course, students will also have completed the unit "Seeing Geometry through Art". In doing so, they will have developed a greater comfort with undefined terms in defined settings. They will have developed a greater ability to visualize abstract concepts. Their depth of understanding will have far surpassed memorizing a few words associated with a diagram. In fact, memorizing will have become less necessary. As students are able to explain what they now can visualize, they will have stated definitions and theorems. Providing these improvements to the students' learning experience is tremendous. Being able to do so inside the context of art has an even greater impact on the learning process. Making the course more relevant will also make it more memorable and more enjoyable. Making fundamental math concepts understood will make future math courses and future problem solving less stressful. Showing students that the complex is merely a compilation of the simple is a life lesson for any class; truthfully almost any setting. Seeing Geometry through Art is a great unit for students. It is also a great unit for educators. It allows great room for personal tastes and preferences to dictate the art that is shared with the class. Great art can be a great inspiration. Having the ability to not only change according to personal taste, but to change just for the sake of keeping things fresh, new, and exciting makes the unit worth the time.

Notes

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This website contains and excellent illustration of how to make a simple perspective drawing with one vanishing point.

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Figure 4

Figure 3