Math, Logic and KENKEN®

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Introduction

Educators in the classroom today are faced with one of the most difficult tasks in decades. Provide each student with tools they will need to master in order to become productive and successful in their life. The current teaching methods educators have in their toolboxes are some of the ones used when their student's parents were in school. In today's times, the successful educator does not have to completely do away with these tools, but he or she must develop and enhance them with new, alternative methods of teaching in order to reach this generation of students.

Throw the words Math and Logic out in the classroom and most students will react in a negative way. These two words wreak havoc with students when used separately but put them together and the word 'intimidating" comes to mind. When beginning lessons involving math and or logic, the teacher has to lead by example, extrude a feeling that this subject may be challenging but reaching the answers through successful use of the process yields positive end results. The teacher must also relate the processes so the students understand that many of these processes apply to activities the student does every day.

One issue many teachers face is that the foundation lessons of math and logic were never completely grasped by the student which inevitably leads to the student struggling throughout their academic career and even when they enter the workforce. A simple test that proves this theory is to go into a store or fast food restaurant where young students work. The simple act of making change can be very challenging to complete without the use of a register or calculator. Though modern machines and technology have replaced the basic need for people to use their mind processes, the educator must reinforce that there will be instances when these machines and technology are not available and good old brain power will have to be used.

When using a new or maybe even an untested alternative teaching method, there are several critical factors that the teacher must ensure are in place. To be effective the educator must ensure their "foundation" or knowledge of the process and materials being presented are solid and will not easily crack. Once students see that the teacher is struggling then it makes it easier for them to decide that subpar work and or failure to gain the knowledge is acceptable or that success is unimportant. Also, today's students are inundated with rules and regulations not only in school but in society as well. A teacher must maintain a balance between being perceived as overbearing compared to no control at all.

The learning environment must be conducive to the lessons and materials being presented. Obviously, there are factors and distractions that the teacher cannot control but there are also many factors or distractions that the teacher has some control over.

Lastly the teacher needs to know that he or she has the backing of their administration to use and implement an alternative teaching method. This support can be a turning point in the comfort level of the teacher, not worrying about repercussions for trying something new and different to deliver the lesson.

Objective

In my years of teaching I have found that students struggle with multiplying and therefore with factoring. I have also notice an increasing lack of work ethic and problem solving skills. Students are more likely to “give up” rather than work in depth on a math problem when the answer isn’t easily found. They seem to just want all the answers given to them without having to think things through. It seems this generation is fast paced and more in the mindset of “just give me the answer”. The students seem to want to take the easy way out. My objective for this unit is to show students that Math can be fun and that there is self satisfaction in solving puzzles by using logic and knowledge.

I teach at a school in an urban district that has approximately 845 students. Of those students, 62% are African-American; 20% Caucasian; 2% Asian; 10% Hispanic; and 4% that are Multi-Racial. My school also has a high percent of free or reduced lunch students. I have noticed that education is not the most important thing in most households of the students. There are some households that do value education for their children, but most seem to think of the teachers and school as just a free babysitting service. Most of the parents or guardians of our students barely finished high school and very few went to college. The students see this and think that there is nothing better for them. They don’t know what the future could hold for them with an education, because they have no encouragement nor support from home. Unfortunately, the only people who discuss a future with many of our students are the criminals who want to use them to better their own “business”. The challenge I have is showing the students that Math is important and can make a difference in their lives. I also need to show them that they use it everyday, in one form or another. The *KenKen®* puzzles are fun and yet challenging. They also allow the student to make mistakes that can be corrected without any damage being done later in the puzzle. I think it is also necessary for students to learn that making a mistake is ok, as long as you can learn from it. Since there is only one solution for each puzzle, the student knows immediately if they have done the puzzle correctly. They don’t have to wait on the teacher to tell them the correct answer.

My end goal with this unit plan is to review factors and to show students that using factors can be fun. I also hope that students will see that solving a puzzle using logic as well as Math skills can be challenging, but rewarding when solved and increase their self confidence.

What is the KenKen® Puzzle?

Most people are familiar with the *Sudoku* puzzle. This puzzle uses a 9x9 grid and 3x3 "cages" within this grid. For each row the solver can use the numerals 1 through 9 only once and can also only use those numerals for each column. The other rule is that the solver can only use those numerals with each "cage" once. If a student is not familiar with this puzzle, then I would first have them work with another student that is familiar and confident about solving it. It is helpful if students are familiar with this puzzle before they try to start solving the *KenKen*®.

There is a new type of puzzle that has been introduced called a *KenKen*® puzzle. This puzzle is similar to the very popular *Sudoku* puzzles. The *KenKen*® puzzle was created in Japan by Tetsuya Miyamoto. He runs his own school and entry is based on a "first-come first-serve" basis. He uses the *KenKen*® puzzles as part of his "teaching by not teaching" method. He gives his students the puzzles to solve with no questions allowed and no solutions given. He simply tells the students when the puzzles they solve are correct. There are others who have helped make *KenKen*® popular in America and other countries. The term "ken" means "wisdom" in Japanese, so the term "kenken" means "wisdom squared".

We will start with the easier puzzles using basic addition and subtraction skills, before moving on to the puzzles that also incorporate multiplication and division. There are various levels of difficultly for the *KenKen*® puzzles as well. This will help when teaching different levels of students. The *KenKen*® puzzle also incorporates several NCTM standards with each puzzle, such as problem solving, analyzing and communicating solutions with others.

The teacher should first explain how the *KenKen*® works and describe its features. The numbers that can be used depends on the number of boxes in each row. In a 4x4 puzzle, you can only use the numbers 1, 2, 3 and 4. In a 5x5 puzzle, you can only use the numbers 1, 2, 3, 4, and 5; and so on. Each number can only be used once in each row and each column. There are heavily outlined squares, which are called *cages*. Each cage has a *target number* in the upper left hand corner. This is the number the students should reach when doing the mathematical operation that is next to the target number. If there is a cage with just a number and no mathematical operation in it, then that is the number that occupies that box.

The teacher will start with the basic level of *KenKen*®. This puzzle is in a 4x4 format and will only involve addition and subtraction. With the *KenKen*® puzzle, there is no absolute position to start. The "trick" is to see if there is an expression that can only have one solution. This is generally the best place to start for any of the puzzles.

The following steps are recommended for the puzzle below. First, label each square with a letter (as shown in the puzzle next to the original puzzle). This will make identifying the square that students want to solve easier. Since there is a cage with a 4 and no math function in the lower left corner, place a 4 there (the square labeled "D"). Then, place a 2 in the cage with just a 2 in the left corner (square "N"). It might be better to give the students some direction for their first puzzle. Ask them to start on the left side of the puzzle and work going to the right side. It will not make a difference in the solution, however. I just find that with their first puzzle, the students need more guidance and specific directions. Next, ask the students what expressions can only have one solution. In the first column, the "5+" must use 3 and 2 in that order because of the "7+" in the third row (the 3 and 4 must be used here). So the order of the first column must be (from top to bottom) 1, 3, 2 and then 4. The second column must start with a 3, to be added to the 1 from the first column to equal 4. The students should be able to see that square "G" must be a 4, which leads square "F" to be a 1 and square "H" to be a 2. Now you can move to the third column. Square "J" must be a 4 and square "K" must be a 3. Students should start to see the pattern and be anxiously calling out the solution to the rest of the puzzle.

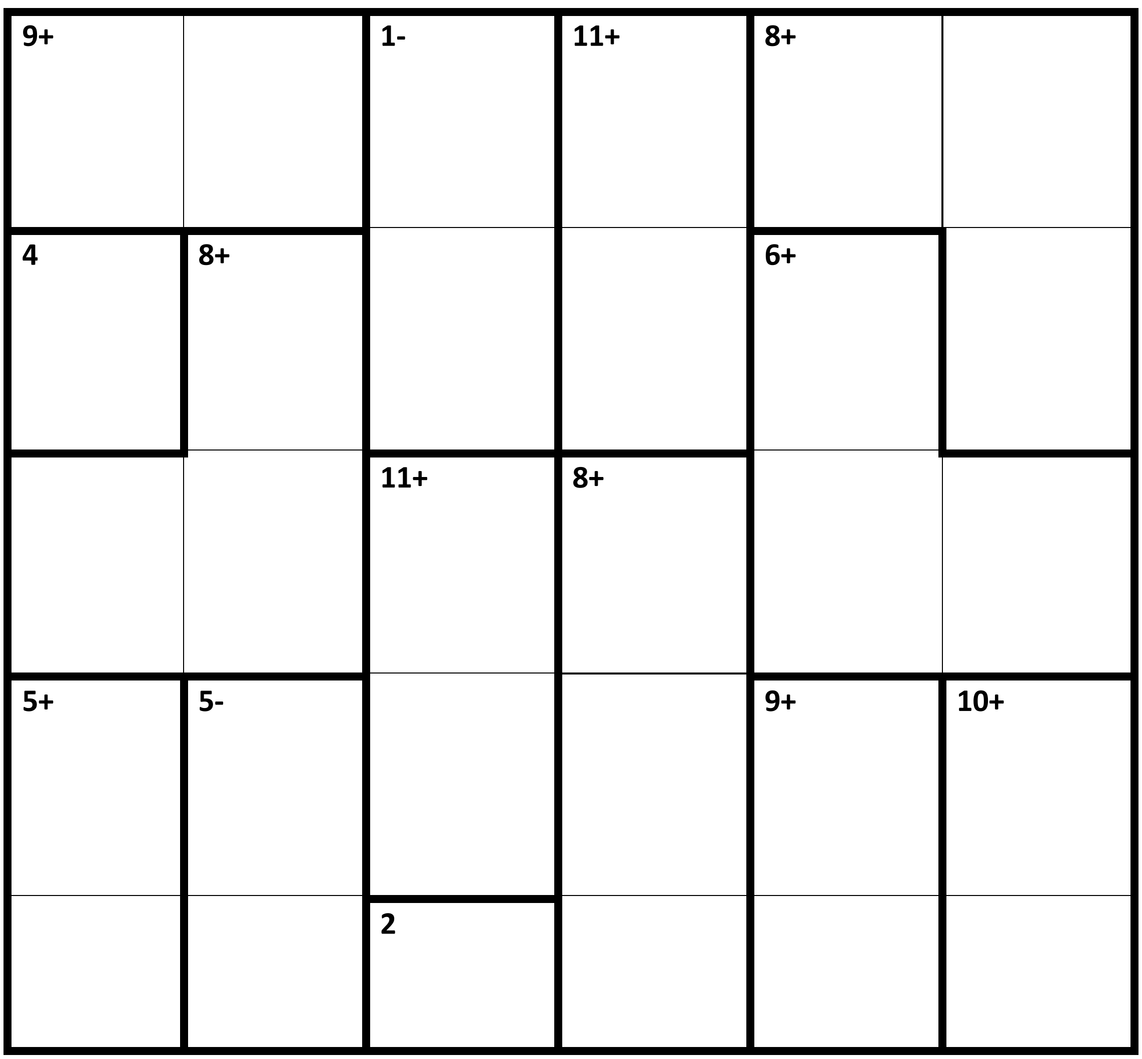
4+		**6+**	
5+	**3-**		**2**
	7+		**2-**
4	**3+**		

4+ A	E	**6+** I	M
5+ B	**3-** F	J	**2** N
C	**7+** G	K	**2-** 0
4 D	**3+** H	L	P

Here is the solution to the puzzle from the previous page:

4+ 1	3	**6+** 2	4
5+ 3	**3-** 1	4	**2** 2
2	**7+** 4	3	**2-** 1
4 4	**3+** 2	1	3

It would be more productive to have the students solve several of the 4x4 puzzles before moving on to a larger puzzle. I also find it helpful to list the numbers that can be used next to the puzzle. It might sound “childish”, but I think it will help for the students to “see” the numbers they can use. It would also be wise to stay with the puzzles that only involve addition and subtraction. This will build their confidence with solving the puzzles and with their abilities to solve mathematical problems and to use reason—not just “guess and check”. On the next page is a 6x6 puzzle, and below it is the same puzzle with the squares labeled. The same reasoning will be used to solve this puzzle.



9+		3+		9+	

9+ A	G	1- M	11+ S	8+ Y	EE
4 B	8+ H	N	T	6+ Z	FF
C	I	11+ O	8+ U	AA	GG
5+ D	5- J	P	V	9+ BB	10+ HH

E	K	**2** Q	W	CC	II
9+ F	L	**3+** R	X	**9+** DD	JJ

Here is the solution to the 6x6 puzzle from the previous page:

9+ 6	3	**1-** 4	**11+** 5	**8+** 2	1
4 4	**8+** 2	3	6	**6+** 1	5
1	5	**11+** 6	**8+** 4	3	2
5+ 2	**5-** 1	5	3	**9+** 4	**10+** 6

3	6	**2** 2	1	5	4
9+ 5	4	**3+** 1	2	**9+** 6	3

Introducing Factoring with the KenKen® Puzzle

Since the puzzles are progressive, I would then ask the students to work in pairs to solve a 6x6 puzzle. As their confidence builds, I would start to introduce the fact that the rest of the puzzles they will solve will include all four arithmetic computations. Before starting any puzzles; however, I would review factors and how they are found. The most important thing would be for the students to know what the term *"factor"* means. I have noticed that students frequently get the term *factor* mixed up with *multiple*. Students will need to be reminded that the factors are the two numbers that are multiplied together to get the product. I will spend time having the students complete the factoring for several numbers before they start the next level of the *KenKen*® puzzle. I have a "circle around" process that I use with my students. I have found that students are less likely to "miss" a factor if this process is used. The process is set up like a table. The students will list the number they are factoring at the top, then list the numbers side by side that multiply together to give you "factor number". When you get to a number on the second column that is consecutive (or almost consecutive) from the last number on the first column, then you know that you have listed all of the factors available. This is why I call it "circle around", because the first column "circles around" to the next number in the second column.

<u>120</u>

1	120
2	60
3	40

4	30
5	24
7 (doesn't work)	
6	20
8	15
10	12

11(doesn't work)

I have the students do the "circle around" method several times, with all sorts of different numbers. After watching the students and working with them, there is another issue I have found when they are trying to factor. It is that they are not strong when it comes to division facts. For example, most students would not see that 120 is divisible by 3. After witnessing this over and over again, I have chosen to ask students to make a divisibility chart in their Math notebook to refer back to and to memorize.

Here is the chart:

NUMBER	***RULE***
2	If the number is even, then it is divisible by 2.
3	If the sum of the digits of the original number is divisible by 3, then so is the original number.
4	If the last 2 digits of the original number are divisible by 4, then the original number is also.
5	If the last digit of the original number ends in a 5 or 0, then the original number is divisible by 5.
6	If the original number is divisible by 2 ***AND*** 3, then it is also divisible by 6.
9	If the sum of the digits of the original number is divisible by 9, then so is the original number.
10	If the original number ends in a 0, then it is divisible by 10.

Now, when students factor a number, I ask them to explain why a number is a factor or why it isn't a factor. This is one of the times when memorization is the best option. If the students memorize the chart above, it will save them time and help them with finding factors.

The next lesson that generally follows finding factors is to find the GCF (greatest common factor) for a given set of numbers. There are at least two different methods that can be used and that I teach my students. The first way is for the student to list the factors of each number in the set of numbers given using the "circle around" method. The students will then list all the common factors and therefore find the greatest of the common factors. Another method is called the "ladder method". To use this method the students set up a division box and place the set of numbers being compared inside the box. I call this a "double division" problem. They will then find a number that both numbers can be evenly divided by and then solve. They will then use the answer from the previous problem and follow the same steps until the answer has 1 as the only common number. To find the GCF, the student will then multiply all of the numbers that are on the "outside" of the division box. For example, to find the GCF of 24 and 36 the students will set up the problem like this:

$$\begin{array}{r|rr} \cline{2-3} & 24 & 36 \end{array}$$

Both of these numbers are even (which is what most students recognized first) and therefore are divisible by 2.

$$\begin{array}{r|rr} \multicolumn{1}{r}{} & 12 & 18 \\ \cline{2-3} 2 & 24 & 36 \end{array}$$

The students should then follow the same procedure and find what number 12 and 18 can be evenly divided by. Again both of these numbers are even and can be divided by 2.

$$\begin{array}{r|rr} \multicolumn{1}{r}{} & 6 & 9 \\ \cline{2-3} 2 & 12 & 18 \\ \cline{2-3} 2 & 24 & 36 \end{array}$$

Now the students need to be able to see that 6 and 9 can be evenly divided by 3 and follow the same process.

$$\begin{array}{r|ll}
 & 2 & 3 \\ \cline{2-3}
3 & 6 & 9 \\ \cline{2-3}
2 & 12 & 18 \\ \cline{2-3}
2 & 24 & 36
\end{array}$$

Since 2 and 3 only have 1 as a common factor, this is where the division ends. To find the GCF, the student will need to multiply the outside numbers (3*2*2) which is 12. Therefore 12 is the GCF for 24 and 36. If there are 3 numbers in the set of numbers give to find the GCF, the students will use the same process, only it will be “triple division” instead of double. It makes it a little challenging to find a number that will “go into” all three, but still possible. I have taught both methods to my students and many find the “circle around” method much easies, while others like the “ladder method” better. This is just one of many examples showing the reason why teachers must teach the objective using different styles. It makes planning a lesson more challenging and more time consuming, but the payoff is that almost all students should be able to solve and answer test questions correctly. I also absolutely love to see the “I got it” light shine on their faces (which is why I love teaching so much!!!).

Creating a KenKen®

Now that the students have had practice with finding the factors of numbers, they should be even better at solving the *KenKen®* puzzles. Since there are different sizes and levels of these puzzles, I would start the students off using a medium level of a 6x6 puzzle and gradually increase the difficulty as the students display success with the previous level.

Once the students feel confident in their ability to solve a 9x9 puzzle I would introduce the culminating activity—for them to create their own *KenKen®* puzzle. This is definitely not something I would introduce at the beginning of the lesson. I would save this activity until year-end. The best time might be during the time after EOG and before the end of the school year, where there is some “down time” from teaching standards. You are still asking the students to use logic, which is always something that needs teaching.

The students may feel intimidated with the idea of creating this puzzle, so it might be best to start them off with creating something a little easier. Having the students create a "mine sweep" puzzle would be a good introduction. Of course having the students solve a mine sweep puzzle would be beneficial as well. Below is an example of a mine sweep puzzle:

	1		1		2		2		2
1		2		3		2		3	
	3		2		3		2		2
1		3		2		1		3	
	3		2		1		1		2
2		4		1		1		2	
	3		3		2		3		1
1		2		2		3		2	

The objective of this puzzle is to find which of the empty boxes contain mines, and which ones do not. The number in the box indicates how many mines there are in the boxes that touch one of the sides of the numbered box. As with the *KenKen*® puzzles, there is no absolute starting place; however, with this particular type of puzzle, it would be best to start with the boxes that surround the box with the 4 in it. This tells you that all 4 of the empty boxes contain a mine. It is also a good idea to place an X in boxes that do not contain a mine and an M in the boxes that do contain the mine. Here is the answer key to the above mine sweep puzzle:

X	1	X	1	M	2	X	2	M	2
1	M	2	X	3	M	2	M	3	M
X	3	M	2	M	3	X	2	M	2
1	M	3	X	2	M	1	X	3	M
X	3	M	2	X	1	X	1	M	2
2	M	4	M	1	X	1	X	2	X
M	3	M	3	X	2	M	3	M	1
1	X	2	M	2	M	3	M	2	X

Once the students have completed this puzzle, they should feel more confident in creating their own puzzle. To do this, you will need to make sure that you keep every other box open for a number to be placed in it at the end of creating. You should first decide where you want to place a mine and where you do not by following the same steps you used to solve the puzzle, only in backwards order. Once you have placed the mines where you want them, you simply place the numbers in the boxes to show how many mines are around it. Once the students have created their puzzle, it would be a fun activity to have student exchange their puzzle with another student to solve.

Now that the students have an understanding of puzzle building they should feel more confident on building their own *KenKen*® puzzle. Again, it would better to have the students start with creating a 4x4 puzzle using only addition and subtraction. Once they have become familiar with building these puzzles, then they can move on to creating the more difficult puzzles one level and size at a time. This process will be similar to creating a mine sweep puzzle, in that the student should work backwards. They should figure out what numbers they want in each box, then what arithmetic function they want to use in order to accomplish that number and how many boxes they want to use to make a cage. It would also be a good idea for the students to then try to solve their own puzzle, to make sure that it works and that there is only one solution.

Creating and solving these puzzles will build their confidence with creating problems for others to solve and it will also build their confidence with solving logic problems. Problem solving skills are being used while creating and solving these puzzles, which is a Math objective in every grade level and during the whole school year. Most importantly, using this type of teaching lesson should result in the students thinking, using logic and having fun all at the same time.