# Kenken and the Problem Solving Classroom

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Throughout my teaching career, as I have met each new group of students, I have discovered that nearly all of them, whether they are a heterogeneous group of fourth graders, sixth grade Honors students, or a group of below average fifth graders, demonstrate obvious difficulties in the broad area of "problem solving." It does not take a lot of probing before it becomes clear that their problem is not "problem solving" per se, but a much larger one which first manifests itself in their inability to recognize even the simplest of patterns. Through casual conversations, observations, and listening to students as they work, it has become evident to me that students do not recognize patterns because they are not looking for them, and they are not looking for them because they do not expect them to be there. In other words, they do not expect mathematics to make sense! This lack of expecting mathematics to make sense seems to be at the root of students' difficulties in all areas of math. When we think of how we feel when aspects of our lives do not behave in a reasonable manner, and do not make sense to us, it throws us into an unpleasant state of confusion and lack of control, of disequilibrium. This is the state many of our children are in when they approach mathematics! No wonder we have a society where the statement "I'm just not good at math" is the norm, and is accepted with an understanding grin from the listener.

For many of the students that I teach, math is just a set of rules, and if you can remember them, you will get the right answer, and, after all, that is what math is all about. As a sixth grade teacher, trying to turn students from rule followers who have no idea how to approach a problem without instructions on how to solve it, to mathematicians who approach a problem with confidence in their own power to figure out a way to solve it, is a daunting task!

John Van de Walle, in his book Elementary and Middle School Mathematics, states that math makes sense is the most fundamental idea that a math teacher needs to know and pass on to his/her students. He sees mathematics as a study of things that have logical order and patterns of regularity. Studying mathematics is all about finding this order and making sense out of it. But first the students must believe this order is actually there!

My goals for my students, beyond the obvious mastery of the North Carolina Standard Course of Study, include the development of logical thinking, the ability to use numbers with fluidity for a purpose, and the empowerment of the student to approach any problem with confidence. None of these goals are possible, without students understanding that mathematics makes sense, not sometimes, but always.

In 2003, a Japanese mathematics educator, Tetsuya Miyamoto, created a puzzle which he calls Kenken ("cleverness squared" in Japanese), which he claims "transforms the brain into a vigorous, problem solving engine." Kenken is a pretty straightforward, seemingly simple puzzle, which involves a 4 x 4 or larger grid. In the 4 x 4 grid, the numbers 1-4 are used, similar to Sudoku. In the Kenken grid, some of the squares are outlined and include a target number with an operation sign. The player is expected to write the numbers in the outlined squares which will combine to make the target number using the given operation sign.

| | | | |
|---|---|---|---|
| 5+ | | 3- | |
| 1 | 2- | 8+ | |
| 2- | | | 4+ |
| | 7+ | | |

It seems simple and it is, but it presents an amazingly rich array of opportunities for logical thinking, reasoning, learning how numbers interact, simply learning standard facts, and working with factors and multiples.

In this unit, I will explore the possibilities of the use of Kenken in the classroom, to accomplish the goal of creating a stimulating and viable problem solving classroom, particularly how it relates to the sixth grade North Carolina standards, as well as the NCTM Standards for School Mathematics.

Despite what Mr. Miyamoto might claim, I do not believe Kenken can do it all by itself. For Kenken to be successful as a strategy for the development of mathematical problem solvers, it must fall on the fertile ground of a problem solving classroom. The creation of an environment where children can be successful mathematicians does not happen by chance. It must be intentionally created by a teacher who is willing to celebrate the importance of problem solving, and to get out of the way as his or her students construct understanding for themselves.

Such a classroom is a place where children feel comfortable making mistakes, asking each other for help, explaining their thinking, and feeling that their own ideas are worth listening to. It is not a teacher-centered environment where students spend their days listening to the teacher, but a place where the teacher listens to the students and the students listen to each other. Often students will come from years of elementary school where it was not expected that they listen to each other, where students who make mistakes are ridiculed, and where there was only one right answer and only one way to approach a problem. Undoing such a background requires a huge commitment from the teacher, but as new ways of looking at problems and "really good mistakes" that students can learn from are celebrated, the students begin to look at each other in new ways and begin to see that math goes far beyond right answers and inside each of them is a quite unique mathematician with a quite unique perspective. Let me emphasize that this takes time and patience, but when it finally happens, the classroom becomes a joyful, exciting, productive place to be.

In a problem solving classroom, children must be given time to struggle with a problem. Several years ago, I had the opportunity to visit Japan with the Fulbright Memorial Fund Teacher Program, and was able to observe Japanese mathematics teachers. A typical class involved one problem, not pages of worksheets and practice, but one challenging, rich problem. The students were expected to, and expected to work at it, struggle with it, talk about it for a good amount of time before the class discussed various solutions. This is such a contrast to our American classes, where teachers are so intent on getting through the curriculum, that the students are never given the opportunity to see what they can do.

The idea of struggle is foreign to our instant gratification society in the United States, so this idea of really taking time to struggle is the hardest piece of a problem solving classroom for students to swallow. Many of the students that we teach have parents that work very hard to ensure that their child never has to struggle. In school, students are used to being given some sort of formula to solve a problem, taking a few minutes to do it, and then either having a right or wrong answer, and some of them have learned to like that. Kenken is a valuable and relatively painless beginning to the idea of struggle. In the classroom of Miyamoto, there is no guidance, only a carefully crafted Kenken puzzle, with no set place to start, and no definite strategies. As the puzzles go from simple to more complex, the students gain confidence in their abilities; they know they can solve a Kenken puzzle, so are more willing to take the time to do it. In other words, they know it makes sense, and that they can uncover the structure within it. This is a huge step for some children.

One of the important goals for students contained in the NCTM Standards for School Mathematics of 1989 is the Communication standard. In a problem solving classroom, it is usually most effective for the students to be organized into groups to facilitate communication. Sixth graders are very willing to listen to and talk to their peers, and it is a foolish teacher who does not capitalize on that. Marilyn Burns points out that social interaction is extremely valuable as students bounce their ideas off their peers in a way that they will not do with adults.

When they come to me, my students are often not familiar with the idea of explaining their thinking, or even thinking about their own thinking. As Kenken is introduced, the teacher must make it clear that this is not a guessing game, and model the justification of every move. As the game is first demonstrated, the children will want to just guess, but it is important that with every suggestion of a number being put in the grid comes a reason why the child thinks it should go there. Students will learn to demand that from each other as they work in their groups. As students solve Kenken puzzles with their peers, it is extremely helpful if each group is called upon to explain how they made the decisions along the way to their solution. As they explain their reasoning, they will be forced to follow the chain of reasoning through the problem, which is an invaluable skill as they tackle complex problems with words.

One of the beauties of Kenken is that it develops the skills students need for word problems without the complications of interpreting the language. This is invaluable for ESL students, who, of course, need to develop their reasoning skills in an arena uncomplicated by their lack of English language skills and vocabulary. When ESL students arrive in sixth grade, they frequently come with fairly well developed computation skills, but come with very low standardized test scores, which most people attribute to their inability to comprehend English language. I would argue that this reasoning ignores the logical reasoning skills that even the youngest children develop through working on challenging word problems. Rich problems are often inaccessible to students with limited English, so the only mathematics they are able to learn in the lower elementary grades is computation. Kenken does develop logical reasoning skills, and the students do not have to be able to read English to benefit from it!

The teacher should be aware of individual children’s style of solving problems as groups are formed. Some students spend time simply staring at the puzzle and forming ideas in their mind; some students like to verbalize all along the way. In working with Kenken, children with similar solving styles should be grouped together to avoid conflicts and frustration.

One of the Process Standards from NCTM’s Principles and Standards of School Mathematics is the Reasoning and Proof Standard, where students recognize reasoning and proving their reasoning as an important part of mathematics. As they justify their Kenken decisions, this proving that their reasoning has worked is being developed.

What is the teacher's role as children struggle with Kenken? The teacher must allow children enough time to discuss, struggle, explain their thinking, and should emphasize over and over with his/her students that persistence is valued over speed. The teacher should question the students as they work about what informed the choices they made, but under no circumstances should the teacher give students the answer—the last message you want them to get is that they are not capable of solving these puzzles on their own.

Children in elementary school spend a lot of time solving problems of various kinds, but very little time creating them. When children are encouraged to write word problems, it gives them an understanding of their structure, and forces them to see through the problem a clear path to the solution. The same is true for Kenken. Children can create their own Kenken puzzles for each other, and should be encouraged to do so in a systematic manner, going from small puzzles with one operation to more complex puzzles.

**Kenken and the Curriculum**

Obviously, Kenken is a valuable tool in the development of children's thinking and their ability to communicate that thinking, but it can also be used to enhance the standard elementary school curriculum.

Countless hours in elementary school are spent drilling on "facts." Many of the students I teach, even in sixth grade, are still very shaky on their facts, and many who can rattle off any fact, have little understanding of what those facts actually mean. Marilyn Burns, in her book About Teaching Mathematics points out the necessity for children to think and reason with numbers and make sound judgments about them. They need to see numbers as useful, and real, with a sound purpose. Without the idea that numbers are useful, children see these numbers as meaningless symbols and if all you are doing is manipulating meaningless symbols, math cannot make sense.

With the experience of solving Kenken puzzles, young children can see that numbers are useful, they can be seen as the means to an end, they can observe what operations make numbers bigger or smaller and to what degree, and they of necessity see that there are multiple ways to, for example, "make" 6, and it is up to them to make the most effective choice. This is a very big discovery for a child—that he can actually choose how to use the numbers to accomplish his goal. It goes without saying that the experience of solving Kenken puzzles is a painless way to drill facts, and can be designed by the teacher to focus on whatever operation needs work by the students in the class.

One of the important areas of study in the sixth grade, which is actually begun in the earlier elementary grades is that of Factors and Multiples. In this area, Kenken can serve multiple purposes. First of all, it is fun, and it is a tangible way for students to utilize what they are learning about factors and multiples to solve a puzzle. As they become more fluent with factors, the puzzles which they solve will begin to include large numbers and larger "cages" (the outlined squares), with grids going up to 9 x 9. As they study factors and multiples, Kenken also presents the opportunity to reinforce the vocabulary of the study as the students explain their solutions in terms of factors and multiples.

The Kenken puzzles which have been distributed by Mr. Miyamoto include only the four basic operations and non-negative numbers, but educators have begun to see even more richness in applying Kenken to new situations. One of the biggest areas that students study in sixth grade is Probability, including the study of Permutations. A California educator has recently posed the Kenken conundrum which asks how many ways there are to arrange 1,2,3,4 in a 4 x 4 Kenken grid. The problem turns out to be a permutation problem, which could easily be a source of a rich sixth grade lesson.

In the sixth grade, we begin to lay the groundwork for the study of algebra, beginning with factors, multiples and exponents, the order of operations and the basic properties of rational numbers. As students become proficient with creating Kenken puzzles with only positive numbers, it would be worthwhile to work with their creating puzzles with negative numbers included if it were possible. I feel as though the math education community has barely scratched the surface of the richness in Kenken puzzles, and there will be more and more applications as time goes on.

The structure of this unit will consist of a series of lessons that focus on the solving of the Kenken puzzles themselves and the creation of puzzles by the students. The focus and purpose of these early lessons will be to practice thoughtful solving of the puzzles, group communication, students justifying their thinking, and simply making sense of how Kenken puzzles function. I visualize these as mini-lessons, which will progress from small grids and small numbers to larger grids and larger numbers, and will take place over a number of weeks. With every step along the way, students will create their own puzzles. After the students are familiar with and fairly proficient with the puzzles, there will be a series of lessons which relate directly to the curricular area with which we start the sixth grade, factors and multiples.

**Setting the Stage**

Ideally, this unit would be taught at the beginning of the year, when the teacher has a chance to set the tone and develop the skills that will make it possible to use Kenken as a rich resource in the classroom. It is important to develop the core value that mathematics makes sense and there is no action taken in mathematics that does not have a reason behind it, as well as the students' own belief that they are powerful, capable mathematicians who can use their own powers to solve problems. Students must also be given the opportunity for metacognition, for thinking about their own thinking and expressing their thoughts verbally and in writing, to the teacher and to their peers. Communicating about mathematics and justifying one's thinking must be the norm.

As I have stated earlier, this is a conscious process. Students come from many elementary school backgrounds, often from backgrounds where there has been little or no conversation, where math class was worksheets and memorization of formulas that made no sense, and where there was only one right answer and no room for discussion. As deadly as that sounds, it is what many of your students are familiar with, and therefore comfortable with. The first time a teacher says "I am not interested in your answer, only how you approached the problem" is often greeted with pure horror—this is nothing they have experienced and it unceremoniously dumps them out of their comfort zone.

It is up to the teacher to create a new comfort zone, one which is also empowering for its inhabitants. From the very beginning, students should be placed either in pairs or in groups, and should be encouraged to communicate about mathematics, to discuss their solutions with their peers, to struggle and to argue. They should be presented with rich, non-traditional problems that stretch them and pull from everything in their mathematical background. See Appendix A for a list of sources for these types of problems.

Class discussion should center around process, not the right answer. Students should see that there is a difference between not knowing an answer and not having found one yet, and be able to expect that they will find it! Students should be asked to justify their method of solving a problem, and it should be emphasized that there must be a reason for choosing to take a particular step. It should always be a cause for celebration if a student has another way to approach the same problem. When this happens every day, it becomes the norm. Creating a supportive community where children feel empowered to share their own thinking is the task of the teacher. Daily interactive math journals are another valuable tool where students can explain their thinking in pictures and in words, giving the teacher the opportunity to read and comment on individual students' thoughts. In both classroom discussion and commenting on journal entries, the teacher must celebrate alternative solutions, valuable mistakes that teach us something, and model supportive, non-harmful criticism. This is key.

The whole idea of students expecting mathematics to make sense is obviously essential to any real learning or any real problem solving, so that is the first thing that must be in place for our Kenken adventures, but there is another idea that is often overlooked that will make it possible for students to solve and in fact, create, Kenken puzzles. This is the idea that numbers are made of other numbers and it is the first cousin of mathematics making sense. Students who don't think math makes sense also do not think of numbers having any meaning at all—they are just symbols to be manipulated in a magical way. Essential to Kenken is the ability to combine numbers in various ways to make other numbers.

For a time before starting to work with Kenken, the teacher should include in his/her classes activities which require students to put a set group of numbers together to make other numbers. For example, students can be asked to make the numbers from 1-20 using adding, subtracting, multiplying or dividing, using only 4 4's, for example

$$(4 + 4 \div 4) \times 4 = 20.$$

The game of 24 is also a valuable tool for this skill. This game asks students to make 24 using sets of 4 numbers drawn from a deck of cards, for example, if the numbers chosen are 8.4, 6 and 3, the solution might be:

$$(8 + 4)\ (6 \div 3) = 24$$

These activities will begin to turn the students' minds to a way of thinking that will be a valuable tool as they approach Kenken. These activities are also a valuable tool for practicing the order of operations, which is another skill that is emphasized in sixth grade.

**Introduction: The First Experience**

Here is a sample of a Kenken puzzle. The combinations of blocks with a heavy black border are called cages. Within each cage is a number and an operation symbol. Our task is to make the number in the cage using the designated operation using the numbers from one through four. The numbers 1, 2, 3 and 4 must appear only once in each row and in each column. Remember, we can only use the numbers one through four and we have a list of the ways to make these numbers.

| | | | |
|---|---|---|---|
| 2 ÷ | | 4 + | |
| 3 - | 6 x | | 1 |
| | 6 x | 7 + | 2÷ |
| | | | |

Begin the students' introduction by showing them a 4 x 4 Kenken puzzle on a transparency. Tell the students that the only numbers in this puzzle are the numbers from 1 to 4, and that they will have to use only those numbers to make other numbers. Have the students make a list of all the sums they can make with 1, 2, 3 and 4, all the differences, all the products and all the quotients. Include strings of three numbers when possible. Have the list visible on the board for reference as they begin.

Ask the students for ideas on how to start. They will undoubtedly suggest just guessing a number and erasing if it does not work. The teacher should tell the students that they are going to approach this puzzle as mathematicians who can justify everything they do, and they are going to do this puzzle with absolutely no erasures. In other words, nothing gets written down until there is a good reason to write it down. For the first puzzle, the teacher should model the thinking required to solve it, using a transparency and having the students observe. It is entirely possible that after the first two steps, the students will be excited and want to chime in as you go. That is great! What follows is the thought process that can be shared with students as they start their first Kenken experience. An example of an empty puzzle is included. The completed puzzle is found at the end of the narrative.

First observe the number that is by itself. Obviously that is going to be a 1. Look at the cage next to the one. How can we make 6 by multiplying? We can only use 3 and 2 but we are not sure which goes in which box, so let's put both numbers lightly in both boxes. Since the 3 and the 2 in that row will go in the second and third columns, you now know that the 4 will go in the first column.

How can you make 3 by subtracting and using a 4? You can use 4 - 1. Now let's look at the 7. The only way you can make a 7 by adding is with a 3 and a 4 but you don't know which goes in which block. However, you do know that you cannot use a 3 in the second row of the third column, so you must use a 2.

How can you make 6 by multiplying three factors? 3 x 2 x 1 is the only possible way. I know there is already a 3 in the second column, so the 3 will have to go in the first column. Now I know I can put a 2 in the first row of the first column. How can I get a quotient of 2 using a 2?. I could divide 2 ÷ 1, but I know I will need a 1 to make a product of 6 in the second row. I could get a quotient of 2 by dividing by 4, so this must be a 4 in the second column.

Let's go back to the product of 6. I know I need a 1 and a 2 to complete the string. There is already a 1 in the third row, so that must be a 2 in the third row. Now let's go to the top row and look at the sum of 4. How can I get a sum of 4? I can't use two 2s because that is against the rules, so it must be 3 and 1. There is already a 1 in the fourth row, so the 1 must go in the third row and the 3 in the fourth.

Let's go back to the sum of 7. Because we now have a 3 in the fourth row, we can place the 4 in the third row and the 4 in the fourth row, and I can now place the 4 and the 2 to obtain a quotient of 2 in the fourth column.

| | | | |
|---|---|---|---|
| 2 ÷<br>2 | 4 | 4 +<br>1 | 3 |
| 3 –<br>4 | 6 x<br>3 | 2 | 1<br>1 |
| 1 | 6 x<br>2 | 7 +<br>3 | 2÷<br>4 |
| 3 | 1 | 4 | 2 |

After the students have been introduced in this way to the process of solving a Kenken puzzle, the next step is to do a puzzle of similar difficulty as a whole class on the overhead, but with the students discussing the justification for each move. The teacher can judge how many times this would be necessary before the students are ready to tackle puzzles on their own. Using four by four puzzles, have the students work their puzzles in pairs and have them explain their solutions to their classmates.

As the students master four by four puzzles, introduce a five by five puzzle and question the students about what sums, differences, products and quotients will now be available to us? Add these to the list they made at the beginning of this exploration. As students begin to work puzzles on their own, the teacher's role is always to question, to retrace the students' steps if they are unsuccessful, and to value students' persistence over their speed in finding a solution. Students must be made to feel secure and comfortable in taking their time to think through their solutions.

**Creating a Kenken puzzle**

After the students have had some experience solving puzzles, they will be ready to try to create a puzzle of their own. This is a good partner activity. Students may create a four by four or five by five puzzle. They should start with their lists of sums, differences, products and quotients. Have a discussion of how to approach the problem. You might also model the creation of a puzzle, but I think students would learn more developing their own strategy. Some will begin with the grid filled in with the numbers, and others will start from the lists of problems they want to include. When they have created their puzzles, have each pair come and explain their strategy. As a class, discuss which strategy is more effective.

**Factors and Multiples: Using Kenken to enrich the curriculum**

Factoring is a skill with far-reaching applications and as such it is worthy of a great deal of attention. In the sixth grade, Factors and Multiples is one of the earliest and most extensive topics to be studied. Students are taught to list factors and to find the prime factorization of numbers, usually using a "factor tree."

Here we will look at an approach to factoring which is often overlooked. Students should be asked to create strings of numbers whose product is, for example, 1050. They will immediately see 105 x 10, and should be led to break the numbers apart further into, for example 5 x 21 x 5 x 2. Challenge the students to see who can come up with the longest string that still has a product of 1050. The longest string will turn out to be the prime factors. The Connected Math Project volume Prime Time includes an excellent lesson entitled Finding the Longest Factor String.

In preparation for the next activity, have students make a list of all the possible factor strings for these numbers: 168, 210 and 630. Ask them to circle all of their strings that only include the numbers 1 through 7. Tell them that they will use these strings to solve the following Kenken puzzle:

| | | | | | | |
|---|---|---|---|---|---|---|
| 3 | 2 ÷ | | 168 x | 210 x | | |
| 18 + | | | | 10 + | | |
| 6 - | | 16 + | | 10 + | | 3 ÷ |
| | | | 2 x | | | |
| 12 + | 1 - | | | 630 x | 5 - | 20 x |
| | | | | | | |
| | 13 + | | 9 + | | | |

Using the strategies for solutions that they learned in the previous lessons, have the students work this puzzle in pairs using the factor strings that they have generated. Their goal should be to complete the puzzle with no erasures! Have each pair talk about their solutions, and the strategies that they used. The discussion should include whether or not there are strategies that are better than other strategies and why. Can you generate an instruction sheet for Kenken puzzles? Is there anything the solver always must do? This has the potential to be a rich discussion and it may well result in the students deciding that there is no set strategy.

This experience should be followed with experience solving more 7 x 7 puzzles, as well as 8 x 8 puzzles and 9 x 9. Students enjoy solving these puzzles as a warmup or when they have finished other work in the classroom and need something to do!

**Using Factor Strings to Create a Kenken Puzzle**

After the students have had experience both with creating factor strings and solving more complex Kenken puzzles, they will be ready to use this knowledge to create more complex Kenken puzzles of their own. Unlike the experience of creating a simple 4 x 4 puzzle, these larger puzzles will require a more thoughtful approach to developing a strategy. Students should discuss possible strategies with their partners and share with the whole group. This discussion should include the place of prime numbers in the creation of the puzzle. Since the prime numbers do not have multiple divisors, they will be useful only as sums. Students should decide whether they want to include all operations or only some of them. For example, the class might discuss what would be the pros and cons of leaving out division? When they have decided which operations to use, it will be helpful if they make a list of the possible sums, differences, products and quotients of the numbers they will be using. When they are generating the list of products, it should include as many factor strings as possible for each product, enabling them to use strings of 3 and 4 in their puzzle. It should be up to the individual pairs what size grid they will be using. The teacher should guide students in their choices, thereby being able to challenge the more advanced students while giving all students the opportunity to work at an appropriate level. All students should be challenged to use more than two numbers in a string to create their puzzle.

**Kenken in the Classroom: Extension Activities**

Once students have had experience with solving and creating Kenken puzzles, they can be incorporated into the classroom in a multitude of ways. Kenken puzzles lend themselves to many challenges, projects, and tasks that can be completed in the normal class period. The following is a list of suggested activities and projects.

Students and families would enjoy a Family Kenken tournament as part of a Family Math night or as a stand-alone event. Students can also challenge students from another class to a tournament—competing to see who can solve the puzzles in the shortest amount of time.

Activities for students:

Make an illustrated book of puzzles for younger children, including instructions on how to go about solving the puzzles.

Assign a numerical value for each letter of the alphabet (A=1, B=2, etc.) and create a Kenken puzzle using letters rather than numbers. For example A + C=D and D x B = H.

Create a 4 x 4 puzzle where all of your sums, differences, products and quotients are multiples of 2.

Create a 5 x 5 puzzle that includes prime numbers as at least two of your sums, differences, products or quotients.

Create a 5 x 5 puzzle that includes two square numbers among your sums and products.

Create a new kind of puzzle that uses only numbers. Give your puzzle a name, and write the rules for solving your puzzle.

Explore the possibilities for creating a 4 x 4 Kenken puzzle that includes negative numbers. What numbers would you use instead of 1,2,3, and 4?

Investigate how many possible solutions there are for a Kenken puzzle that has 1, 2, 3, 4 in order across the top row.

Investigate how many possible solutions there are for a 3 x 3 Kenken puzzle.

As you can see, there are many possibilities for using Kenken to supplement and enrich the standard mathematics curriculum. Students love Kenken, and working on Kenken puzzles takes the tedium away from practicing "math facts" for sixth graders who are often embarrassed by their inability to recall these facts and as such are reluctant to practice. With Kenken, students can apply their knowledge of factors in a way that has meaning for them rather than using their time for endless activities where students must list factors in a meaningless way. Kenken can tap the competitive nature of children as well as their love of puzzles and their creativity, while helping to teach them that math makes sense.

**Appendix A: Implementing District Standards**

The State of North Carolina has a Standard Course of Study, which may be viewed at the North Carolina State Department of Public Instruction website. This unit addresses the following standards in the sixth grade Standard Course of Study.

**1.04** Develop fluency in addition, subtraction, multiplication, and division of non-negative rational numbers.

a. Analyze computational strategies.
b. Describe the effect of operations on size.
c. Estimate the results of computations.
d. Judge the reasonableness of solutions.

The solving and creation of Kenken puzzles enables students to practice, and therefore develop fluency in computing with non-negative numbers. They are required to analyze the strategies they develop for solving a puzzle, and judge the reasonableness of their solutions.

**1.05** Develop fluency in the use of factors, multiples, exponential notation, and prime factorization.

As students learn about factors, multiples and prime factorization, they gain the tools to analyze the structure of Kenken puzzles and to create them themselves. The successful solving and creating of these puzzles requires a knowledge and understanding of factors.

**1.07** Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil

The solving of Kenken puzzles is a problem solving activity that demands that students develop a strategy and test that strategy in a practical situation. Students are required to compute mentally, as well as with calculators and paper and pencil to complete puzzles and/or create new puzzles.

**Appendix B: Sources for Good Problems**


Cook, Marcy. *Detective Problem Solver*. Balboa: Marcy Cook, 1990.

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Kenken puzzles are from the Kenken website. http://www.kenken.com.
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