The Positives and Negatives of Integers

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Objective

The unit I have written is based on integers. Students who I teach need to be able to add, subtract, multiply and divide using positive and negative numbers; as stated in the North Carolina Standard Course of Study. My goal is for students to not only understand the concept of integers but also to correctly apply their knowledge to real situations. I will use different strategies and approaches for students to get a strong understanding of these concepts.

I teach seventh grade mathematics at Carmel Middle School, which is a part of a large urban district, Charlotte-Mecklenburg Schools. There are two levels of math in seventh grade, standard plus and honors. The honors level consists of students who are above grade level. Along with learning the seventh grade curriculum, these students are also responsible for learning eighth grade goals as well as a few topics from the Algebra 1 high school course. The standard plus level includes an academically diverse group of students ranging from below grade level to some students even a bit above grade level. To add to the different levels of students, the population I teach is culturally diverse. Carmel Middle School is approximately 60% Caucasian and 40% minority; Hispanic, African-American and Asian. I specifically teach a high population of students who are either Limited English Proficient or English is not their first or primary language. There are many students I teach that have learning disabilities and have learning accommodations as described in their Individualized Education Plan or 504 Plan.

One of the most difficult topics in mathematics for my students to grasp is operations with integers. Students need to have a strong understanding of integers in order to do higher levels of math. This topic is the backbone of so many other goals and topics in mathematics that ensuring my students truly have a good understanding of it is my number one priority. I want to enable my students to be as successful as possible in mathematics throughout the years.

In my experience, I have found my students usually understand the concept of ordering integers. They are able to put integers on a number line or list them in order from least to greatest. Through the introduction of integers, we discuss a lot about where integers are actually used. Examples include ideas such as temperature, money and bank accounts. The students are able to visualize and understand, for example, that -10 is less than -5 when they see it on a number line. They are able to identify the reason is because

-10 is further away from zero than -5. That shows me the students have somewhat of an understanding of absolute value. Operations with negative numbers are a new concept for them because they have not worked much with negative numbers up to this point.

A misconception when integer operations are first introduced is the students do not fully understand the negative sign and its meaning. For example, they get confused with the idea that a negative sign means 'subtract' but that it can also make a number 'negative'. Helping the students understand that they both mean the same thing. Adding and subtracting are the two most difficult operations for them. The students are able to understand, when looking at an integer such as -3, that it is negative 3. When the students are expected to begin doing operations with the integers is where the confusion begins.

In this unit, I will use activities to teach my students how to add, subtract, multiply and divide with integers but with a larger focus on the adding and subtracting. I will incorporate a few new strategies such as a puzzle known as KENKEN®, heaps and wholes method and The Singapore Model Method for Learning Mathematics approach to help the students think mathematically about these operations.

Strategies

A misconception students have later on in mathematics, specifically algebra, is whether the '- sign' means it is a negative number or it is a minus sign. In order to help them understand this, I constantly show examples to my students so they can see that the word 'minus' and 'negative' essentially mean the same idea; plus a negative is the same as minus.

$$6 - 7 \text{ is the same as } 6 + (-7)$$

My students have prior knowledge of how to combine like terms. When I teach my students to combine like terms I make sure they take the sign (addition or subtraction/positive or negative) that is in front of the number with it. I have the students circle like terms and box other like terms to separate the different terms. Then they simplify the expression.

For example: $3x \boxed{+ 7} - 2x \boxed{+ 2}$

$$3x - 2x + 7 + 2$$
$$x + 9$$

As they are simplifying, they are able to see that they are adding each of the terms together. They can see from this way of modeling that it is 3x + (-2x) and 7 + (+2). I will use this prior knowledge to help my students grasp adding and subtracting with integers better in several of the strategies that I will describe below. Because my students have a more difficult time with adding and subtracting of integers, my main focus in the unit is to use strategies to help them get a better understanding of how to perform the operations when given integers.

I have used many strategies in the past to help teach this tough topic of integers. I start by introducing situations in the real world. For example, I may ask the students, "If I borrowed $20 from my friend and then I got my paycheck for $35, how much money do I have after I pay my friend back?" When I relate the idea of adding negative numbers to money, it really seems to help the students understand the application. From that point though, it is difficult for students to do the correct operation when given the numbers, -20 + 35.

I have used a method using integer chips to help model operations to students. The chips are colored red on one side and colored yellow on the other. The red chips represent the negative numbers and the yellow chips represent the positive numbers. This method works by 'canceling out'. I will have my students always make addition problems by taking the sign that is in front of the number. I will have the students do the same technique they used when combining like terms, as I described previously, to circle each of the terms with the sign in front of it. If there is no operation left in between the two circled terms, they are to put in an addition sign. They could use the integer chips as a strategy for the following example.

-4 + 8 could be modeled by
r r r r
y y y y y y y y

The four red chips would cancel out four of the yellow chips, which would leave four yellow chips left or positive 4. Another example would be:

-6 – 5 or -6 + -5 could be modeled by
r r r r r r r r r r r

Since all of the chips are the same color, the students would count the total amount and would end up with -11.

This method is a great way for students to see what is happening on a concrete level. The only down side to this strategy is that it really only works when dealing with smaller integers. When the problems start having larger integers (absolute value of the integers), then it is time consuming to write out all of the *r's* and *y's*. If a problem states -25 + 72, it would take a lot of time to write down 25 red's and 72 yellow's.

I show the students that they can still use this method for larger absolute value integers because we will focus on place value. Instead of having 25 *r's* to represent the integer -25, we could use a different variable. The students can use *R* to represent the value in the ten's place and *r* to represent the value in the one's place. One R is equivalent to ten r's. The same idea could be used with the positive numbers also, Y could represent the amount in the ten's place of a positive number and y could represent the amount in the one's place. This type of strategy may take a bit more time because of all of the regrouping that is necessary, but the students will be able to see the importance of place value. For example:

-25 + 72 could be modeled by

R R r r r r r

Y Y Y Y Y Y Y y y

The two *R's* and two of the *Y's* would cancel out (a). From that point the students could cancel out two *y's* and two *r's* (b). The students will have to use their understanding of place value and that 10 *y's* are equivalent to 1 *Y* to finish out the model. The goal is to get either all positive or all negative in order to have a final answer. There are still three *r's* left. The only way to cancel them out is to rewrite *Y* using ten *y's* (c). From that point the three *r's* will cancel out three of the *y's* (d). The final answer would then be positive 47.

(a) r r r r r
Y Y Y Y Y y y

(b) r r r
Y Y Y Y Y

(c) r r r
Y Y Y Y y y y y y y y y y y

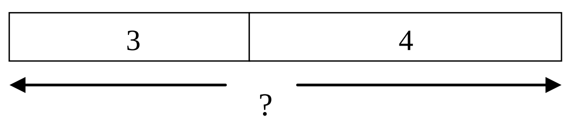
(d)
Y Y Y Y y y y y y y y

Again, this strategy may not be a very efficient one to use all the time, but it is a great way to introduce to the students adding and subtracting of integers. It also gives them a chance to review the topic of place value which is always an important topic my students could understand better.

Another strategy I will use is The Singapore Model Method to help students understand how to add and subtract integers. As stated in the book, *The Singapore Model Method for Learning Mathematics*, states, "…the Model Method is used to develop students' understanding of fundamental mathematics concepts and proficiency in solving basic mathematics word problems." The main idea with the Model Method is for students to use pictorial models to help them visualize the problem. Since my students struggle with abstract ideas, using the method will make the problems concrete. I will use the idea of the Model Method to illustrate adding and subtracting integers.

The way that I will show this is a lot like the integer chips that I previously spoke about. This will help the students to solve problems that deal with integers that have larger absolute values. The students will be able to use the idea of 'canceling out' as they did with the red and yellow chips, but instead of drawing out so many *r's* and *y's* they will draw bars to represent each of the integers. Because my students have not ever used this method before, I would start out with some basic examples to ensure they understand the process.

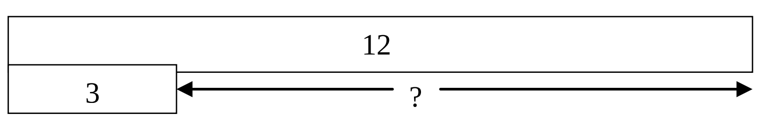
For example: 3 + 4



The students can make a bar that represents 3 and make a bar that represents 4 just a little larger, it is not necessary for the bars to be drawn to scale. The point is to be sure the larger number (or larger absolute value when dealing with integers) has a larger bar. From that point they can see that the total would be both of the bars together which would give an answer of 7.

Subtraction is shown a little bit differently. Because you know the total amount, the bars will be drawn on top of each other so that the difference can be observed.

For example: 12 – 3



The larger bar that is on top represents 12 and the smaller bar on the bottom represents 3. The arrows represent the space or amount that is left when finding the difference between 12 and 3. The students will quickly be able to identify the answer as 9.

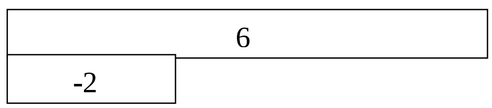
Using the Model Method with integers will be a bit different. I will have my students always make addition problems by taking the sign that is in front of the number. I will again have the students do the same technique they used when combining like terms, as I described previously, to circle each of the terms with the sign in front of it. If there is no operation left in between the two circled terms, they are to put in an addition sign. In order to make a subtraction problem become an addition problem, the students would do the following:

-7 – 3 would become -7 + -3

Students will use their prior knowledge to help them change a subtraction problem to an addition problem. From that point, the discussion will revolve around the idea of ‘canceling out’. But now, instead of using the integer chips, they can use bars to represent each of the integers. The following are two examples of adding and subtracting integers

Example A: -2 + 6

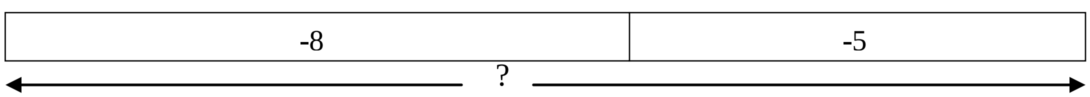
$\boxed{-2} + \boxed{+6}$



Answer : 4

Example B: -5 – 8

$\boxed{-5} + \boxed{-8}$



Answer: -13

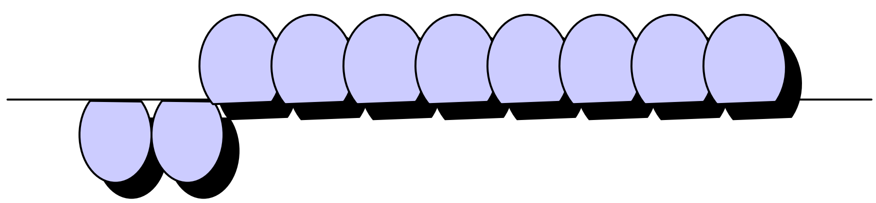
After students make sure the problems are set up as addition, they then need to look at the signs of the integers they are adding. If the signs are the same, the bars need to be on the same level; therefore they will use the addition model as shown in Example B. If the signs are different, the bars need to be on different levels so they will need to use the subtraction model as in Example A, which deals with canceling out. The way they draw the bars is the same as what they did with the integer chips. All of the y’s were on one level and all of the r’s were on another. It is necessary for the students to look at the absolute value of each of the integers in the problem when drawing the bars. In Example B, -8 is smaller than -5. However, the absolute value of -8 is larger than -5 therefore making -8’s bar larger than -5’s bar. The bars do not need to be drawn to scale. They just need to be based on the absolute value of the integer, like in Example B, -8’s bar needs to be larger than -5’s.

In Example B, the students would draw a bar that represents 6 and draw a bar beneath it that represents -2. The bars are not on the same level because they have different signs. After canceling out, they are left with 4. They will work to understand the rule that if the signs are the same then they are to add the absolute value of the numbers together and then take the common sign. If the signs are different, subtract the absolute values of the numbers

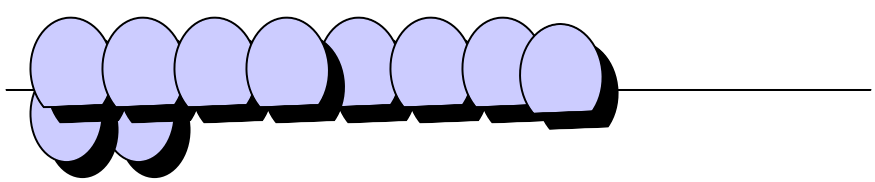
Students tend to really struggle with figuring out whether the final answer will be a positive or a negative. Using the Model Method should help to eliminate this problem. In Example A, the students should be able to see that 6 has a larger absolute value than -2 so therefore, the sign of the number in the largest bar will be the sign of the answer. Again, this is for the subtraction method. On the other hand, for the addition method where the bars are on the same level, the answer has the same sign of the two integers in the problem. Since both of the integers are negative, then the answer must also be negative.

Another strategy I will use to introduce adding and subtracting integers is the ‘heaps and holes’ method. This method or strategy was found in Classroom Strategies on the North Carolina Department of Instruction website. This strategy deals with having students think about digging holes and filling them back in.

-2 + 8 or -2 + 8 could be modeled by:



There would be two holes to represent the negative and eight heaps to represent the positive. The heaps need to fill all of the holes so that it can to cancel them out. Therefore, it is going to look like the following:



After canceling out, there would then be 6 heaps left so the answer would be positive 6. The students are able to conceptualize the idea of digging holes and filling them back in.

As the students work through each of these strategies, the hope is that the students will be able to get to the point to actually understand the rule of adding and subtracting integers. In the past, I have taught just the rules that the students were to memorize, but they were never able to have a strong understanding of 'why' they rules work. As I work through the strategies the students will be able to see that if the signs are the same, they are to add the absolute values and keep the sign that is common between both of the integers. On the other hand, if the signs are different, they are to subtract the absolute values and keep the sign of the number that has the larger absolute value.

Another strategy I plan to use with my students is a puzzle, known as KENKEN®. KENKEN's® are mathematical puzzles that are set up as a large square containing smaller squares inside. In a normal puzzle, for example a 4-by-4, it is necessary to fill it in using the numbers 1 through 4. When filling it in, there cannot be a repeated number in any row or column. The numbers in each heavily outlined set of squares, called cages, must combine (in any order) to produce the target number in the top corner of the cage using the mathematical operation indicated. Cages with just one box should be filled in with the target number in the top corner. A number can be repeated within a cage as long as it is not in the same row or column.

3-	**7+**		**2**
	2÷		**9x**
6x	**4**		
		2÷	

When solving the KENKEN® above, identify the operation and number in each cage. For example, the cage in the top left corner of the puzzle has a 3- which means two numbers have to subtract to get three. Since the numbers 1 through 4 are the only ones that can be used, it is easy to identify that it will need to be filled with a 4 and a 1. The 4 and the 1 can go in either box, but in the end there can only be one 1 and one 4 in each row and column.

I will use puzzles that are either 4-by-4 or 6-by-6 with my students. Usually these puzzles only consist of using positive whole numbers. I have modified some of the puzzles to include negatives as well. This will allow my students to get the practice they need in integer operations as well as in mathematical logic.

I have worked with my students on KENKEN's® since the beginning of the school year so they have a great understanding of how the puzzle works. In the beginning, my students tried to fill in the puzzle using the guess-and-check method. As they got more comfortable with the puzzles, they are able to think about ways to solve it more mathematically and not just by chance. I work a lot with my students on these so they get to the point that they only fill in an answer when they are completely certain that it can fit in the box. The guess-and-check method is no longer needed.

I will not use these puzzles with negatives to introduce the topic of adding and subtracting integers, instead, I will use them to reinforce their skills. In order for the KENKEN® puzzles to have negatives, I had to make some adjustments. With the new puzzles, for example a 4-by-4, the students can still only use the digits 1 through 4. However, the difference is they can choose to use either the negative or the positive form of the digit to produce the target number in the top corner of the cage. Although it won't be the best tool for multiplication and division, it will be great practice of adding and subtracting integers; which tends to be the more difficult operations for students to understand anyway. The reason it won't be a lot of practice for multiplication and division is because a negative times/divided by a negative is a positive. In the end, it doesn't matter whether the numbers are two positives or two negatives. It will not change the outcome of the answer. Adding and subtracting is different in that it does matter whether or not a negative or a positive number is in each box.

Classroom Activities

Before I begin the lessons with the students, there are a few prerequisites that I would be sure to cover to be confident that my students understand the topics. The students will need to have a strong understanding of combining like terms. When they use that information they will be able take any subtraction problem and turn it into an addition problem. Once it is an addition problem, they can then use one of the strategies described.

Activity 1

Objective

The students will be able to correctly model adding and subtracting of integers using integer chips and the Singapore Model Math Method.

Procedure

To introduce adding and subtracting integers, I will begin speaking with the students about different scenarios. For example, " If my mom gave me $10, but I owed my sister $20, how much money do actually have?", or, "Johnny borrowed $25 from Sally to buy a shirt. He then got his paycheck in which he made $40. After he pays Sally back, how much money will Johnny have?" These examples will get the students thinking about real situations in which negatives and positives are together. Because the students have been in these situations before, it makes it easier for them to figure out the amount of money each person will have at the end of the scenario. As we discuss each of the scenarios, I will be sure to write each of them down using integers. For the first scenario, I would write the following on the board, as the students explained what was happening.

$10 – $20 = -$10

As we continued talking about Johnny and Sally, the students would instruct me to write the following on the board.

-$25 + $40 = $15

That discussion would lead us into talking about positive and negative numbers and operations involving them. I would discuss with the students and have them describe how to combine like terms and what the importance is of taking the sign that is in front of a number along with it. I would begin by showing them that we want to make all problems into addition. In other words, all subtraction problems can be easily transformed into addition. We will do several examples together.

-5 – 8 would become -5 + (-8)

3 – 9 would become 3 + (-9)

-14 +2 would become -14 + 2

The next part of the lesson will focus on modeling the information using integer chips. I would pass out integer chips to all of the students and explain what they are. I would be sure to write on the board that yellow represents positive while red represents negative. I would then have the students come up with ways to model the three problems given above.

Assessment

As an informal assessment of the students' understanding of the topics, the students would fill out a 'ticket out'. They would need to describe to me why 6 – 8 was the same as 6 + (-8). The students could draw pictures or use either of the two strategies they were introduced to.

Prior to Activity 2, students will become very familiar with ways to model adding and subtracting of integers. They will have used several different models to show the adding and subtracting concretely.

Activity 2

Objective

The students will be able to add, subtract, multiply and divide with integers. Students will also be able to model different methods for adding and subtracting.

Procedure

Students will be working in pairs to construct a model of adding and subtracting integers. The students will be given the opportunity to construct a three-dimensional model of a way to show these operations. The students will be able to have the opportunity to be creative in coming up with a way to identify and show adding and subtraction. I will have the following materials for the students to use: shoe boxes, tissue boxes, cotton balls, tape, glue, scissors, blocks, markers, scraps of material, string, etc. The students will also be able to bring in their own material.

Students will work with their partner on the first day to do some brainstorming. I will lead them through this process. I want the students to think about which models work better for each person and how to show it in a 3-dimensional way. After the students brainstorm and come up with a plan, they are expected to get it approved by me. The students can then begin building it.

Assessment

The students will be assessed in a few different ways. There will be stations set up where students can set up their models. One of the partners will 'man' the station while the other partner goes around and plays with the other models made by classmates. The partner that is left back at the station will be in charge of explaining how the model works. They will also be expected to give each student plenty of examples to try in order to identify if it correctly models the operations. Once the first partner has gone around seeing each of the models, they will switch places so that whey will both be expected to explain their model and also get the opportunity to see all of the other models. In the end, the students will evaluate their peers on the models created. The students will also be assessed by me.

Activity 3

Objective

Students will be able to take their knowledge of adding and subtracting and apply it and their mathematical logic to construct a KENKEN® puzzle.

Procedure

Students are given a blank 6-by-6 grid to construct a KENKEN® involving negative numbers. The students are to work individually to figure out a way to make a puzzle involving operations with negative numbers. The students have been working on KENKEN's® throughout the year with positive numbers and operations. Only recently have the students experienced completing a puzzle where there is an option to use either positives or negatives. The following are expectations that they must use in order to complete the task:

1. The numbers 1 – 6 or their opposites can be used to fill in the puzzle.
2. Students need to be sure each number fits correctly into each of the cages.
3. Another student will be expected to complete the puzzle that is the final product. A peer evaluation along with a teacher evaluation will be completed.

When students complete the task, they will give it to a partner in the class to complete and essentially check the other student's work. Once each of the students receives a KENKEN® from one of their classmates they are to be sure they can solve it and that it actually works.

The next day, the students will sit down with the student who made the puzzle and discuss whether or not it works and give suggestions of possible changes. Once the puzzle maker is confident with his puzzle, he will turn it in. I will make copies of all of the new KENKEN's and get them laminated. I will then have a class set of KENKEN puzzles involving operations with negative numbers that my students can use throughout the year.

Assessment

The students will be assessed by the completion of their construction of a KENKEN puzzle involving operations with negative numbers. They will be assessed by a peer and the teacher. The idea is that the students show a strong understanding of problems solving and integer operations.

Appendix

KENKEN® 1

4x		**-8+**	
	-8+		
48x		**-1+**	
		2÷	

KENKEN® 1 answer

4x 1	2	**-8+** -3	-4
2	**-8+** -3	-4	-1
48x 4	-1	**-1+** 2	-3
3	4	**2÷** 1	2

KENKEN® 2

3+		2+		15x	
-10+			4+	2	-5+
	2			3÷	
-10+		1	3÷		80x
	9x	2÷			
2			5	3÷	

KENKEN® 2 answer

3+ -2	4	**2+** -6	1	**15x** 5	3
-10+ -6	-5	3	**4+** -4	**2** 2	**-5+** 1
-4	**2** 2	5	3	**3÷** 1	-6
-10+ -5	-6	**1** 1	**3÷** 2	3	**80x** 4
1	**9x** 3	**2÷** 2	6	4	5
3	1	4	**5** 5	**3÷** 6	2

Annotated Bibliography

Curriculum Planning & Development Division Ministry of Education, Singapore. (2009). *The Singapore Model Method for Learning Mathematics.* Singapore: EPB Pan Pacific.
This book focuses on explaining the process of using the Model Method in mathematics problems.

Enright, Brian and Baker, Leslie. *Algebraic Thinking Part One Teacher's Edition*. Greensboro: National Training Network, Inc., 2004
This book shows many examples of using integer chips in order to show operations with negative numbers.

www.kenken.com (accessed 09/10/09).
KENKEN® is a registered trademark of Nextoy, LLC, ©2009, KENKEN Puzzle, LLC, all rights reserved. This website publishes KENKEN puzzles and has a sign-up to get them emailed to you on a regular basis.

North Carolina Department of Instruction. Grade 7 Instructional Resources. http://www.dpi.state.nc.us/curriculum/mathematics/middlegrades/grade07/ (accessed 09/15/09).
This is a site that has all kinds of resources for teachers to use. Specifically, operations involving integers under the classroom strategies section is a great resource.