Ticktacktoe as an Introduction for Problem Solving

Craig Willis

Introduction

Most students are not instructed in how to analyze or solve problems. Ticktacktoe is a good example of a game we think we know, yet few students or adults have explored the depth of the game. Other games, puzzles or problems could also be presented in illustrating how to semantically solve difficult problems. For this short presentation, we will only look at Ticktacktoe.

Math Can be Fun

Since most students and most elementary classroom teachers do not enjoy problem solving and the depth of study this requires, math is considered by many as a dull subject. Some math instructors have reversed this trend, and in their classrooms math is fun. These teachers use concrete objects to illustrate abstract concepts. They explore problems from different points of view. Difficult problems can be simplified and simple problems can reveal complex mathematical concepts when studied deeply. Through this process the instructors and the students can unlock the fun in problem solving.

"Do you enjoy doing math? Do you like to solve challenging problems?"

Unfortunately most Americans would answer "no" to these questions. In my household of three, we have an engineer and an accountant who works with mathematical problems every day. Both of them hate to do the math. It is common for people to admit they do not like math or even that they can't do math. Almost no one would admit they don't like to read or they can't read.

Math for fun

On the other hand, many people find math problems to be entertaining. There are many books such as Innumeracy and Beyond Innumeracy that people buy for pleasure reading about math. Martin Gardner made a career on writing about how fun math can be.

Gardner wrote over 60 books during the years 1955 to 1985. Why is this subject so distasteful to some while being delightful to others?

A floor painter

Bob Richardson paints concrete floors for a living. His company is doing very well because he understands the math involved in estimating the cost of the projects he bids to work on. He developed his math skills through problem solving. To him computation is fun; he enjoys demonstrating that he can multiply two three digit numbers in his head in seconds. These skills did not come directly from any math class. He figured out his methods for solving problems on his own. Developing this kind of understanding of problem solving should be the goal of all math classes.

Memorization vs. Understanding

In America's classrooms, students are taught how to do math. This includes memorizing facts and algorithms. Most of our elementary classroom teachers are required to teach math and have their students' perform well on standardized tests. There is pressure on the teachers and the students to demonstrate more than a year's growth each year. Because of this pressure and because of the mandates given to classroom teachers, there is little effort given allowing students to reach beyond what is perceived as the standard curriculum. The result is there are far too many students who can pass tests but do not understand the math. They are expected to find the answers but not how the process works. This leads to adults who do not like math because they never took the time to understand it.

Are we teaching math in a boring way? In many classrooms, this emphasis on computation makes learning dull for the students and teachers alike. What if we taught reading and English using just flash cards, diagramming sentences, and studying grammar? Would students still learn to read and write? Computation should be part of a good math program just like the study of grammar, but it should not be the focus of the program.

Problem solving should be the core of the curriculum. Mathematics should be understood as part of the language and tool to solve problems. With a focus of problem solving a math curriculum can become interesting instead of some kind of mental exercise. Solving problems should be the center of any math class. The problems do not have to be real life problems, but just interesting problems that catch the interest of the teachers and students.

Ticktacktoe is an example of a problem that is in the real world, but does not have any long time real value outside the learning of how to solve it. We don't buy and sell Ticktacktoe problems. Instead people have been playing with solving Ticktacktoe hundreds of years.

With the constant emphasis on computation, there is the pressure of getting the correct answer. This can create an atmosphere where students repeat what they hear and are afraid to speak or take a chance on anything they are not sure is right. There are many ways to solve most problems. Students need to be encouraged to take chances. As John Allen Paulos wrote, "The popular belief that all wrong answers are equivalent, or even that all right answers are equivalent, lessens the need to think critically, and this accounts for its prevalence among students and sad to say, among many teachers as well." [1] Math should be about thinking and applying our skills and knowledge to new situations or problems that we find interesting.

Could math be a "fun" subject and still fundamental to a good education? Teaching a series of lessons based on the ancient game of Ticktacktoe, would be time well spent in elementary classrooms. This is not about playing games for the sake of fun. Fun and enjoyment in solving problems should be a cornerstone to an effective instructional math program. People who are highly motivated learn concepts at a deeper level than those who are told "facts" or asked to memorize information without understanding it. Bill Tilden wrote in 1950, "A person who really learns to concentrate, so that the mind does not wander to extraneous things during study, can do a job in about half the time that the person of average attention can." One does not develop that level of concentration unless he/she is highly motivated. Students who are having fun solving problems do have this high level of concentration. [2]

Build an atmosphere where problem solving is studied. Ticktacktoe can be a useful format to the introduction of complex problem solving program. Yes, the game can be mastered with an hour or so of study. Yet, how many students, teachers, or adults have the skill, patience, or motivation to study a problem for an hour? In today's America, few people are inclined to study problems deeply. Students who have experienced problem solving lessons based on games, puzzles, and interesting mathematical concepts are more likely to be motivated to understand math at a deeper level. They also may develop a lifelong liking to solving problems.

North Carolina Standard Course of Study

The North Carolina Standard Course of Study (NCSCS) has five competency goals for

the third, fourth, and fifth grades in math. The first through fourth competency goals deal mostly with computation. Of course all students need to master these skills if they are to be proficient in math. Gifted students rarely have trouble picking up these first four competency goals and the algorithms that are standard procedure in most arithmetic classes.

Competency Goal 5

In the third and fourth grades the NCSCS states: "Competency Goal 5: The learner will recognize, determine, and represent patterns and simple mathematical relationships." [3] To be successful at the game Ticktacktoe, the student must master the skills in Competency Goal 5. The patterns within the game must be determined. The student must recognize these patterns. While studying these patterns, the students should represent these patterns in a methodical method. To do this, the student needs to represent the patterns in some written form. These first representations need to be recorded in a methodical form so they can be studied by the student.

Direct instruction or Indirection?

The game may be simple, yet the analysis of Ticktacktoe is not. The skill of problem solving or the analyst of a problem needs to be taught and is essential in using the computational skills. How this instruction is carried out may be beyond the scope of this paper. It should be noted there are times for direct instruction and times to allow the students to be able to figure out the problems themselves. This question of instructional style depends on the objective of the activity and the judgment of the teacher.

Different games have different objectives. In the game of Chess, certain skills and moves, and strategies are needed to help students improve their level of play. For the serious Chess player, the objective is improvement in the game. Chess is more complicated than any game presented here. A player may read several books all on themes of the same opening moves. Problem solving is just a by product of learning the game of Chess. So, in Chess direct instruction is highly valued in attacking or defending specific situations.

Should these strategies be taught directly or should the students be allowed to discover the game's solutions on their own? Well, that depends on the objective of the lesson. If the lesson is an introductory lesson, direct instruction may help the students start on the path of discovering problem solving. Some of the more advanced games based on

Ticktacktoe may be more beneficial for the students to discover solutions or strategies of their own. The focus of the lessons centers around developing problem solving skills not just mastering the game.

When the game is secondary to the skills of problem solving, allowing students to explore the process may be a preferred method. It is the instructor's call on how much information should be presented.

How to Play the Game

For this paper, instructions will be given. Most teachers do not want to do the studying themselves. There will also be variations of the game that may be complicated and they may not even look like Ticktacktoe. These games could become challenging or frustrating to adults. Challenging students and teachers may be a good thing while frustrating them is not. Solutions will be addressed because we do not want anyone to be frustrated reading this paper or in any math class.

Effective Play of Ticktacktoe

To play the game effectively one must know some strategies. Of course, the rules must be followed. Example: X always goes first. This gives X an advantage in every game. This advantage can be overcome. Most of us know that well played games of normal Ticktacktoe end in a draw.

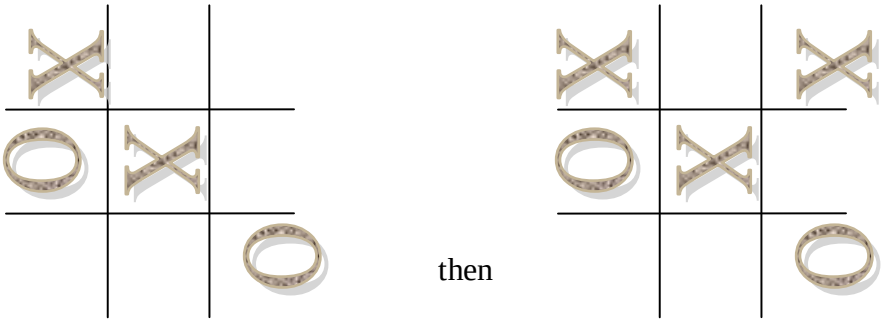
Still, not everybody knows these strategies or the reasons behind them. The board stays the same no matter how it is turned. There is no top, bottom, right, or left, because it can be rotated and it is still the same.

There is a middle space, four corners and four edges. As we will see, they do not have the same value when trying to win the game. Like chess and other board games, some locations on the board are stronger than others.

Where to Start in Ticktacktoe?

Some parts of the board are more powerful than other parts. Where should X move first?
(In regular Ticktacktoe it is always an advantage to go first.)
Middle (See figure 1)

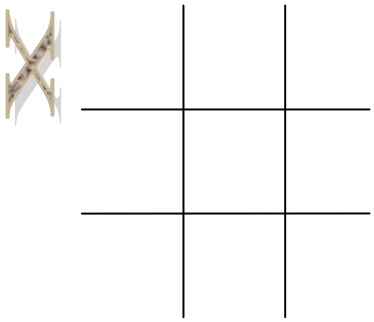
(1)

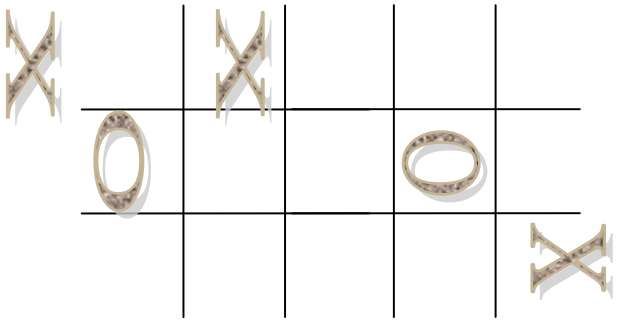


then

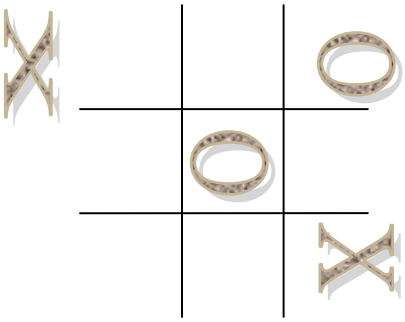
Most people choose the Middle spot. If O moves in any of the Edge spots on the next two moves, X will have a chance to win by taking either corner spot next to O's move. Good choice, but not the best.

Corner (See figure 2)

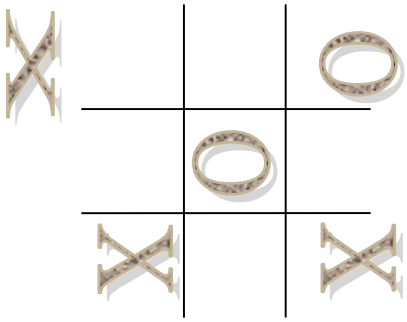




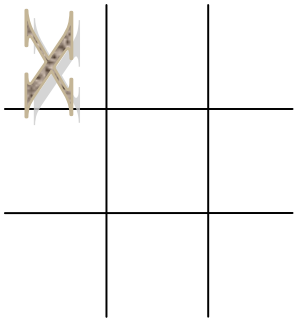
(2)



This is a mistake for O.

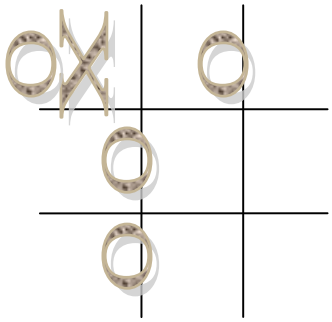


The Corner spot is the best choice, because it gives O the chance to choose 8 different spots on the board and 7 of those spots loose. Only if O chooses the Middle spot will O have a chance of a tie.
Still there is a trap for O. If X chooses a corner diagonal to the first X then O must choose an Edge on their second move.
This is the exception to the rule of choosing the most powerful spots!
Edge (See figure 3)



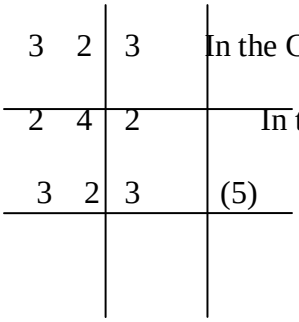
(3)

The Edge spot for the first move is a poor but possible choice. For O to tie the game, O must choose one of the following moves as their second move. [4] (See figure 4)



(4)

All of this is because some spots are more powerful than other spots on the board.
If possible, stay away from the edges!



(5)

In the Corners, winning passes three ways.
(See figure 5)
In the Edges, winning passes two ways.
In the Middle, winning passes four ways
It is clear some spaces are stronger than others.

What about other games based on Ticktacktoe? Let's look at some other games based on this simple game of Ticktacktoe. These games are more than just analogies to Ticktacktoe; they represent a basic math concept called isomorphism which deals with equivalence.

"15"
A word game that does not look anything like Ticktacktoe
Symbols
Haunted House
Reverse Ticktacktoe
Wild Ticktacktoe and Reverse Wild Ticktacktoe
Ticktacktoe with different number of squares

All of these games are from Martin Gardner's writings. Other sources can be found, but for the purposes of this paper there is enough information to have a good time and still become totally confused.

The Game of 15

15 is a great game and a simple one too. The goal is to choose three numbers that add up to 15. You choose from the numbers available, that have not been chosen already, from 1,2,3,4,5,6,7,8,9. You and your partner play against each other taking turns choosing numbers. There are times you must block your partner or he/she will choose their winning number. [5] (See figure 6)

15

1 2 3 4 5 6 7 8 9

(6)

This game should be played many times, before any clues are given as to what is behind the game. Of course it is Ticktacktoe it just doesn't look like it.

If you want to play "15" well just follow the illustration and then play Ticktacktoe. . (See figure 7)

2	7	6
9	5	1
4	3	8

(7)

Isomorphism sure is a fancy word. Isomorphism is a mathematical term meaning equivalence. Two mathematical systems are isomorphic if the sets of objects are the same size and the operations on these objects are equivalent. The names of the objects may be different, but the process is the same.

Isomorphism is a major concept in problem solving and in math is to figure out a difficult problem by making it simpler. Sometimes this is reversed by taking a simple problem and expanding on the core concept of the problem and applying it to a more

complicated problem. Ticktacktoe and other simple puzzles and games have been the basis of much more complex puzzles and games.[6]

Nine Words

Nine Words is a made up name of a word game which you try to get three words that have common letters from a list of nine words.[7] Included are two lists, one written by Martin Gardner and one by the author of this paper. Play Nine Words with each of the lists and then try to make up one of your own. This too is an isomorphism of Ticktacktoe.

Martin Gardner's list	The Author's	Your list of words
HOT	home	
TANK	use	
TIED	told	
FORM	tummy	
HEAR	sore	
BRIM	hut	
WOES	heart	
WASP	map	
SHIP	sat	

Nine Words can be written in any language. It just takes a little practice. [8]

Yes, you should also make up your own list of words that fit into this game.

In the diagonal shown, H is the common letter. In the line across, A is the common letter. (See figures 8 and 9)

Nine Word answer lists

HOT	FORM	WOES
TANK	HEAR	WASP
TIED	BRIM	SHIP

(8)

home	map	tummy
sore	heart	use
told	sat	hut

(8)

Nine Words isomorphism becomes a new game, Symbols

Nine Words can change into a game; let's call it Symbols that just has eight symbols. This is a great example of how to play Ticktacktoe again. To win the game, one must pass through three spaces on the board that have the same symbol and are in line.[9] For this game of Symbols the symbols are! @ # $ % ^ & *, but they could be anything.

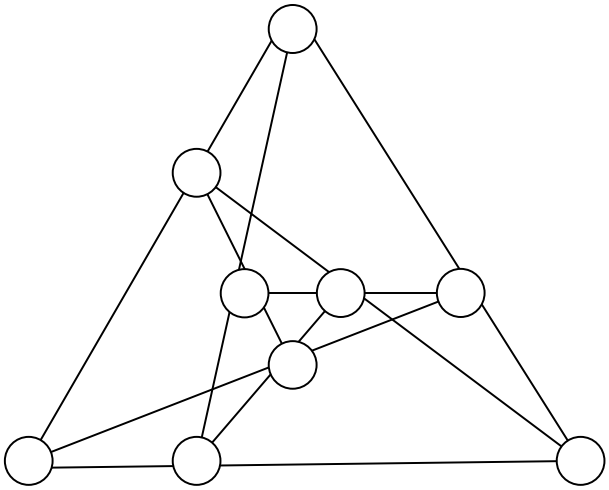
Once again, the symbols represent the eight ways it is possible to win. Refer to page 4 to see three ways to win in the corners, two ways to win in the edges, and four ways to win in the middle. (See figure 10)

! @ #	! $	! % ^
@ &	$ # & ^	% &
^ @ *	$ *	% # *

(10)

Tri-Hex

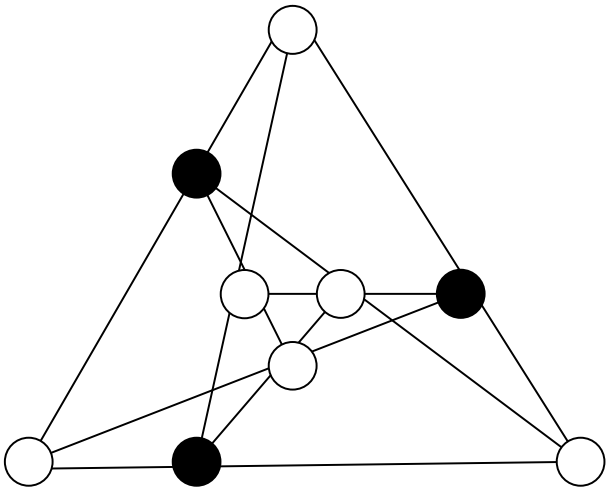
To play Tri-Hex each player needs four counters, pennies and dimes work well. The player who goes first chooses a place to put their coin and of course they take turns. There is no fifth move for the first player. The goal is to get three coins in a row.
Does the first player have an advantage in this game or is it going to be a draw each time it is played?
Take note the distance between the lines is divided by a space which reflects the lengths of the golden ratio. [10] (See figure 11)



(11)

Who wins and how?

Tri-Hex can be won by the first player if he/she moves correctly. The board does not have an up or down. It can be turned anyway and still be played the same.
The first player should be able to win if he/she chooses the middle outside space. After the second move, the first player should be able to force the second player into a weak space and win on the fourth move. Below is an example of where the first player should move first. Move to any of the black circles first. (See figure 12)
A very clever fifth grade student explained that these spaces will win because all of the ways to win pass through these three spot
It is still a complicated game.

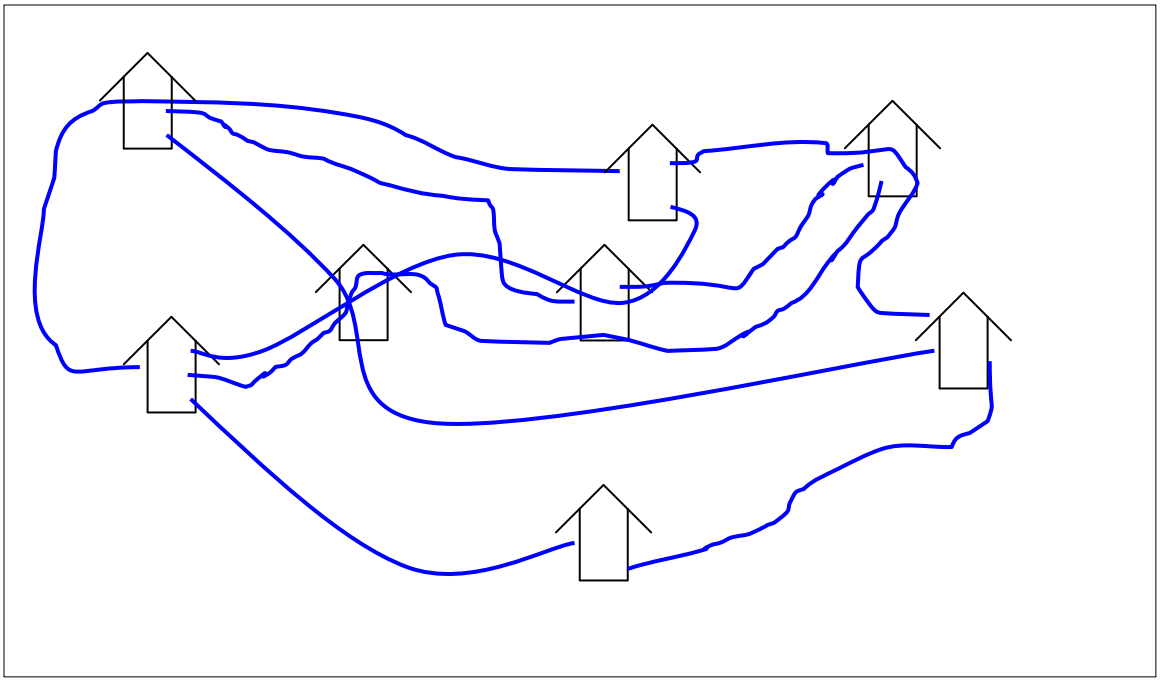


(12)

Houses and Roads

This does not look anything like Ticktacktoe. You and your partner take turns choosing roads using colored pencils. Notice some roads pass through houses and go on. Some just stop at the house. The first to take three roads to the same house is the winner.[11]

(See figure 13)



(13)

Solution

Isomorphism from Ticktacktoe can be discovered by following the roads and numbering them. They will be numbered 1 through 9. Just like the nine spots on a Ticktacktoe board. The houses, of which there are eight, stand for the eight ways to win in Ticktacktoe.

This is complicated and difficult to see as Ticktacktoe. It may be easier to copy this illustration or copy the one in Martin Gardner's book Aha! on page 120 and play the game several times. Study and analyzing the game is useful, but so is playing the game!

Wild Ticktacktoe

A simple variation of Ticktacktoe is Wild Ticktacktoe which only changes one rule. Each player can choose to draw an X or an O each time it is his/her turn. The player who completes three in a row wins.[12] Strategy has to change for this game.

As mentioned, in regular Ticktacktoe, all games should be ties if played correctly by both players. What about this game? Who has the advantage the first player or the second

player? There are two ways to find out who has the advantage and how. Either study the game or read the solutions on the next page.

Wild Ticktacktoe solutions

The first player should win every time. After the first player plays an X in the center spot the second player must play an O or the game will be over on the next move. No matter where the second player moves, the first player can respond with another O on the opposite side. Soon the second player will run out of places and the first player will win.[13]

This reminds the author of another game of circular ten penny Nim. The play of both games follows the same strategy by mirroring the other player's moves. This idea of symmetry comes up in many games.

Wild Ticktacktoe Reversed

What if we play the same game and the first player who gets three in a row loses? Again symmetry plays a role in the solution. If both players play logically, the game will be a tie every time. Just play the opposite of regular Ticktacktoe and follow the symmetry. This plan also works in reverse regular Ticktacktoe.[14]

There are many variations of Ticktacktoe. Some have boards that not just three across, but many "slots" across. Other forms are three dimensional. Rules can change and boards can change. Like many old games, people have playing with and rearranging them for many years.

The goal of these games is for them to be intellectually engaging. For a game to be engaging, it must be fair and challenging. A game that is easily mastered, or gives an advantage to one of the players will loose the interest of the players right away. Since there are so many forms of Ticktacktoe, and since it has been around for hundreds of years, it is difficult to come up with something new and still worthwhile.

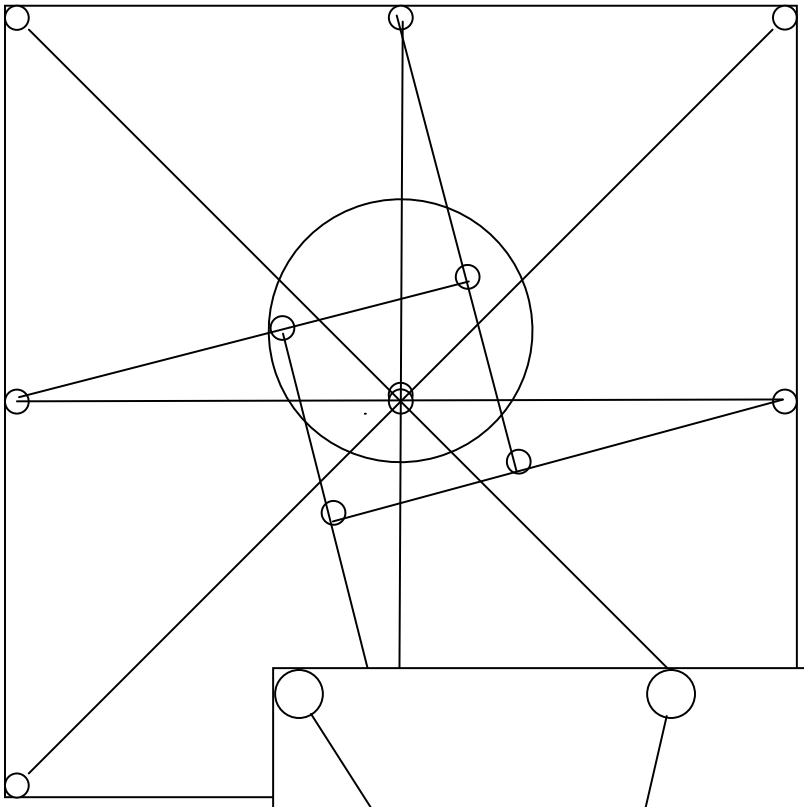
In our seminars, we have looked at many engaging puzzles, shapes, and models which are three dimensional. Transferring 3 D puzzles to two dimensions seemed to be a good place to start in developing a new game.

By placing squares on top of squares a simple game was discovered. This game is

similar to Tri-Hex, but with squares and circles instead of triangles. Still the object of three in a row is still in play.

The small circles are the spaces occupied by the players. Of course, players must play defense as well has offense. With most games, the player who goes first has an advantage.

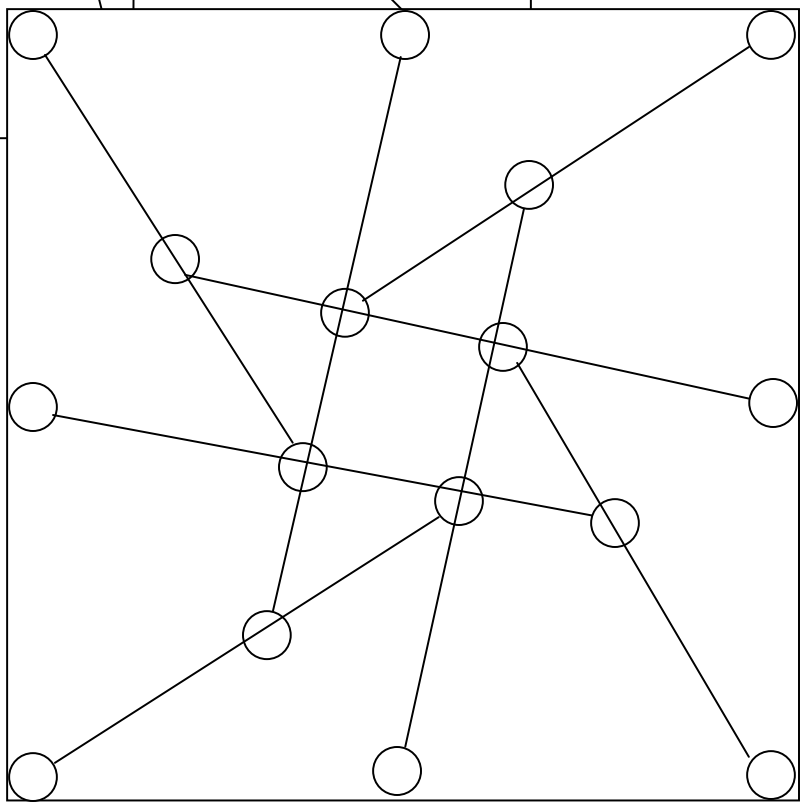
This is an example of a game that looks good, but is unfair, because the center spot has so much power. Figure (14)



Not a fair game.

(14)

Even when one adjusts the rules so the player which goes first may not choose the center spot, the game is still unfair.

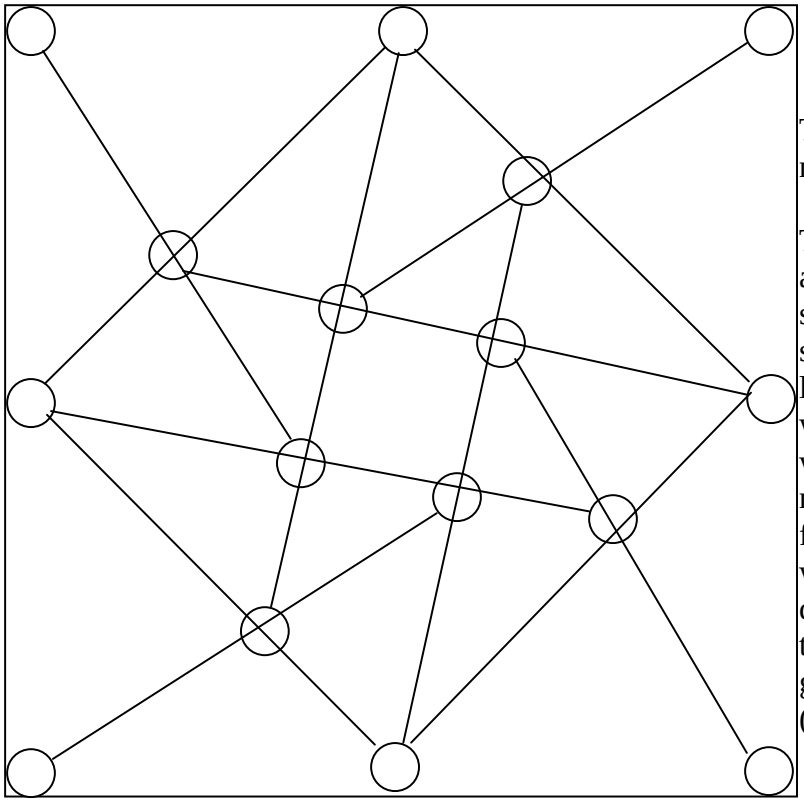


Whoever chooses the center spot will have an unfair advantage. Notice each of the spots except the center spot has three winning ways attached to it. The center has eight ways and it makes the game unfair. The center space must go!

Figure (15)
By adding another square between the center square and the outer square, we have a more challenge game and one that is fair with some simple rule changes.

The first player may not choose one of the center four spaces on his/her first move.
(15)

There are traps in this game, and many options, but it is a fair game. If played well, the first player and the second player should end with a tie. Currently, the game does not have a name. There are copies of this game in the appendix. With a little effort, you can master this game too.



This version of the game is not a fair game!

There is another square added between the outside square and the small square in the middle. It looks and is similar, but with the extra lines for winning. This change makes the player who goes first a sure winner. One will only see this difference if one studies the game and plays the game a number of times.
(16)

This game is similar to Ticktacktoe, but the two games do not have a true isomorphic relationship. The introduction of a possible four in a row in the new game makes this a

little different from Ticktacktoe. It does meet the goal of being intellectually engaging as demonstrated in classroom in lessons. It is difficult to draw, so it may be more useful in the classroom as a board game or a wooden model. A wooden model will be in the appendix if that is acceptable.

The game is not important in itself. It is only a vehicle or a means to an end. Educators need to understand the game of Ticktacktoe or any game or puzzle can be a learning tool. Our goals should help students learn how to solve problems, and to enjoy solving problems. If we can do this, students will take responsibility for their own education, because they will want to. We will also have productive and happy adults.

1 Beyond Numeracy page54 by John Allen Paulos

2 How to Play Better Tennis page13 by Bill Tilden

3 North Carolina Standard Course of Study

4 Hexaflexagons and other Mathematical Diversions page38 by Martin Gardner

5 Aha! Page118 by Martin Gardner

6 Aha! Page119 by Martin Gardner

7 Aha! Page119 by Martin Gardner

8 Aha! Page119 by Martin Gardner

9 Aha! Page119 by Martin Gardner

10 Mathematical Magic Show Page 69 by Martin Gardner

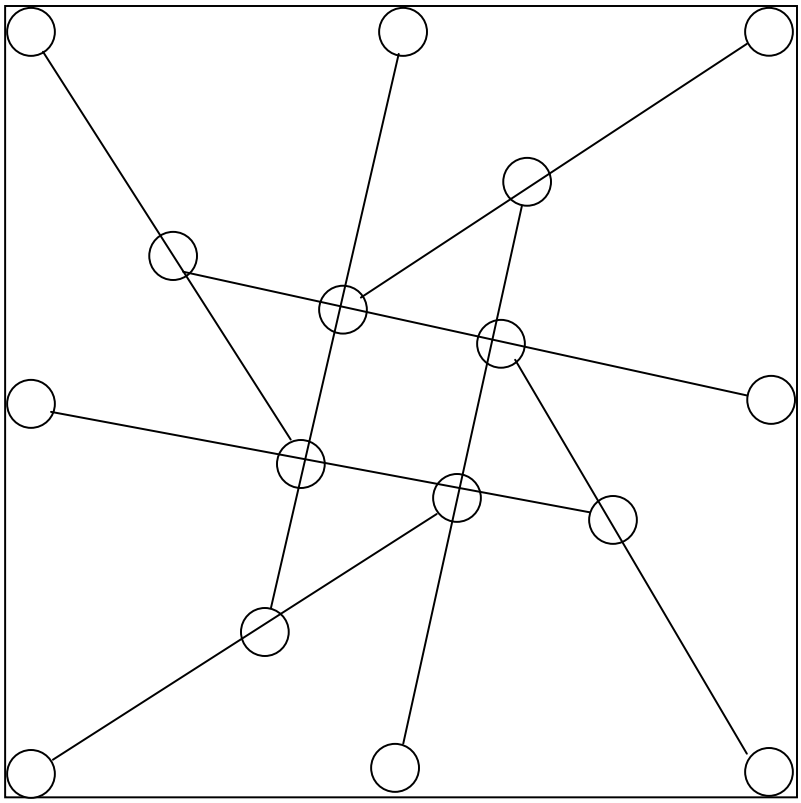
11 Aha! Page119 and 120 by Martin Gardner

12 The Colossal Book of Short Puzzles and Problems Page314 by Martin Gardner

13 The Colossal Book of Short Puzzles and Problems Page326 by Martin Gardner

14 The Colossal Book of Short Puzzles and Problems Page326 by Martin Gardner

Appendix



The player who goes first may not take any of the inside four spaces on their first move.