The progression of accurately solving a multi-step equation

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Introduction

This curriculum unit is designed to give students a more complete and comprehensive understanding up the steps involved to solve multi-step equations. Students often become disengaged from an activity if they do not fully understand how to solve it. Multi-step equations can be difficult for some students and this can be completely avoided if they have a solid foundation and understanding of the concept of a balanced equation. To avoid any confusion they must start with a solid foundation on the process involved to solve a single-step equation. Once this is in place then the students can advance to the next level of a multi-step equation. This unit will start with the basics of solving a single step equation and progress to a multi-step equation with variables on both sides.

To start the unit I will spend one to two days on the fundamentals of balancing an equation. A typical problem we will work on is a simple one-step equation. For example,

$$y + 9 = 12 \text{ or } 3x = 9$$

It is important to start with the addition and subtraction of solving a equation first. Once we have mastered the concept of a one step equation then we will move to a two step equation. For example,

$$3x + 9 = 21.$$

We will spend two days on this as well. It is important to make sure the students understand and know how to balance an equation using addition, subtraction, multiplication and division before these two step equations are introduced. Isolating the variable is an important concept that will be introduced at this time. Since Algebra is a large part of our End Of Grade(EOG) testing in North Carolina it is helpful to keep the multi-step problems in the slope-intercept form so that when that concept is introduced

later on in the year the students are familiar with the form and are comfortable solving it. I will also introduce word problems at this point and will most likely spend two days on them. A typical word problem would be the following;

> "Ted is paid $7 an hour to work at his dad's ice cream shop. If he made $51 on Sunday which included $9.00 in tips, how many hours did he work?"

The application of multi-step equations to real world problems is vast and the students can really get a sense of the importance and value of knowing how to solve and use these equations. I will then progress the class into multi-step equations with variables on both sides. For example,

$$3y + 1 = 8 + 2y$$

The students must have an understanding of combining like terms. This could have been introduced prior to solving multi-step equations or it could be introduced now. This is the hardest part of teaching students multi-step equations. The majority of students that will run into difficulty will find it here.

Background Information

I teach 8th grade math in a urban middle school in Charlotte, North Carolina. In this particular class, I have relatively few behavior problems. Talking at inappropriate times, and not paying attention are my students' biggest fault. Ranson Middle School is on a 5 block schedule and my math classes meet every day for approximately 70 minutes. I have a pacing guide, created by the county I teach in, that I follow. There is a moderate amount of time set aside for solving multi-step equations. It is a rather large part of the 8th grade curriculum, so, we spend a significant amount of time on it. I typically have about 32 students in math classes and I have approximately 20 to 25 students in my Algebra classes.

The population at my school is as follows: 78.5 % African American, 11.1 % Hispanic, 2.2 % Asian and 4.4 % Caucasian. Seventy five percent of our students are enrolled in the free and reduced lunch, while 15% of our students have some degree of disability. We are currently an equity-plus school serving over 1200 students.

The percent of students at or above grade level in math is 39%, and the students at or above grade level in reading is even lower- 31.5%. These percentages put us below the Charlotte Mecklenburg District average in both categories. However, over the last year Ranson Middle School had 97% of Algebra students performing at or above grade level. We also made 29 of 29 AYP goals last year.

The District itself is growing larger and larger every year; it currently enrolls 122,000 students and employs 10,050 teachers. Over the last couple of years, the Charlotte-Mecklenburg School System has increased the number of students by approximately 6,000 students per year. The district population is 43% African American, 37% Caucasian, 12% Hispanic, and 4% Asian

Strategies

Many students want to memorize a set of steps to solve a muti-step equation. In their eyes it is easier to do this than to spend the time understanding the how, what and why behind solving an equation. What they don't realize is that if they have a deeper understanding of the fundamentals behind why they make the steps that they do then they will significantly decrease the time spent on the problems, the errors they will make, and the frustration they may experience.

First, The students must fully understand the step involved in solving a one step equation, for example, $y + 8 = 12$. In order to do this they have to understand the concept of a ***balanced equation***. I will start with

$$7=7$$

This is a balanced equation just as much as,

$$y + 8 = 12.$$

Then we will talk about what happens to $7 = 7$ if you add a number, like 5, to one side of the equation. For example,

$$7 = 7$$
$$7 = 7 +5$$

The equation is no longer balanced. Therefore, what ever you do to one side you must also do to the other is the common phrase. Point out that if two sides of an equation are equal you can add or subtract the same amount from both sides, and they will still be equal. I have a SmartBoard in my classroom and many of my ideas in this unit will be centered around the various interactive SmartBoard activities. You can create a balance on the SmartBoard and show the kids how to balance an equation. There are also many

different websites that offer interactive balance beams(Please refer to the website in the bibliography listed as "balance beam"). The point is to show the students that to keep the equation balanced you must also add a 5 to the other side of the equation. For example,

$$7 + 5 = 7 + 5$$

The class can then use this information to help them transition into solving a single step equation using addition and subtraction. The students will be required to complete 4 simple steps throughout the mult-step unit. The steps are as follows:

1. Identify the variable being isolated(circle it)
2. List the actions you will take to solve
3. Solve
4. Check your solutions.

The students will be asked to complete these 4 steps for each problem they solve. This is time consuming and will most likely be met with some complaints. However, in the end the students will find solving problems easier and with careful planning they should be more accurate in their problem solving. The students can be weaned off of this process eventually.

The first problem to model for the students is $y + 8 = 12$. The 4 steps are as follows

1. Y is the variable to be isolated
2. Subtract 8 from both sides
3. Solve
4. $4 + 8 = 12$ CHECK!

Using addition and subtraction model for the students an example of a single step equation and the process of solving the equation. It is important to mention at this point the concept of isolating the variable. For example

$$\begin{aligned} y + 8 &= 12 \\ -\ 8 &\quad -8 \\ y &= 4 \end{aligned}$$

I will mention that the variable is "isolated". Before, the 'y' variable had a +8 next to it and was not "isolated". By eliminating the 8 the variable is all alone and we are able to discover what the unknown variable is equal to. The next typical problem to be introduced to the students is,

$$-8 + y = 12$$

If the students are able to solve this problem correctly then you know they are beginning to comprehend the material correctly. However, if they do not understand the problem they will most likely solve this problem in the following manner;

$$\begin{array}{r} -8 + y = 12 \\ -8 \qquad -8 \\ y = 4 \end{array}$$

Students will look at the operation that is going on between the variable and the constant and determine their next step based on this. It is important to point out that you must first identify the location of the variable. Then, you must determine what is in the way of isolating the variable. In this case the student would identify the 'y' as the variable and the -8 as the constant to be eliminated. Students that have problems understanding problems suddenly become completely aware if you relate it to money. I would make the reference that if you had -8 dollars in your bank account, what would it take to get that balance back to 0 dollars? You would have to add 8 back into the account. For example,

$$\begin{array}{r} -8 + y = 12 \\ +8 \qquad +8 \\ y = 20 \end{array}$$

I will give the students a number of problems involving addition and subtraction and allow them some time to independently solve the problems on their own. I will collect the classwork assignment and give them two problems as their ticket out the door.

$$y - 12 = 24$$

$$-7 + x = 19$$

Day two, the students will have a warm-up on the board involving some simple one –step equations. We will then transition into going over the previous nights homework. The students will come to the boards and write the answers down to their homework. They must show all their work. We will check the problems for accuracy and answers any problems they may have. Today the objective is to solve simple one-step equations using multiplication and division. A typical set of problems I will model include,

$$3x = 9$$

$$b / 7 = 5$$

The steps to solve this problem should be listed as such,

1. x is the variable to be isolated
2. Divide by 3 from both sides
3. Solve
4. 3 x 3 = 9 CHECK!

1. b is the variable to be isolated
2. Multiply by 7 on both sides
3. Solve
4. 35 / 7 = 5 CHECK!

This process of identifying the problem, naming a solution and checking the answer is very powerful. It will make it difficult for the student to get the problem wrong and will hopefully, make the student better at solving the problems.

To solve the problems listed above, identify the variable, find the number or constant that is in the way of "isolating" the variable. For the first problem, the students must realize that in order to isolate the variable they have to divide by 3 on both sides. This process undoes the multiplication. For example,

$$\frac{3x}{3} = \frac{9}{3}$$
$$x = 3$$

The multiplicative inverse is the solution. I also mention doing the opposite of what operation the coefficient is doing to the variable. Likewise, for the one-step division problem. For example,

$$7 \text{ x } b / 7 = 5 \text{ x } 7$$
$$b = 35$$

The variable, b, is being divided by 7. To undo this both sides of the equation must be multiplied by 7 to isolate the variable. Once I have modeled a couple of examples of this then I will allow the students to work in groups of two to solve a set of problems. I can pull the students that did not demonstrate proficiency levels on the previous days "ticket out the door" and work with them one-on-one to get them up to speed. I will again collect the classwork at the end of class to check for accuracy. I will also give another "ticket out the door" to check for comprehension of the days lesson.

Day three, the students will have a warm-up with a combination of addition, subtraction, multiplication and division problems. We will repeat the process for the homework, student volunteers, show all their work, etc. Assuming the class is prepared to move on then I will introduce solving multi-step equations of the form $3x + 4 = 16$. This is another great opportunity to use the SmartBoard to have the students interact with the equation. The website mentioned in the bibliography as balance beam is another great hands-on activity where the students can get a great visual imagery of what is happening. I will point out that the students need to follow the same basic steps as before, find the variable, identify what is in the way of isolating the variable and determine the steps needed to solve it. I will model a couple of different examples similar to this,

$$\begin{aligned} 3x + 4 &= 16 \\ -4 \quad & \quad -4 \\ \frac{3x}{3} &= \frac{12}{3} \\ x &= 4 \end{aligned}$$

It is important to continue with the 4 steps as the problems get more and more complicated. They should look something like this:

1. x is the variable to be isolated
2. Subtract 4 from both sides, divide by 3
3. Solve
4. $3(4) + 4 = 16$ CHECK!

I will allow the students to do a trial and error type problem. They need to make a couple of mistakes so they can learn. You can see above the first step is to subtract the 4 from both sides. They will then have $3x = 12$ remaining. However, some will want to divide by 3 as the initial step. I will allow them to do that so they can see what a more complicated mess the problem can be. For example,

$$\frac{3x}{3} + \frac{4}{3} = \frac{16}{3}$$

Hopefully, the students will see that they have bunch of fractions or decimals. Explain what would happen if they subtracted 4 from both sides first. They will see what a more appealing and simpler equation it is. It is important to explain to them that they have

options when solving these equations and that in some cases, as the problems get more involved, their process for solving an equation may not be the same order of steps as their neighbor. The students need to practice doing a couple of problems. I will give them their classwork assignment and allow them to work in pairs. I will circle the room and provide any help or explanation they need. Using the data from the previous days assessment I will re-loop some students that are struggling with the multiplication and division problems.

I will spend two days, or more if needed, on these multi-step equations. It is important that they have a firm grasp on how to solve them before moving on.

Day five, the students will come in and complete a warm-up that involves only multi-step equations. We will review the correct answers and then spend some time going over the previous night homework. The students will come to the board and show all their work for each problem on the homework. We will spend the next two days going over the application of multi-step equations in the real-world. Typical problems to review would include,

> Stuart charged $5.00 for every ticket sold to his rock concert. The venue he was playing at gave him $100 to play there. If Stuart made $200 on his first night how many tickets did he sell?"

It is important to point out to the students the importance of being able to solve the equation correctly. If you don't you could be out a significant amount of money or you may be expecting more than you will actually receive. It is also difficult for students to set this up as an equation. Visually, this is an excellent way to look at the problem. For example if he sold 2 tickets it would look like,

$$\$5.00 + \$5.00 + \$100 = \ ?$$

Then you could then continue to increase it by $5.00 up to a certain point. Ask the students if there was a faster way to find out how much he made in ticket sales rather than adding up a bunch of $5.00 amounts each time. With some luck you will have a student that tells you to multiply. Then ask what they would do if they didn't know how many tickets they sold, what could they use? A variable? They could then multiply that variable times the $5.00 amount. Hopefully, they see it is the same concept as before, but, now you are using a variable in place of how many tickets you sold.

$5.00x + $100 = $200

1. x is the variable to be isolated, the number of tickets sold
2. Subtract 100 from both sides, divide by 5
3. Solve
4. 5 x 20 + 100 = 200 CHECK!

Students need to solve a number of problems like this in order to get a good feel for how to set them up. Creating an equation to help them solve a problem is an extremely powerful concept. This will give them insight as to the necessity of being proficient in solving and creating these types of problems. There are typically not a lot of great problems in the text we use for these types of problems. Creating your own problems and incorporating the students into the problems is usually entertaining for the students and can draw their attention into the problem.

Once the students are completing the problems with accuracy then it is time to progress to solving multi-step equations with variables on both sides of the equation.

$12x – 18 – 3x = 12x – 36$

This is a good equation to start with. The activity for this is a hands on one. You can either use a SmartBoard or little whiteboards to do this problem. If you choose the SmartBoard option you can create each term listed above as a separate piece. Make each of the like terms the same color. For example the 12x, 3x and 12x would be blue. The 18 and 36 would be red. This would identify the like terms. Then allow the students to come to the board and move the like terms on one side of the equation around to each other. For example,

$12x – 3x – 18 = 12x - 36$

Do not allow the students to move terms around from side to side. This is a good time to talk about how the sign in front of the equation goes with the number, variable or coefficient. That is a common middle school student mistake. If you do not have access to a SmartBoard you can use small white boards. Use 6 different small whiteboards and put a term on each, including the equal sign. This way the students can manipulate the terms around and put them in order so that the like terms are side by side. This makes it a little less confusing for the students. It is important to stay with identifying your 4 steps here again.

1. x is the variable to be isolated
2. Combine like terms, add 18 to both sides, subtract 12 from both sides, divide by -3 on both sides.
3. Solve
4. 12(6) – 3(6) – 18 = 12(6) – 36 CHECK!

The students have to be good at solving these types of problems to be anticipating the steps in their head. This will take some practice.

Lessons

The objective of the first lesson is to solve simple one-step equations using addition, subtraction, division or multiplication. The big picture is for the students to understand the 4 steps to solving the equation and not just memorize the process of solving an equation. These first few days are critical to the success of the student as the days progress. Using terms like balancing equations, operations, inverse operations, and isolating the variable are terms that should be introduced and expected as common vocabulary for the students to use. The second day should involve solve simple one step equations using multiplication and division. Using the same vocabulary as the previous day the students will need to practice solving these problems repeatedly in groups and on an individual basis. Re-looping is key to the success of the student during these first few days. Therefore, collecting classwork or some sort of ticket out the door piece is important to be able to assess student comprehension. Stress the entire time to show all work so that it is easier to see their errors. The third day is combing the operations learned on days one and two together so that the students can solve a multi-step equation. Again, practice solving these problems so that they can build an arsenal of knowledge of solution strategies. Most students will need to spend more than one day on multi-step equations. Depending on the make-up the number of days you will spend on multi-step equations may vary. The lesson should involve a series of questions where the students can see real-world applications of the problems. This could also take a number of days to get the students to a comfortable place. The student work must remain consistent. Do not allow short-cuts and no work shown. Make sure they students are identifying the 4 steps. Staying consistent will ensure that the students have success in future.

Appendix

One Step equations

1. -5 + x = -14 (-9)
2. s – 5 = -11 (-6)
3. k – 10 = 9 (19)
4. -6 + r = -2 (4)
5. m + (-4) = 12 (16)
6. -49 = -7r (7)
7. x / -5 = - 5 (25)
8. -6x = 24 (-4)
9. a / 4 = 5 (20)
10. 6x = -36 (-6)

Two step equations

1. 2x + 1 = 9 (4)
2. 2a – 7 = -9 (-1)
3. 3 – 4k = -1 (1)
4. 2r + 1 = -1 (-1)
5. -3a + 4 = 7 (-1)
6. 2x – 7 = 7 (7)
7. 5y – 2 = 28 (6)
8. 2x + 3 = 9 (3)
9. w/5 – 2 = -1 (5)
10. x/3 – 1 = 5 (18)

Multi-step equations

1. 3a – 10 = 4 + a (7)
2. 5y + 4 = 4y + 5 (1)
3. b + 3 = 2b – 7 (10)
4. 2x – 1 = x – 3 (-2)
5. 2w + 3 = 4w – 5 (4)
6 . 6n + 4 = n – 11 (-3)
7. 29r + 4 = 28 + 13r (1.5)
8. 15y + 14 = 2(5y + 6) (-.4)
9. 15 – x = 2(x + 3) (3)
10. 4(3d -2) = 8d – 5 (3/4)

Word Problems

Stuart charged $5.00 for every ticket sold to his rock concert. The venue he was playing at gave him $100 to play there. If Stuart made $200 on his first night how many tickets did he sell?"

Teddy rented a bike for $10.00 an hour. The bike store also charged him an additional $25 insurance fee. If he was charged a total of $125, how many hours did he rent the bike?

MJ designs clothes for women. She charges $500 per evening gown plus $50 an hour for tailoring. How long did she spend tailoring an evening gown if she was paid $850 for the gown?

If Blockbuster charge $4.50 to rent a movie for one week and $.30 for every day it is later after that, how late was Ben's movie if he paid a total of $7.50 for the movie?

Matt's father is 45. He is 15 years older than twice Matt's age. How old is Matt?

Your dad took you shopping for some new clothes this past weekend. He purchased jeans and shirts. The jeans cost $45 each and the shirts cost $25 each. If he bought twice as many shirts as jeans, and he spent $190 on clothes for you, how many jeans and how many shirts did he buy?

Tom's father is 3 times old as Tom. 4 years ago, he was 4 times older. How old is Tom?

Ben's father is 4 times old as Ben. 4 years ago, he was 5 times older. How old is Ben?

Scott bought three bags of candy with 75 pieces in each one. He plans to divide all the candy evenly among seven friends. How many pieces of candy will Scott have left for himself?

Anne earned $3 an hour baby-sitting, and $4 an hour working in the garden. Last week she did baby-sitting for 5 hours and garden work for 3 hours. How much more money does she need to buy a game that costs $35?

Tom divided $360 among his six children for them to use for Christmas gifts. His daughter Kate added $20 to her portion, then used the money to buy 16 gifts that each cost the same amount. What was the price of each of Kate's gifts?

Implementing District Standards
Charlotte Mecklenburg Schools Mathematics Standards

8th grade Math
Competency Goal 5: The learner will understand and use linear relations and functions

5.01a Identify relations and functions as linear or nonlinear.
5.01d Interpret and compare properties of linear functions from tables, graphs or equations.
5.04 Solve equations using the inverse relationships of addition and subtraction, multiplication and division
5.03 Solve problems using linear equations

Competency Goal 1: The learner will understand and compute with real numbers.

1.02 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, paper and pencil.

Algebra
Competency Goal 1: The learner will perform operations with numbers and expressions to solve problems.

1.02 Use formulas and algebraic expressions, including iterative and recursive forms, to model and solve problems.

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